

# A Quantitative Analysis of the US Housing and Mortgage Markets and the Foreclosure Crisis<sup>1</sup>

Satyajit Chatterjee and Burcu Eyigungor  
*Federal Reserve Bank of Philadelphia*

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<sup>1</sup>Corresponding Author: Satyajit Chatterjee, Research Department, Federal Reserve Bank of Philadelphia, 10 Independence Mall, Philadelphia, PA 19106. Tel: 215-574-3861. Email: satyajit.chatterjee@phil.frb.org. The views expressed in this paper are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of Philadelphia or of the Federal Reserve System.

## **Abstract**

We construct a quantitative equilibrium model that accounts for the salient features of the US housing market, namely, the homeownership rate, the average foreclosure rate prior to the crisis and the distribution of home-equity ratios across homeowners. Given this steady state, we examine the consequences of a 3 percent unexpected increase in the supply of housing on house prices and foreclosures. We analyze the impact of the tax code, namely the the fact that the implicit rental income from housing is not taxed and that the mortgage interest payment is tax deductible, for the steady state of the model as well as for the impact of the unanticipated supply shock. We show that the model is able to account for the observed decline in house prices with a modest increase in the cost of new mortgages following the shock (crisis) and predicts a large increase in foreclosures.

**Key Words:** Leverage, Foreclosures, Mortgage Crisis

# 1 Introduction

The analysis is motivated by the recent experience of the housing boom and bust and rising defaults on mortgages in the United States (and elsewhere). The goal of this project is two-fold. First, to construct a quantitative model that can account for key features of the US housing market, namely, the homeownership rate, the average foreclosure rate prior to the crisis and the home-equity distribution observed in the US data. Second, to see if such a model can account for the steep rise in foreclosures following a modest over-building shock.

The key elements of our model environment are as follows. We imagine a city with an exogenously given but potentially time-varying stock of housing. The city is populated by a continuum of infinitely lived residents subject to uninsured idiosyncratic shocks to earnings. Residents can buy consumption goods, save in the form of a risk-free savings account with an exogenously given and constant interest rate, and purchase or rent their housing space. If a resident chooses to purchase housing space, he or she can borrow funds from a mortgage market to do so. Residents must pay tax on earnings and interest income. As in the US federal tax code, interest payments on mortgages and property taxes can be deducted from taxable income and the implicit rental income from owner-occupancy is excluded from taxable income.

We model the mortgage market as competitive, with every borrower being charged an interest rate that exactly reflects the borrower's objective probability of default. A key determinant of this default risk is the level of down payment made on the loan - a level that we assume is freely chosen by the borrower. The endogeneity of the down payment will imply that when the risk of default is perceived to be low, the down payment chosen will may be low and homeowners may be highly leveraged.

We show that this basic model is capable of accounting for the observed facts regarding the US housing market noted above. The tax treatment of housing plays a huge role in bringing the model close to reality. Without the deduction of mortgage interest income, the model would predict way more home equity than we see in the data. And without the exclusion of implicit rental income from taxable income, the model would predict a homeownership rate of zero. The model accounts for the dispersed distribution of home-equity observed in the US data because mortgages are nominal

contracts and there is steady increase in the general price level over time, which erodes the real value of mortgage debt, relative to the value of housing.

The second goal of this paper is to use the model to study the foreclosure crisis in the US. We locate the proximate cause of the foreclosure crisis in “overbuilding”: a modest increase in the supply of owner-occupied housing that fails to be matched by an increase in demand at the going price. In equilibrium, the price of owner-occupied housing must fall to absorb the increase in supply. Given the transactions costs of selling and buying homes, the decline in price in the short-run has to exceed the long-run decline in price. We show that a 3 percent excess supply of owner-occupied space along with a 1 percent increase in the cost of new mortgages can account for the observed large drop in house prices. This drop in house prices in turn generates a large increase in foreclosures because homeowners with low or negative home equity find it optimal to take advantage of low rents and low house prices in the future by dumping their house back to the lenders.

We also use the model understand how different factors contributed to the rise in foreclosures and the drop in house prices. In particular, we quantify the impact of

- foreclosures on the house price drop,
- the substantial lengthening of the time to foreclosure during the crisis (during which time a defaulter can stay on in the house without making any mortgage or rent payments) on the foreclosure rate
- the disruption of the flow of funds into the mortgage market on foreclosures and house prices,
- an unexpected drop in the inflation rate during the crisis on foreclosures and house prices, and
- we examine how the over-building shock would have affected the economy if the deductibility of mortgage interest did not exist.

## 2 Literature Review

There is a rapidly growing literature on the financial crisis and the mortgage crisis. Much of this literature is empirical. To the best of our knowledge, ours is the first quantitative-theoretic investigation of the interaction between the jump in foreclosures (the mortgage crisis) on the one hand and the crash in house prices (the housing crisis) on the other. We build on a small but growing quantitative-theoretic literature on the housing sector. We have in mind studies such as Gervais (2002), Nakajima (2005), Jeske and Krueger (2005), Guler (2008), Ríos-Rull and Sánchez-Marcos (2008), Corbae and Quintin (2009), Iacoviello and Pavan (2009), and Chambers, Garriga, and Schlagenhauf (2009).

Among these papers, Corbae and Quintin (2009) is also an attempt to gain a quantitative understanding of the “foreclosure boom” (as they call it). But, unlike our paper, they take the drop in house prices as given and focus on the role of innovations in mortgage financing, namely, the engineering and diffusion of subprime mortgages, on the foreclosure crisis. We do not focus on mortgage innovation per se but recognize that high leverage (low or negative home equity) played a key role in engendering the foreclosure crisis. In our model, the fundamental driver for the rise in foreclosures is over-building.

In terms of modeling the mortgage market, we follow Jeske and Krueger (2005) and Guler (2008) in assuming that each loan is competitively priced to reflect the objective probability of default on the loan (individualized or risk-based pricing) and in assuming that a borrower controls the objective probability of default by her choice of down payment. In terms of modeling the housing market, we follow Gervais (2002) and Chambers, Garriga, and Schlagenhauf (2009) in conceiving of the housing market as a market for homogeneous housing *space*, as opposed to houses. This conceptualization is attractive in that there is *one* housing market price – the price per square foot of housing space – to be determined each period.

The role of aggregate shocks in a model of mortgage lending and housing has been analyzed in Ríos-Rull and Sánchez-Marcos (2008). Their analysis is explicitly stochastic, and they employ a forecasting equation (for future house prices) of the type developed in Collusi (2006) to get around the high dimensionality of the state vector. In contrast, our analysis is not explicitly stochastic –

we analyze the impact of an unanticipated shock to housing supply – but we work out the perfect foresight equilibrium path following the shock.

### 3 Environment

We will study the housing equilibrium in a representative city. Time is discrete and indexed by  $t = 0, 1, 2, \dots$ . The city has a fluctuating stock of housing space  $H(t) > 0$ .

#### 3.1 People

There is a fixed continuum of individuals. Individuals derive utility from the consumption of a homogeneous consumption good and the service flow from housing space. Let  $c_t$  denote consumption of the homogeneous good in period  $t$ , let  $h_t$  denote the consumption of housing space in period  $t$ . Then an individual values the consumption stream  $c = \{c_0, c_1, c_2, \dots\}$  and  $h = \{h_0, h_1, h_2, \dots\}$  according to:

$$U(c, h, e) = \sum_{t=0}^{\infty} \beta^t u(c_t, h_t), 0 < \beta < 1, \nu \geq 0. \quad (1)$$

We assume that

$$u(c_t, h_t) = [c_t^{1-\theta} h_t^\theta]^{1-\gamma} / (1-\gamma). \quad (2)$$

We assume that people must either own their housing space or rent it.

Each resident independently draws an earnings level  $w$  from a finite-state Markov process with non-negative positive support  $W \subset R_+$ . The probability that  $w_{t+1} = w'$  given  $w_t = w$  is  $F(w', w)$ .

#### 3.2 Market Arrangement

The homogeneous consumption/endowment good is the *numeraire* good. Period  $t$  prices are expressed in period  $t$  consumption goods. There are four markets in this economy.

1. There is a market for owner-occupied housing in which the price per unit of housing space in period  $t$  is  $p(t)$ . An individual who buys  $k'$  units of housing pays a purchase price of  $p(t) \cdot k'$ .

We assume that owner-occupied housing comes in discrete sizes given by the finite set  $K$ . We assume that the depreciation rate on housing is random: houses depreciate by  $\delta_j$  with probability  $\xi_j$ ,  $j = H, L$ ,  $\delta_H > \delta_L$ . That is, houses either depreciate at a high rate or a low rate.

2. Second, there is a market for rental housing in which the rent per unit of housing space in period  $t$  is  $z(t)$ . An individual who rents  $h$  units of housing space in period  $t$  pays  $z(t) \cdot h$  as rent. We assume that rental housing space depreciates non-stochastically at the rate  $\Delta$ .
3. Third, there is a market for risk-free deposits. The nominal interest rate on deposits is  $i(t)$ , where  $(1 + i(t)) = (1 + r)(1 + \pi(t))$ . The real interest rate  $r$  and the inflation rate  $\pi(t)$  is taken as exogenous.
4. Finally, there is a market for mortgages where individuals borrow in nominal terms to fully or partially fund the purchase of a house. We assume when a person takes out a mortgage, he agrees to make a sequence of geometrically declining mortgage payments  $\{X, \mu X, \mu^2 X, \dots\}$ . Thus, each mortgage lasts for forever, unless the mortgagee defaults or the mortgage is terminated due to the sale of the house. In the case of default, the financial intermediary gets ownership of  $k$  and the defaulter pays a utility cost  $\kappa$  that is drawn from a continuous distribution  $\Phi(\kappa)$ . In the case where the mortgage is paid off, the lender receives the present value of the remaining promised sequence of nominal payments,  $\{\tilde{X}, \mu\tilde{X}, \mu^2\tilde{X} \dots\}$ , discounted at the nominal risk-free rate. The geometrically declining payments implies that the value of the individual's obligations to the lender declines over time. If we denote by the present discounted value of the stream  $\{X, \mu X, \mu^2 X, \dots\}$  starting next period by  $Q(X)$ , the value of debt bought back by the borrower is given by  $(1 - \mu)Q(X)$ . Consequently, the portion of the payment  $X$  that is payment of interest is  $X - (1 - \mu)Q(X)$ . Because of the possibility of default, the (unit) price  $q$  of a mortgage depends on the amount of housing pledged as collateral  $k'$ , the initial payment amount  $X$  whose real value in the current period is  $x$  (since the price of consumption goods today is normalized to 1), the individual's post-purchase savings  $a'$ , his current earnings  $w$ , and the time period  $t$ . An individual who takes out a mortgage in period  $t$  obtains  $q(w, a', x, k', t) \cdot x$  in the current period.

### 3.3 Taxes

There is also a government sector that levies taxes on income. For simplicity, we assume that government consumption of goods does not have to provide any benefits to households. The amount of taxes  $G$  to be paid by an individual in nominal terms is modeled after the US tax code. The individual's nominal taxable income  $I$  is given by:

$$I = \max\{0, W + \omega i(t)A - \max[X(t) - (1 - \mu)Q(X) + \rho P(t)k, S]\} \quad (3)$$

where  $S$  is the standard deduction,  $\rho P(t)k$  is property tax,  $1 - \omega$  is the fraction of assets on which interest earnings are tax deferred. Thus, an individual's taxable income is wage and the taxable portion of interest earnings less the greater of the standard deduction  $S$  or the sum of the interest payment on the mortgage and property taxes. Then, the individual's nominal tax liability is then given by

$$G = \int_0^I T(Y) \cdot Y dY$$

where  $T(\cdot)$  is the marginal tax rate and weakly increasing in taxable income (we assume that tax brackets move up with inflation). In real terms, an individual's tax liability is given by:

$$g(w, a, x, k) = \int_0^{\max\{0, w + \omega ia/(1+\pi) - \max[x - (1-\mu)\bar{q}(x)/(1+\pi) + \rho p(t)k, s]\}} \tau(y) \cdot y dy, \quad (4)$$

where  $\bar{q}(x)$  is the discounted value of the stream  $x, \mu x/(1 + \pi), \mu^2 x/(1 + \pi)^2, \dots$  using the real interest rate,  $\tau(\cdot)$  is the marginal tax rate when income is measured in current period consumption good.<sup>1</sup>

### 3.4 Financial Intermediaries

Financial intermediaries take in deposits, sell mortgages, and own the housing space rented by people. All intermediaries can borrow or lend funds in a world credit market at a given risk-free interest rate  $\bar{r} > 0$ . We will assume that there is one representative risk-neutral intermediary that takes all prices as given.

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<sup>1</sup>In the computation we approximate the return on assets that are subject to income tax as  $\omega a(r + \pi)$ .

## 4 Decision Problems

### 4.1 People

The state variables for individuals are  $w, a, x, k, \delta_h, \kappa, t$  and whether the individual is excluded from the mortgage market or not. Consider first the decision problem of an individual who does not own housing and who is not excluded from the mortgage market due to prior default. For this person  $k = 0$  and  $x = 0$  and the person may choose to rent or she may choose to buy. Since the person is not excluded from the mortgage market she can borrow to purchase a house. In this case, if the individual chooses to purchase, she solves:

$$M_1(w, a, x = 0, k = 0; t) = \max_{c \geq 0, k' \in K, x' \geq 0, a' \geq 0} \{u(c, k') + \beta E_{w', \delta', \kappa' | w} V(w', a', x', k', \delta', \kappa'; t + 1)\}$$

$$c = w - g(w, a, x = 0, k') + a(1 + \omega r + (1 - \omega)\tilde{r}) - a' - p(t)[1 + \chi_B]k' + q(w, a', x', k'; t) \cdot x'$$

where  $\chi_B$  is the percentage transactions cost of purchasing a house,  $\tilde{r}$  is the rate of return on assets on which taxes are deferred. When the household buys a house of size  $k'$ , he consumes the house in that period. We assume that homeowners consume all of their house; we do not allow homeowners to rent out their owned space.<sup>2</sup> Given that the buyer consumes the home in the period of buying, it pays taxes over the property in the current period, but the payment of the chosen mortgage  $x'$  starts next period, so  $x = 0$  in the tax calculation function  $g(w, a, x = 0, k')$ .

If the individual is excluded from the mortgage market due to a prior default but chooses to purchase a house, she solves:

$$M_1^D(w, a, x = 0, k = 0; t) = \max_{c \geq 0, k' \in K, a' \geq 0} \{u(c, k') + \beta E_{w', \delta', \kappa' | w} V(w', a', x' = 0, k', \delta', \kappa'; t + 1)\}$$

$$c = w - g(w, a, x = 0, k') + a(1 + \omega r + (1 - \omega)\tilde{r}) - a' - p(t)[1 + \chi_B]k'.$$

We assume that if an excluded individual purchases a house, he is no longer excluded from a mortgage market (the default flag is gone).<sup>3</sup> The only difference between the value functions  $M_1$  and  $M_1^D$  is that, in  $M_1^D$  the household cannot get a mortgage and  $x' = 0$ .

<sup>2</sup> This is without loss of generality, since a home-owner who has a second home and rents it out, there are no owner-occupancy benefits. His return from owning a second home and renting it out would be the same as that of the business sector, which is no greater than the return on savings.

<sup>3</sup>This assumption is really without any loss of generality because given the substantial transactions costs of

If the individual is not excluded from the mortgage market and chooses to rent, she solves

$$M_0(w, a, x = 0, k = 0; t) = \max_{c \geq 0, h \geq 0, a' \geq 0} \{u(c, h) + \beta E_{w'|w} V(w', a', x' = 0, k' = 0; t + 1)\}$$

$$c = w - g(w, a, x = 0, k' = 0) + a(1 + \omega r + (1 - \omega)\tilde{r}) - a' - z(t)h$$

Note that there are two types of future value functions, one for home-owners and one for renters. The number of arguments in each is different. For renters, future value is  $V(w', a', x' = 0, k' = 0; t + 1)$ , and depends on next period's wage level and next period's asset level. Home-owners' value function is  $V(w', a', x', k', \delta', \kappa'; t + 1)$ . In addition to wage and financial assets, this value depends on the mortgage and house sizes and on the housing depreciation shock  $\delta'$  and the cost of defaulting on the mortgage next period  $\kappa'$ . We do not distinguish between homeowners who are excluded from the mortgage market (due to a prior default) but bought their homes with cash and homeowners who bought their homes by taking out a loan. The implicit assumption is that once an individual purchases a house, his credit record is automatically set clean.

If the individual is excluded from the mortgage market and chooses to rent, she solves

$$M_0^D(w, a, x = 0, k = 0; t) = \max_{c \geq 0, h \geq 0, a' \geq 0} \{u(c, h) + \beta E_{w'|w} \lambda V^D(\cdot; t + 1) + (1 - \lambda)V(\cdot; t + 1)\}$$

$$c = w - g(w, a, x = 0, k' = 0) + a(1 + \omega r + (1 - \omega)\tilde{r}) - a' - z(t)h$$

where  $\lambda$  is the probability the individual remains excluded from the mortgage market (as noted earlier, the "exclusion flag" is followed only for renters").

Then

$$V(w, a, x = 0, k = 0; t) = \max \{M_1(w, a, x = 0, k = 0, ; t), M_0(w, a, x = 0, k = 0; t)\}$$

And

$$V^D(w, a, x = 0, k = 0; t) = \max \{M_1^D(w, a, x = 0, k = 0, ; t), M_0^D(w, a, x = 0, k = 0; t)\}$$

Consider next the decision problem of an individual who owns a house and has an outstanding mortgage. The household may choose to keep the current house, sell it, or default on the mortgage. 

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 purchasing and selling a home, individuals purchase homes for a long duration of time. By the time they need to make another purchase, their exclusion flag would typically be gone. Thus the individuals will behave as if they do not have a default flag.

If he chooses to keep the house, he solves:

$$K_0(w, a, x, k, \delta; t) = \max_{c \geq 0, a' \geq 0} \{u(c, k) + \beta E_{w', \delta', \kappa' | w} V(w', a', x\mu/(1 + \pi(t)), k, \delta', \kappa'; t + 1)\}$$

$$c = w - g(w, a, x, k) + a(1 + \omega r + (1 - \omega)\tilde{r}) - a' - x - \delta h$$

If he chooses to sell, he solves:

$$K_1(w, a, x, k, \delta; t) = \max_{c \geq 0, h \geq 0, a' \geq 0} \{u(c, h) + \beta E_{w', | w} V(w', a', x' = 0, k' = 0; t + 1)\}$$

$$c = w - g(w, a, x, 0) + a(1 + \omega r + (1 - \omega)\tilde{r}) - a' - x + p(t)[1 - \chi_S]k - \mu\bar{q}(x/(1 + \pi)) - z(t)h - \delta h$$

Here  $\chi_S$  is the percentage cost of selling a house. Selling the house requires the individual to buy back his promised sequence at the nominal risk-free interest rate.<sup>4</sup> We assume that the house is sold at the beginning of the period and the household has to rent to consume housing in that period. For the payment of mortgage obligations, the household pays the current flow mortgage obligation  $x$  and also needs to buy back the outstanding future (geometrically declining) mortgage obligations at risk-free rate  $i$ . The taxing function  $g(w, a, x, 0)$  reflects all these assumptions, that is, the period mortgage payment  $x - (1 - \mu)Q(i, \mu, x)$  is deducted from taxes, and as the household sells the house beginning of period, it does not pay property tax.

If the household chooses to default on the mortgage, he solves:

$$K_D(w, a, x, k, \kappa; t) = \max_{c \geq 0, h \geq 0, a' \geq 0} \{u(c, h) - \kappa + \beta E_{w' | w} [(1 - \lambda)V(\cdot; t + 1) + \lambda V^D(\cdot; t + 1)]\}$$

$$c = w - g(w, a, 0, 0) + a(1 + \omega r + (1 - \omega)\tilde{r}) - a' - z(t)h$$

Foreclosure results in the individual losing the house as well as the mortgage and being excluded from the mortgage market for some random length of time. A random utility cost  $\kappa$  is added to the value function if the household defaults. This cost is observed before the household makes its decision of default or not. This cost is added for technical reasons, without it the solution algorithm can fail to converge. It ensures smoothness in default decision and so smoothness in the pricing function.

Then for  $k > 0$  and  $x \geq 0$

$$V(w, a, x, k, \delta, \kappa; t) = \max \{K_0(w, a, x, k, \delta; t), K_1(w, a, x, k, \delta; t), K_D(w, a, x, k, \kappa; t)\}$$

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<sup>4</sup>Ideally we should require the individual to buy back the perpetuity at  $\tilde{q} \cdot x$  where  $\tilde{q}$  is the price the individual paid when he issued the perpetuity. But implementing this would require expanding the state vector.

## 4.2 Financial Intermediaries

The representative financial intermediary accepts deposits, buys mortgages, and buys and rents housing. We will assume that the intermediary chooses how much to engage in these activities on the basis of the expected return in each of these activities. Denote the intermediary's expected net rate return (profits) on a deposit of size  $a$  by  $\nu(a)$ , the net expected return on a mortgage with characteristics  $w, x', a', k'$  and  $t$  by  $\nu(w, a', x', k', t)$  and the net expected return on the purchase of  $h$  units of housing in period  $t$  by  $\nu(h, t)$ . Correspondingly, let  $m(a)$ ,  $m(w, x', a', k', t)$  and  $m(h, t)$  denote the measure (more precisely, the density or pdf) of such contracts acquired by the financial intermediary. Then, the financial intermediary's decision problem is:

$$\nu(t) = \max_{\{m(a), m(w, x', a', k', t), m(h, t)\}} \left\{ \int \nu(a)m(da) + \int \nu(w, x', a', t)m(dw, dx', da', t) + \int \nu(h, t)m(dh, t) \right\}$$

For there to be a (bounded) solution to this problem the net expected returns on each type of asset must be non-positive. For deposits this requires  $\nu(a) = a - [a(1 + r)]/(1 + \bar{r}) \leq 0$  for all  $a$ . This requirement reduces to

$$(1 + r) \geq (1 + \bar{r}). \quad (5)$$

For housing, this requires that  $\nu(h, t) = z(t) \cdot h - \rho p(t)h + [p(t + 1) \cdot (1 - \Delta)h]/(1 + \bar{r}) - p(t)h \leq 0$  for all  $h$  (financial intermediaries do not pay any cost for selling houses). This requirement reduces to

$$p(t) \geq \frac{z(t) + [p(t + 1)(1 - \Delta)/(1 + \bar{r})]}{1 + \rho} \quad (6)$$

For mortgages, the expression for net return is more involved. When the intermediary acquires a mortgage it gives up  $q(w, a', x', k'; t) \cdot x'$  in goods. Next period, if the individual defaults, the intermediary receives  $p(t + 1)[1 - \chi_D]k'$  where  $\chi_D$  is the cost of foreclosure to the intermediary; if the individual sells the property, the intermediary receives  $x + \mu\bar{q}(x/(1 + \pi))$ , and if she neither defaults nor sells, the intermediary receives  $x$  plus the value of the continuing mortgage, which is

then given by  $q(w, a', \mu x / (1 + \pi), k'; t + 1)$ . The requirement that the expected net return from a mortgage  $\nu(w', a'', x', k', t)$  be non-positive becomes:

$$\begin{aligned}
q(w, a', x', k', t) x' &\geq (1 + r)^{-1} \times & (7) \\
E_{w', \delta', \kappa' | w} \{ &d(w', a', x', k', \delta', \kappa'; t + 1) p(t + 1) [1 - \chi_D] k' + \\
s((w', a', x', k', \delta', \kappa'; t + 1)) &(x' + \mu \bar{q}(x' / (1 + \pi))) + \\
(1 - d(\cdot; t + 1)) (1 - s(\cdot; t + 1)) &(x' + q(w', a'', \mu x' / (1 + \pi), k', t + 1) \mu x' / (1 + \pi)) \}.
\end{aligned}$$

## 5 Equilibrium

An equilibrium consists of a sequence of rental pricing function  $\{z^*(t)\}$ , a sequence of housing price functions  $\{p^*(t)\}$ , a deposit interest rate  $r^*$ , a sequence of mortgage price functions  $\{q^*(k, x, a, w, t)\}$ , a sequence of distributions  $\mu^*(k, x, a, w, t)$  of people over the state space, and a sequence of decision rules for excluded and non-excluded individuals such that:

1. The decision rules are optimal given  $r^*, z^*(t), p^*(t), q^*(t)$ .
2. The net returns (5)-(7) are zero.
3. Demand for housing equals supply, that is,  $\int h^*(w, a, x, k, \delta, \kappa; t) \mu^*(dw, da, dx, dk, d\delta, d\kappa; t) = H(t)$
4. The sequence of distributions  $\mu^*(w, a, x, k, \delta, \kappa; t)$  is implied by the decision rules.

## 6 Parameter Selection and Calibration

Turning first to the Markov process for earnings, we assume that log earnings follow an AR1 process:

$$\ln(w_t) = \rho \ln(w_{t-1}) + \epsilon_t \quad (8)$$

Several studies have estimated log earnings processes for the US using PSID earnings data.<sup>5</sup> Esti-

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<sup>5</sup>These processes are typically modeled as sum of a fixed random effect, an AR1 process and purely transitory shock. For reasons of tractability, we ignore the fixed random effect and the purely transitory shock.

mates of  $\psi$  and the standard deviation of  $\epsilon$  ( $\sigma_\epsilon$ ) vary across studies. We follow Storesletten, Telmer and Yaron (2004) in setting  $\sigma_\epsilon = 0.129$  and  $\rho = 0.97$  (which is the near the lower bound of  $\rho$  estimates reported by the authors). This AR1 process is then approximated by a 17-state Markov chain.

Setting aside the parameters of the income tax schedule, our model economy has 18 other parameters. These include 3 preference parameters ( $\beta, \theta, \gamma$ ), 8 parameters related to housing transactions ( $\chi_S, \chi_B, \psi, \Delta, \{\delta_j, \xi_j\}, j = H, L$ ), 1 related to the mortgage contract ( $\mu$ ), 2 related to costs of foreclosures ( $\lambda, \chi_D$ ), and 3 related to the asset market ( $\omega, \tilde{r}, r$ ) and, finally, the steady state inflation rate ( $\pi$ ).

Of the preference parameters,  $\gamma$  is set to 2, which is a standard value in macro studies and the value of  $\theta$  is set 0.20, based on reported share of rents in expenditures for renters.<sup>6</sup>

Of the housing transactions parameters, Gruber and Martin (2003) find (from the Survey of Consumer Expenditures) that transaction costs account for around 7 percent of the house value; we split this into 6 percent selling cost and 1 percent buying cost which fixes  $\chi_S$  and  $\chi_B$ , respectively. The average property tax rate in the US in 2007 was 1.38 percent, so  $\psi$  was set to 0.0138.<sup>7</sup> The value of the depreciation rate for rental housing space,  $\Delta$ , was set to 1.5 percent, which is line with the estimate reported in Shilling, Sirmans and Dombrow (1991) for rental properties built more than 10 years ago.<sup>8</sup> The value of  $\lambda$  was set to 0.5 which implies that exclusion from the mortgage market upon default lasts 2 years, on average. The loss in value of house that goes into default is set to 15 percent, which fixes  $\chi_D$  to 0.15.<sup>9</sup> We also assumed that the high depreciation shock for homeowners is equal to  $\chi_D$ , so  $\delta_H = 0.15$ .<sup>10</sup>

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<sup>6</sup>According to Consumer Expenditure Tables (<http://www.bls.gov/cex/csxshare.htm#1989>) the share of rents in total expenditures varied from 19.7 percent in 1989 to 21.6 percent in 1999.

<sup>7</sup>This figure comes from Moody's Economy.com and was reported in <http://www.nytimes.com/2007/04/10/business/11leonhardt-avgproptaxrates.html>

<sup>8</sup>The authors report that the average depreciation rate for owner-occupied properties is roughly between 1.93% in Year 1 and 1.06% in Year 10, while that for rental properties ranges from 2.54% in Year 1 to 1.66% in Year 10. The findings are robust to alternative specifications of the hedonic price model

<sup>9</sup>Shillings, Benjamin and Sirmans (1990) estimate that price per sq foot of foreclosed properties is about 10 percent less than that of non-distressed properties. In addition, there are other costs borne by lenders that further lower the net realized value from foreclosure.

<sup>10</sup>The motivation for this assumption is simply that in steady state the high depreciation shock leads to a default

We set the average inflation rate to 2.5 percent, which set  $\pi = 0.025$ .

Turning to the asset market parameters, we set the real pre-tax return on financial assets to 4 percent, which fixes  $r$  to 0.04. Regarding the tax treatment of interest earnings, we recognize that only a portion of the nominal returns on financial assets is taxed at the relevant individual income tax rate; the remaining portion is taxed at a (potentially) lower rate because some of the return on assets is in the form of capital gains (which is typically taxed at a lower rate) and both capital gains as well as dividends and interests on assets which are in retirement accounts are not taxed until the the individual reaches retirement. We assume that the portion that is taxed at the relevant income tax rate is 40 percent, which sets  $\omega$  (the fraction of returns from long-term investments) to 0.60.<sup>11</sup> We assume that the returns from this latter portion is taxed at a flat rate of 23 percent after a period of 10 years. Given an inflation rate of 2.5 percent and a real return of 4 percent, this is equivalent to annual after-tax earnings from long-term investments of 2.18 percent, which fixes  $\tilde{r} = 0.281$ .<sup>12</sup>

These values are summarized in Table 1.

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when the home-equity ratio is low enough. Thus, we assume the same loss in home value as in a foreclosure.

<sup>11</sup>In 2009, the fraction of household financial assets in retirement account was 35 percent (Investment Company Institute (2009)). Of the remaining 65 percent, we assume that 70 percent is allocated to equity. The return on equity that due to capital gains has been about 58 percent (Ibbotsen and Chen (2003, Figure 1)). Thus, the portion of return on financial assets that are taxed at a lower rate is  $0.35 + (0.65)(0.70)(0.58) \approx 0.60$ .

<sup>12</sup>The nominal gross after-tax return on a dollar invested in the long-term asset is  $[(1.025 \times 1.04)^{10} - 1](1 - 0.23) + 1 = 1.6890$  and the real return is  $1.6890 / (1.025)^{10} = 1.3194$ , which implies an after tax real rate of return of  $1.3194^{1/10} = 1.0281$ . Hence  $\tilde{r} = 0.0281$ .

Table 1

Parameter	Value	Description
$\lambda$	0.5	probability of re-entry after default
$\bar{r}$	0.04	risk-free real interest rate
$\rho$	0.97	autocorrelation of earnings
$\sigma$	0.129	sd of innovation to earnings shock
$\delta$	0.015	depreciation of rentals each year
$\delta_H$	0.15	high depreciation rate for homeowners
$\theta$	0.20	exponent to housing consumption
$\gamma$	2.0	risk-aversion coefficient
$\pi$	0.025	steady-state inflation
$\chi_B$	0.01	cost of buying
$\chi_S$	0.06	cost of selling
$\chi_D$	0.15	foreclosure cost
$\psi$	0.0138	property tax rate
$\tilde{r}$	0.0281	real after-tax annual return on long-term investment
$\omega$	0.60	portion of asset return in long-term tax-deferred investment

We need to specify the tax schedule  $\tau(\cdot)$  and the standard deduction  $s$ . The tax schedule is chosen to match the tax table for 1998. In our model, people are viewed as individuals (this seems consistent with the earnings data). But we will take individuals to be married. Hence, the tax table we use is the tax table for married filing separately. According to the Census Bureau, male median income of year-round full-time workers age 25 and older in 1998 was \$37,906 and that of females was \$27,956. We use the average of these two numbers, which is \$32,931, as the median income of an individual filing for taxes. Normalizing the tax brackets for 1998 by this estimate of median income, we obtain the following tax schedule  $\tau(\cdot)$ :

Table 2

Tax Brackets	Tax Rate
0 - 0.64	0.15
0.64 - 1.55	0.28
1.55 - 2.37	0.31
2.37-4.23	0.36
4.23 -	0.396.

And normalizing the standard deduction for a married person filing separately by median income gives  $s = 0.1116$ .<sup>13</sup>

The remaining 4 parameters  $(\beta, \delta_L, \xi_H, \mu)$  are determined by computing the steady state of the model so as to match the ratio of mean financial asset to mean income, the homeownership rate, the steady-state default rate and the fraction of homeowners with home equity ratio less than or equal to 25 percent. In computing the ratio of financial wealth to income we used the 1998 Survey of Consumer Finances and ignored the top 3 percent of the wealthiest households (by net worth) since it is well-known that this class of models cannot match the upper tail of the wealth distribution.<sup>14</sup> The homeownership rate we target is the 10-year average ending in 2003 (we stopped at 2003 so as to not to distort our parameter choices by booms and busts in the residential real estate markets). The foreclosure rate is the average foreclosures started each year as a percentage of all mortgages for 1991-1998. The facts about the home equity ratio distribution was obtained from the 1998 Survey of Consumer Finances<sup>15</sup>

The results of the matching exercise is displayed in Table 3. The model matches the target statistics almost exactly. The parameter values that achieve this match are listed in the final column. Although these statistics are jointly targeted, the parameter listed in each row is the parameter that is the key for determining the corresponding statistic. The table also lists statistics that were not targeted but are relevant for understanding model mechanics or for analysis of the mortgage crisis presented later in the paper. The ratio of mean housing wealth to mean income is 1.27 whereas it is 1.33 in the data. The CDF of the model's home equity distribution rises faster than the CDF for the data for low values of the home-equity ratio. By construction, the CDFs cross at home-equity ratio of 25 percent which then implies that the model underpredicts the cumulative distribution for low values of the home-equity ratio and over-predicts it for higher values.

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<sup>13</sup>Our tax schedule overstates the taxes paid by low-income people because we ignore the earned income tax credit. However, what is important for our study is the tax benefit of owner-occupied housing and this benefit is *not* affected by the earned income tax credit. This is because the credit is calculated on a person's adjusted gross income and, therefore, does not depend on whether the household rents or owns.

<sup>14</sup>Financial wealth is define as financial assets - credit card balance - margin loans, loans against pensions, loans against life insurance - other lines of credit not secured by equity in home - educational installment loans.

<sup>15</sup>The home equity ratio is defined as of the value of the home minus housing debt to the value of the home.

Table 3

Targeted Statistic	Data	Model	Parameter Values	
Avg asset to avg income ratio	1.52	1.50	discount factor, $\beta$	0.9556
Homeownership rate	0.664	0.664	low depreciation rate for homeowners, $\delta_L$	0.006
Steady state foreclosure rate	0.0135	0.0135	probability of high depreciation rate, $\xi_H$	0.063
Frac of homeowners with home equity $\leq 25$ %	0.19	0.18	Prop of mortgage maturing each period, $\mu$	0.015
<b>Non-targeted statistics</b>				
Avg Inc of homeowners/Avg income of renters	2.02	1.97		
Avg housing wealth to avg income	1.28	1.27		
Average home equity ratio	0.64	0.67		
Frac of homeowners with negative equity	3.03	4.07		
Frac of homeowners with less than 10 % equity	7.83	10.15		
Frac of homeowners with less than 20 % equity	13.73	15.53		
Frac of homeowners with less than 30 % equity	22.95	20.49		

## 7 Steady State

We solve for the steady state by first normalizing the price of rentals. Given the Cobb-Douglas form in the utility function, a renter will always spend a fixed proportion of his current expenditures on rents, and a fixed portion on the other good. Given the stream of rental income to the risk-neutral landlord, subtracting the cost of depreciation and property taxes, gives a discounted value of a unit of rental to the landlord. Although we have not modeled a construction sector explicitly, it is assumed that in steady state the return on a unit of space should be the same whether it is sold to a landlord who has an intention to rent it out or it is bought by a home-owner. This implies that if a construction sector were to exist in the model, it would be indifferent toward providing an extra unit of housing to the home-owner market or to the rental market (but landlords do not pay cost of real-estate agents, while home-owners pay). Thus, normalizing the rent fixes the price of housing space so that the housing stock is effectively endogenous.<sup>16</sup> When we compare alternative steady states, we follow the same procedure. Thus all steady state comparisons should be viewed

<sup>16</sup>This also implies that the price-rent ratio is independent of the normalized rental price. If the normalized rental price goes up by 1 % all households (whether they are renters or home-owners) would consume 1% less housing units and nothing else would change.

as comparisons of long-run equilibrium in the housing market, where the cost of providing a unit of housing space is constant. If the cost of constructing houses does not change from one equilibrium to the next, this would be exactly the long run equilibrium.

First, one may ask, why do people in our model have an incentive to purchase homes? We have assumed that there is no direct benefit to homeownership and in our calibration there is no “rental externality”: The average depreciation of rental properties is 0.0150 and that of owner-occupied units is 0.0153. Furthermore, in computing the solution to our model, we assume that households that rent can choose their  $h$  freely, while those that purchase their homes must choose from a finite set of house sizes. Thus, homeownership affords no direct utility benefit, involves somewhat faster depreciation and less flexibility (in terms of house sizes) than renting, and involves the payment of significant transactions costs in its acquisition. Nevertheless, the majority of people choose to purchase their homes. This is because of a key tax benefit of homeownership: the implicit rental income from ownership is not counted as part of income and therefore not taxed. This exemption means that people – especially those in the higher tax brackets – have a strong incentive to purchase their homes.<sup>17</sup> The deductibility of mortgage interest payments encourages these households to *borrow* to finance the purchase of their homes as opposed to paying for the purchase from accumulated assets, which, in fact, is what people do in reality. Both of these effects kick in more strongly for richer households because their tax rate is higher and the mortgage interest payment deductions are more likely to exceed standard deductions as they buy bigger homes. Given this, home-owners are concentrated among richer households. In our model, the average income of homeowners is 1.97 times the average income of renters, which compares very favorably with the data where it is 2.02. Our model is also consistent with the observation that owner-occupants consume more housing space, on average, than renters. In our equilibrium the per capita housing space of renters is 54 percent of per capita housing space of owner occupants. There are two reasons for this: First, high earners choose to buy houses which makes the housing space of owner-occupants larger than that of renters. In addition, the tax benefit of owner-occupancy makes owner-occupants consume more

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<sup>17</sup>Let’s say the household is deciding between saving in risk-free asset or saving in a home. When the household saves in risk-free asset it pays taxes over the nominal interest return. If the household saves by buying a house, the return to that saving comes as (implicit) rental income and appreciation in the value of the house, both of which are not taxed. So there is additional tax benefit of home-ownership.

housing than renters.

To see how tax incentives encourage households to buy bigger homes, Table 4 shows how the steady state is altered if these tax benefits were to be eliminated. If the mortgage deduction is eliminated, the homeownership declines from 0.664 to 0.583. Average equity rises to more than 95 percent since there is no benefit to taking on leverage. In addition, average housing consumption in the whole economy (including home-owners and renters) declines by 7.3 percent. If, in addition to getting rid of mortgage tax deduction, the implicit rental income from owner-occupancy is taxed in exactly the same fashion as returns from financial assets, the incentive to own homes goes away entirely. Furthermore, average housing consumption declines by 14.5 percent relative to the baseline model.

Table 4

<b>Statistic</b>	<b>Baseline</b>	<b>No Mtg Ded</b>	<b>No Mtg Ded &amp; Taxes on Impl Rental Inc</b>
Homeownership rate	0.664	0.583	0
Avg Home Equity	0.64	0.960	0
Avg Housing Cons	0.893	0.903	0.876

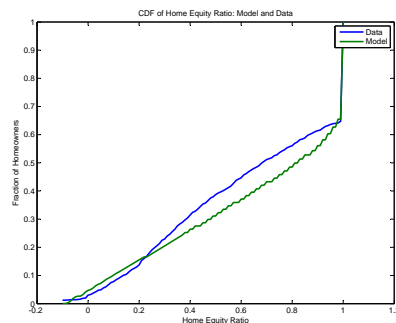
Given the tax incentives of home-ownership why do households not hold a much bigger portion of their wealth in housing? In the model, we constrain households to own only one house, but given that the mentioned incentives apply only to the first homes of households, this is without loss of generality. In the steady state economy, there is no appreciation of house-prices, so the benefit to having home equity occurs because the implicit return in rental income from owning the house is not taxed, while if that equity was transferred to other assets, the returns would have been taxed. But still the size of the house people wish to own is bounded by the utility they get from the consumption of the house. Although the implicit rental income from home-ownership is not taxed, it comes with the constraint that this non-taxable income has to be spent on housing consumption. So although home-owners consume bigger houses than renters (as it is implicitly subsidized), how much they invest in housing is bounded by how much of their income they would like to allocate to housing consumption. The share of housing in the Cobb-Douglas utility function is 0.20, which results in an average home equity to average income ratio of 1.27 in the model, which is almost exactly what it is in the data.

And why is there default in the steady state economy? One reason is that we have random de-

preciation of homes that hit home-owners with some given probability. We interpret this random depreciation as random changes in local house prices. But the existence of this random depreciation shock is not sufficient to generate default. The additional assumption is that after such a depreciation, if there is a default on debt, the further foreclosure cost to the financial intermediary is small (in the paper we assume it is zero). So in the beginning of the contract, the financial intermediary expects that if the depreciation shock happens early on in the contract (when the equity of the home-owner in the house is still low) the owner will dump the house to the lender. In return, the lender charges the home-owner with a higher interest payment spread through the years. Given that the lender is risk-neutral and the depreciation shocks are independent across homeowners, it is optimal for the lender to take on the risk of a bad depreciation shock. If there is a further cost of foreclosure, however, the optimal arrangement will deviate from this, with the bank asking for higher premiums to bear the additional expected cost. The higher cost of loans will motivate households to borrow less and, therefore, default with lower frequency (or not default at all).

What factors determine the home-equity distribution in the model? The home-equity distribution at the time of the buying of a house is bi-modal (U-shaped). A large portion of households buy houses with very little equity and these are mostly high-income households who wish to utilize the tax deduction of mortgage interest payments since the interest payments are likely to exceed the standard deduction. The middle is relatively empty, with the second big mass coming at the other end with households choosing very high equity rates. These are middle to low income households who wish to buy smaller houses, and for whom – had they taken out a big mortgage – the interest payments would have fallen below the standard deduction. When the mortgage payment is below or very close to standard deduction, it is more beneficial for the household to invest more of its existing savings into the house, and get the tax-free implicit rental returns from that savings. The average income of people taking out (new) mortgages with less than 50 percent home-equity is 1.62 times the average income of households taking out new mortgages with more than 50 percent home-equity and the average house size of the former group is about 1.37 times the average house size of the latter group. Aside from these two important effects, the decision of how much to borrow to finance the purchase of the house also depends on the household's financial wealth: Some households get a large loan because they wish to become home-owners but do not have the financial wealth for a large down-payment. The second important factor affecting home-equity distribution is inflation.

Because the mortgage contract is nominal, inflation reduces the real value of debt over time and, therefore, increases the real value of home-equity. The fairly uniform nature of the home-equity distribution results from this steady erosion of the real value of mortgage debt. Figure 1 displays the home-equity distribution in the model and in the data.



How does steady state inflation effect the housing market? Table 5 compares the baseline steady state, which has an inflation rate of 2.5 percent, with the steady state when the inflation rate is 4 percent. There are two effects: First, a higher inflation rate increases the nominal interest rate and, therefore, the tax benefits of the mortgage deduction. The benefits of the mortgage deduction increase with inflation because nominal interest payments are tax-deducted, not real. This encourages households to take on more debt and buy bigger houses. On the other hand, higher inflation erodes the value of debt faster and thus causes households to accumulate home-equity at a faster rate. In the experiment, the second effect dominates and average home-equity goes up along with house-size and the home-ownership rate. Higher inflation also makes saving in housing more attractive as the effective tax rate on financial assets become higher when inflation is higher since those taxes also depend on the nominal return, not the real return.

Table 5

<b>Statistic</b>	<b>Baseline</b>	<b>SS inflation of 4 percent</b>
Homeownership rate	0.664	0.664
Average equity	0.64	0.66
Average housing consumption	0.893	0.90

## 8 Quantitative Analysis of the Foreclosure Crisis

### 8.1 Baseline

In this section, we use the model to study the foreclosure crisis. Our story for the foreclosure crisis is the following: During the boom, when house prices were inflated, more houses were built than is consistent with demand. When this excess supply became evident, house prices fell to clear the market and the decline in house prices induced leveraged households to default. While this story has intuitive merit (at least to us), the question we answer in this section is whether it has *quantitative* merit. We model the over-supply of housing as a permanent, unanticipated increase in the stock of housing space in the hands of the business sector in the initial period. We compute the perfect foresight transition path to the new steady state with the higher stock of housing. As we explain below, the unanticipated increase in supply results in a drop in price and an increase in foreclosures in the period of the shock. Following the period of the shock, house price rises and the foreclosure rate falls toward their new steady state values. In reality, house prices peaked in the 2006 Q2 and reached a trough in 2009 Q2. During the same time, the foreclosure rate rose above its norm (of 1.35 percent per year) and has remained elevated to date. The cumulative fall in house prices from peak to trough is about 19 percent, the cumulative foreclosure rate since 2009 Q2 is 15 percent and the cumulative foreclosure rate in excess of normal foreclosure rate is 10 percent.<sup>18</sup> The specific question we ask is whether the model can generate the observed cumulative drop in house prices

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<sup>18</sup>The house price drop is computed from LoanPerformance house price index excluding distressed sales. According to this series, house prices peaked in 2006Q2 and hit its trough 2009 Q2. We use 5 quarter averages centered around these peak and trough quarters to calculate the percentage decline in price due to the crisis. For the foreclosure rate, we simply sum the quarterly foreclosure rate between 2006 Q2 to 2010 Q2. We cumulate up to the current quarter because foreclosures take time to process and the foreclosures that are occurring in 2010 Q2 presumably started around 2009 Q2.

and the cumulative jump in gross and net foreclosures for a plausibly-sized excess supply shock.

We start with estimates of over-supply in the housing market. McNulty (2009) reports that between 2005 and 2007, the housing stock increased by 3.8 million units but the number of occupied housing units increased by only 1.8 million units. Thus, about 2 million housing units were added that did not find occupants. Since houses typically sit on the market for sometime before they find occupants, part of the increase in unoccupied housing units is simply a reflection of “frictional” vacancy. McNulty estimates that the increase in unoccupied units due to frictional vacancies to be about 0.28 million units, which leaves an excess of 1.72 million units. As a percentage of the stock of owner-occupied housing units in 2005, this is about 2.3 percent. In our analysis, we assume that the excess supply of owner-occupied housing is 3 percent. We chose a somewhat higher excess supply in order to compensate for the fact that our model leaves out features that, in the real world, tends to lower the elasticity of housing demand with respect to the price of housing space.<sup>19</sup>

To give the model a reasonable shot at explaining the data, we make 3 additional assumptions. First, starting with the period of the shock, we assume that the owner-occupied segment of the housing market is segregated from the rental market. All previously owner-occupied housing space and the additional 3 percent increase in this space must remain owner-occupied during the transition to the new steady state as well as in the new steady state. This assumption is meant to capture the fact that the rental market for housing meant for owner-occupancy is thin. Following the shock, the supply of housing in each of the two markets stays constant and the price in each market is determined by demand and supply for housing space in that market.<sup>20</sup>

Second, we assume that for the first 4 years following the shock, households that choose to foreclose on their homes get to live in their house rent-free with some fixed probability. This assumption is meant to capture the fact that the foreclosure process takes longer when there is a large number of households declaring foreclosure (as will, in fact, happen following the housing supply shock).

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<sup>19</sup>For instance, new household formation is one channel through which increases in the supply of housing is absorbed, and the rate of household formation is presumably not very responsive to the price of housing space.

<sup>20</sup>As in steady state, we assume that whichever entity owns the housing space must incur the depreciation expense which keeps the stock of housing constant through time. The segregation implies that in the post-shock world, the return from renting a unit of housing space and selling a unit of housing space will not be equal. We think this is reasonable given that the shock is unanticipated.

In normal times a foreclosure takes about 6 months to complete. During the crisis, foreclosures have been taking an additional 7 – 8 months, on average. This extra time for foreclosure is taken into account by setting the probability with which a person declaring foreclosure gets to stay in the house rent-free for another additional year to 0.63.

Finally, we assume that following shock, there is an additional cost that must be paid to get a mortgage. This cost is meant to capture the disruption in the flow of funds to the mortgage market following the mortgage crisis. We assume that the cost remains constant for the first 4 periods following the shock, then declines at the rate of 80 percent, per year. Since it is difficult to pin down the relevant cost by direct observation, we chose it so that the model produces a decline in the price of owner-occupied housing in the initial period that is close to what is observed: the model produces 18 percent and the data has 19 percent.<sup>21</sup> Calibrated in this way, the additional cost is roughly equal to a 1 percentage point increase in the cost of funds beyond the risk-free rate.

Table 6

	<b>SS</b>	<b>Post-shock SS</b>	<b>Shock Period</b>
House Prices	1	0.98	0.81
Rents	1	0.98	0.88
Foreclosures (%)	1.35	1.39	12.25

Table 6 displays the equilibrium outcome regarding house prices, rents and foreclosures for the new steady state and for the initial period (the period of the shock). In the new steady state, the increase in the supply of owner-occupied housing has benign effects: The 3 percent increase in housing supply leads to a roughly 2 percent decline in the price of owner-occupied housing and a 2 percent decline in rents. The excess supply is absorbed through increase in the average housing space occupied by owners and an increase in the fraction of homeowners. There is only a slight increase in the foreclosure rate. In the period of the shock, however, the gross and net foreclosure rates rise to 12.25 percent and 10.9 percent, respectively. These predictions are reasonably in line with what actually transpired. Because it is costly for owner-occupants to increase the demand for

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<sup>21</sup>Note, however, that there is a natural lower bound to how low price of owner-occupied housing space can fall. Since landlords always have the option to leave the space empty for one period, the price of housing in period  $t$  cannot fall below  $(1 - \Delta)p(t + 1)/(1 + \bar{r})(1 + \rho)$ . In equilibrium, the price of owner-occupied housing is greater than this lowest price possible in each period, so no housing space meant for owner-occupancy is ever left empty.

housing space – due to the transactions costs of selling and buying new homes and the additional cost of getting a new mortgage – the demand for owner-occupied housing space does not rise as much with a decline in the price of housing. Consequently, house prices have to drop substantially to motivate renters to become homeowners and absorb the excess supply of housing. Even though the higher foreclosure rate increases the demand for rentals (a quarter of defaulters must rent, the rest get to stay in their house), the movement of renters into owner-occupancy reduces demand for rental space in the period of the shock and thereby lowers rents as well.

We now turn to understanding the incentives to default following the shock. Ignoring for the moment the chance to live rent-free for one year, a homeowner who has positive home equity net of the transactions costs of selling the house will never choose to default. Default results in loss in value of the house as well as utility costs stemming from exclusion from the mortgage market. The homeowner can avoid these costs by selling the house. However, the chance to live rent free for one year may tempt some homeowners with slightly positive home equity (net of transactions costs) to default. In general, low or negative home equity is a necessary condition for default. The 19 percent decline in house prices along with the selling costs of 6 percent means that homeowners with about 25 percent or less home equity prior to the shock satisfy the necessary condition for default following the shock. In our model, the fraction of such homeowners is 18 percent, which is calibrated to be close to what it is in the data. Whether such a person defaults depends on his next best alternative. If selling dominates keeping the house (and continuing on with the mortgage payments), the homeowner must choose between selling and defaulting. But we have already argued that for a low or negative home equity household, default would be better than selling so such a household will generally choose default. For such households, then, the incentive to default boils down to understanding why selling dominates keeping the house. Since selling incurs a capital loss, it can dominate keeping the house only if the household is no longer happy with the size of its house or the size of its mortgage. This can happen if, since purchasing its house, the household has received income shocks (good or bad) that make it desire a different sized house or a different sized mortgage. In this situation, the utility gain from adjusting house and/or mortgage size can compensate the homeowner for the capital loss from selling. Because rents decline in the period of the shock and house prices stay low for some length of time, these incentives are highest for homeowners who desire a bigger house or smaller mortgage payments. Such households can increase housing consumption

or lower housing expenditures by renting cheaply during the exclusion period and then get a bigger sized house or a smaller loan when the exclusion period is over. If keeping the house dominates selling, then the household may or may not choose to default. But the logic for default would be the same: A household that is unhappy with the size of its house or the size of its mortgage may wish to default to take advantage of the low rents and low house prices.

In the balance of this section, we quantify the contribution of different factors to the decline in the price of housing and to the rise in foreclosures in the initial period (figures appear in sequence in the back of the paper).

## **8.2 Do Foreclosures Depress House Prices?**

As we have noted, the drop in the price of owner-occupied housing space creates the necessary conditions for the foreclosure crisis by increasing the fraction of homeowners with low or negative home equity. One may also ask if there is feedback from the rise in foreclosures to the decline in the price of owner-occupied space? In particular, if we prevented foreclosures would house prices fall less? The answer depends on a defaulter's next best alternative. If the defaulter is choosing between selling and defaulting, preventing him from defaulting would push him to sell. This would increase the supply of space in the owner-occupied market. The reason is that when a person defaults, there is only 1/4 chance that person will have to move out of the house and the space would be offered for sale. In contrast, if the homeowner is forced to sell then the space would be offered for sale with probability 1. If the next best alternative to default is to keep the house, then preventing default will decrease the supply of space. It turns out that in most cases, the next best alternative to default is to keep the house. Thus, foreclosures are a depressive force on house prices: if we prevented households from defaulting, the drop in the price of owner-occupied housing space would be 16 percent as opposed to 19 percent.

## **8.3 The Role of Lengthened Time to Foreclosure**

The fact that the foreclosure process has lengthened considerably during the crisis may have contributed to the crisis itself. We can examine what equilibrium default and price decline would be

like if the probability of staying “rent-free” for one year is set to 0. The fraction of mortgages that default in the period of the shock is only 6.9 percent, which is about half of what we see in the baseline model. So, the lengthened time to foreclosure is an important factor in the rise in foreclosures. The drop in the price of housing, however, is not much affected: the price decline is 19 percent as opposed to 18 percent in the baseline model. The reason for the small change in prices, despite a large change in foreclosures, is that there is a natural lower bound to how low price of owner-occupied housing space can fall. Since landlords always have the option to leave the space empty for one period, the price of housing in period  $t$  cannot fall below  $(1 - \Delta)p(t+1)/(1 + \bar{r})(1 + \rho)$ . In the baseline model, the price of owner-occupied housing is greater than this lowest price possible in each period, so no housing space meant for owner-occupancy is ever left empty. When we do not allow “free rentals”, this lower bound constraint binds. In the period of the shock, about 0.5 percent of owner-occupied housing space is left empty. The constraint does not bind in any other period.

#### 8.4 The Role Mortgage Market Disruptions

In the model, as well as in the real world, the foreclosure crisis disrupted the flow of funds into the mortgage market. As noted above, in the model we take this into account by incorporating a 1 percent additional cost, beyond the risk-free rate, of getting a new mortgage. The additional cost is present for 4 model periods and then declines rapidly at a geometric rate. What happens if this additional cost is eliminated? In the period of the shock, the foreclosure rate rises to 9.14 percent, as opposed to 12.25 percent in the baseline model, but the decline in price is a lot less. Now, in the period of the shock, the price of owner-occupied housing declines by only 5.5 percent. The reason is that the renters are more willing to jump in and buy houses when the cost of mortgages is lower and, when they do buy houses, they buy bigger ones. Also, because there are fewer foreclosures, there is less downward pressure on the price of owner-occupied housing space. On all these counts, the decline in house prices is much more moderate. Although the price decline is moderate, the foreclosure rate is still quite high. The reason for this is that lower cost of mortgages makes defaulting attractive also: Some defaulters default with an eye to buying a house in the near future when the exclusion period is over. A lower mortgage cost makes this strategy more attractive and thus increases foreclosures.

## 8.5 Unexpected Disinflation and Foreclosures

In this subsection, we study of the effects a lower inflation path on house prices and foreclosures. We assume that in the period of the shock, the anticipated inflation rate going forward falls to 1 percent for 5 years and then recovers back to the steady state value of 2.5 percent. To do this we need to be clear about the nature of the mortgage contract. We assume that the contracts written prior to the shock stipulate that upon sale of the house, the present discounted value of the outstanding payment stream is evaluated at the current market interest rate. In steady state this is equivalent to the present discounted value being calculated at the time the mortgage is written. However, when the shock hits, the nominal interest rate at which the payment stream is evaluated is now (unexpectedly) lower – because anticipated inflation is lower.

The lower inflation path increases the default rate from 12.25 percent to 17.07 percent. With a lower inflation rate, the real value of mortgage debt does not erode as rapidly as in the baseline model. Thus, the value of keeping the house is lower. And, the value from selling the house decreases as well because the present discounted value of the outstanding loan to be repaid upon sale is now higher. For both reasons, more households find default a better option. The higher default rate does not have much of an impact on the price of houses. House prices fall by about 20 percent now as opposed to 18 percent in the baseline model. It appears that the supply of home-buyers is fairly elastic at this price: As the price drops slightly, the excess supply of housing stemming from higher default is soaked up by new buyers and existing buyers buying bigger homes.

## 8.6 Mortgage Deduction and the Mortgage Crisis

In this section, we study how the crisis would have fared if there was no mortgage incentive to take on leverage. As noted in the discussion of the steady state of the baseline model, eliminating the mortgage deduction lowered the incentive to own homes and greatly lowered the incentive to take on mortgage debt. Thus average home equity is much higher and the average size of owner-occupied housing is also lower. Also, given the very high home equity there is no default in steady state. A 3 percent increase the supply of owner-occupied housing lowers the price by about 10 percent. The drop in price does not create any incentive to default, so there is no increase in foreclosures in the

period of the shock. The drop in price in the period of the shock exceeds the steady state drop for the same reasons as in the baseline model: The costs of selling and buying homes make the demand for owner-occupied housing space insensitive to a change in the price in the short run.

## 9 Conclusion

This paper developed a quantitative model of the US housing and mortgage market. We calibrated the model to be consistent with a small number of long run facts in these markets. The model turned out to be consistent with a range of other facts as well. We pointed out that the federal tax code has important implications for these markets. The non-taxability of implicit rental income is key for getting a large number of homeowners. The deductibility of mortgage interest payments in computing taxable income is key for getting people to borrow to purchase their homes. Also, because taxes are calculated on nominal income, higher steady state inflation encourages more consumption of housing.

We used the model to understand the foreclosure crisis, where the proximate cause of the crisis is an over supply of housing. We showed that a modest level of over-supply in the housing market, coupled with a modest increase in cost of new mortgages, can account for the steep decline in house prices.

Given the decline in house prices, the model can account for much of the observed rise in gross and net foreclosures. Given this explanation of the foreclosure crisis, we used the model to perform counterfactual experiments to get a quantitative understanding of the importance of different factors that go into generating the drop in price and the rise in foreclosures. We found that the lengthening of the time to foreclosure (which allows homeowners who default on their mortgage to continue to stay on in the house “rent-free”) is an important factor increasing the foreclosure rate but not an important factor in the drop in house prices. In contrast, the modest increase in the cost of obtaining new mortgages is an important factor in generating the decline in house prices but not very important for generating the increase in foreclosures. We also argued that foreclosures are a depressive force on house prices in that if foreclosures are prevented altogether, the decline in house prices would be less. Finally, we also noted that leverage played a big role in the foreclosure

crisis. If the tax code did not encourage leverage by making the interest payment of mortgages tax deductible, home equity would be much higher. In this case, the oversupply shock would not cause any foreclosures and the price drop would be a lot less.

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