The Bank Lending Channel and Monetary Policy Transmission when Banks are Risk Averse

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Abstract

I examine to what extent the banking system produces a special bank lending channel mechanism by embedding a model of risk averse banking in the financial accelerator model of Bernanke, Gertler, and Gilchrist (1999). I find that the banking system does produce a lending channel that amplifies the effects of shocks on impact and attenuates their effects in subsequent periods. I also offer a partial explanation for the recent build-up of excess reserves in the U.S. banking system by showing that risk averse banks accumulate excess reserves in response to an unanticipated rise in loan defaults.

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1 Introduction

The financial system has long been known to influence how monetary policy is transmitted through the aggregate economy[1]. Bernanke, Gertler and Gilchrist (1999) study the relationship between intermediated finance and monetary policy by embedding a micro-founded model of non-bank financial intermediation in a dynamic new-Keynesian model. A monetary expansion, in their model, reduces friction in the flow of intermediated credit, giving rise to a credit channel or financial accelerator mechanism. By operating together with the standard transmission channels, the credit channel amplifies the effects of monetary policy on the real economy. The Bernanke et al. model has become the canonical framework for studying the link between monetary policy and the financial system, but its exclusive focus on non-bank financial intermediation omits the special role of banking in the money supply process. Accordingly, their model may incorrectly characterize how financial factors affect monetary transmission at the margin.

Kashyap and Stein (1994) argue that a special bank lending transmission channel exists for two reasons. First, bank loans are special and many borrowers are constrained at the margin by the supply of bank loans. These borrowers cannot freely substitute between bank loans and alternative sources of credit and are forced to reduce their asset holdings in response to a contraction in bank lending. Second, banks themselves may face funding frictions that make reservable transaction deposits a less expensive funding source at the margin than alternatives like large-denomination certificates of deposit (CDs). Funding frictions are more likely to apply for smaller banks with less liquid balance sheets (Kashyap and Stein 2000). But if sufficiently many banks find deposits to be without close substitute, then monetary policy can directly influence the supply of bank loans when it changes the supply of reservable deposits.

Empirically identifying whether a bank lending channel exists poses a challenging iden-

[1] Gertler (1988) provides an excellent review of the early literature and Bernanke and Gertler (1995) review some of the more recent evidence.

tification problem that arises, for example, because a monetary contraction may lead to a reduction in both the supply and demand for loans, while bank balance sheets would reveal only a decline in lending. With this in mind, Kashyap and Stein (2000) argue that lending by banks holding fewer liquid securities to buffer against deposit outflows should be more sensitive to monetary policy. They test their proposition using disaggregated bank balance sheet data and find evidence of a lending channel: a monetary contraction does indeed reduce the supply of loans by banks in the bottom 95th percentile of asset size. But these banks hold only about a quarter of all assets held by the banking system. Therefore, while Kashyap and Stein find microeconomic evidence of a lending channel, they cannot conclude that the lending channel has macroeconomic significance.

One reason that Kashyap and Stein cannot argue that the bank lending channel is important for the aggregate cycle is that there is not currently a consensus theory of banking in the aggregate economy. Goodfriend and McCallum (2007) and Canzoneri, Cumby, Diba and López-Salido (2008) have recently proposed interesting models of banking in the new-Keynesian framework and simulations from both models suggest that the banking sector contributes non-trivially to aggregate dynamics. But neither model, however, addresses the complex risk-management problems that banks confront. The practical banking environment is characterized by uncertainty and a theory of banking in the aggregate economy should carefully account for how banks manage risk.

In this paper, I study the relationship between the banking system and the aggregate economy in a model that emphasizes the practical banker's risk management problem. I use the model from Bernanke et al. as the basis for my model for two reasons. First, they study a debt-contracting environment that allows for heterogeneity across borrowers, while still yielding equilibrium conditions convenient for aggregation. Second, the non-financial side of their model is similar to the new-Keynesian models that are currently popular for monetary policy analysis so that the marginal effects of the financial factors can be seen more clearly.

The theoretical contributions that I make in this paper appear as two modifications to the Bernanke et al. model. First, I transform the non-bank intermediary in Bernanke et al. into a representative bank that issues reservable transaction deposit liabilities and that solves an explicit optimization problem. For reasons that I describe below, I model the bank as a risk averse agent. Second, I modify the loan contracting problem in Bernanke et al. by restricting the set of feasible loan contracts. Bernanke et al. model an intermediary that escapes systematic risk on its loan portfolio by writing debt contracts with state-contingent loan repayment conditions. I force the bank in my model to bear systematic risk by prohibiting state-contingent contracts. Since the bank is risk averse and cannot write loan contracts to escape systematic risk, it faces a non-trivial problem when allocating its asset portfolio across loans and other assets.

I model the representative bank as a risk averse agent because I am interested in understanding what might motivate a bank to increase its excess reserve holdings. A distinguishing feature of the recent financial crisis in the U.S. has been the dramatic and prolonged accumulation of reserves by the banking system that began well before the Federal Reserve began paying interest on reserves. In this paper, I examine to what extent risk aversion can explain the sudden build-up of excess reserves. Of course, by modeling the bank as risk averse I depart from the typical assumption that firms are simply expected profit-maximizers. But my assumption makes sense if a bank's behavior reflects the preferences of its management. If the management's compensation were sufficiently correlated with the bank's performance, then it would be plausible that the bank's behavior would reflect the preferences for risk of the management.

In Section 2, I describe the economy in the lending channel model. I carefully layout the banker's intertemporal optimization problem. Among other things, the bank chooses the nominal size of its loan portfolio while taking the stochastic properties of the aggregate loan pool as given. Next, I examine how the bank originates the individual loans in its portfolio. The bank solves a contracting problem for each borrower and, through the terms of each contract, is able to influence the distribution of the stochastic return on the aggregate loan pool. The remainder of the model is similar to the Bernanke et al. model, but for completeness I describe the full lending channel model in the text. For reference, I include an abbreviated discussion of the Bernanke et al. model in Appendix B.

I examine the dynamic properties of the lending channel model in Section 3. I linearize the model around a deterministic steady state. Then I solve the linear system and compute impulse responses to several exogenous shocks: an aggregate productivity shock, a monetary policy shock, and a shock to loan defaults. Several broad conclusions emerge. First, relative to the Bernanke et al. model, the additional mechanisms in the lending channel model tend to amplify the real effects of exogenous shocks on impact. This is a direct consequence of the restricted loan contracting structure I study because exogenous shocks produce unexpected fluctuations in loan defaults. Second, in periods following the shock, the impulse responses of the lending channel model are attenuated relative to Bernanke et al. The attenuating effect is caused by a combination of mechanisms and I discuss them in Section 3. Finally, the bank actively uses excess reserves to manage the expected return on its asset portfolio. In particular, the bank chooses to accumulate excess reserves in response to an unexpected rise in loan defaults.

At this point a caveat is in order. For the computational exercises in Section 3, I use a simple two-part strategy for selecting parameter values. First, for the parameters that are common to the lending channel and Bernanke et al. models, I borrow directly from Bernanke et al. Second, for the parameters that are unique to the model that I have proposed, I have used parameters that produce reasonable implications for the steady state. Clearly this is a subjective strategy and so I emphasize that my results should be interpreted as preliminary. The next step in my project is to calibrate the model parameters rigorously so that I can more confidently interpret the quantitative implications of the model. However, in spite of my parameter-selection strategy, the results from the simulation exercises still suggest that the lending channel has an important effect on macroeconomic dynamics.

2 A Model of the Bank Lending Channel

In this section, I build a bank lending channel into the Bernanke et al. framework. I replace the simple financial intermediary in their model with a risk averse representative bank that solves an intertemporal optimization problem. Specifically, I model the bank as if it were operated by an agent called a banker who receives a flow of utility in each period that is a concave function of the bank's period profits. Each period, the banker maximizes the discounted sum of its expected lifetime period utilities by acquiring an asset portfolio using funds obtained by issuing reservable deposit liabilities. There are three assets available to the bank: loans to entrepreneurs, loans to other banks on the interbank market, and reserves held on account with the central bank.

In the model, the central bank requires the bank to hold a minimum quantity of reserves against its deposits. While the central bank does not pay interest on reserves, the bank still finds it useful to hold reserves in excess of the minimum required for three reasons. First, because the bank is risk averse, it uses excess reserves as a tool for managing the riskiness of its asset portfolio. When confronted with an exogenous increase in the volatility of the return on its loan portfolio, the bank increases the share of excess reserves in its asset portfolio. This reduces the volatility of the return on its overall asset portfolio but also reduces the expected return on the portfolio.

Second, the bank uses real resources to originate loans and by holding excess reserves, the bank reduces its marginal cost of lending. In practice, excess reserve holdings keep bank balance sheets liquid at the margin. Some banks hold excess reserves to more easily accommodate unanticipated deposit outflows (Kashyap and Stein 1995)[2]. I avoid unnecessary

[2]This motive for holding excess reserves is quantitatively weak given the highly liquid markets for T-bills and other money market instruments. Between January 1959 and August 2008, the ratio of excess reserves held by U.S. depository institutions to the demand deposit component of M1 was 0.0031. Data Source: FRED, Federal Reserve Economic Data, Federal Reserve Bank of St. Louis: Currency Component of M1 [CURRSL] ; Board of Governors of the Federal Reserve System: H.6 Money Stock Measures; http://research.stlouisfed.org/fred2/series/CURRSL; Excess Reserves of Depository Institutions [EXCRESNS] ; Board of Governors of the Federal Reserve System: H.3 Aggregate Reserves of Depository Institutions and the Monetary Base; http://research.stlouisfed.org/fred2/series/EXCRESNS; accessed August 13, 2010.

complication by not explicitly modeling intra-period deposit fluctuations. Instead, I assume that excess reserves, along with labor, are arguments in a production function for loans; giving the bank incentive to hold excess reserves in the nonstochastic steady state. This is consistent with the long-run behavior of the banking system and enables me to approximate the model around a steady state with positive excess reserves.

Third, the banker's optimization problem gives rise to a motive for intertemporal "return-smoothing." In a given period, the bank's profit is determined by the net return from the portfolio it originated in the previous period less the cost of originating new loans that will mature in the following period. If the return on the bank's portfolio from the previous period is unexpectedly low, the banker holds excess reserves to reduce the costs of originating new loans. This also reduces the bank's period-ahead profits because excess reserves do not earn interest. In effect, the banker uses excess reserves to borrow against its future profits.

The bank's loan portfolio is a collection of loans to individual entrepreneurs. The bank writes a unique loan contract for each entrepreneur. A contract is characterized by a nominal principal and a fixed nominal repayment rate. Each entrepreneur uses its borrowed funds to invest in a capital project. Individual capital projects earn a nominal return that is subject to both idiosyncratic and systematic variation. If an entrepreneur cannot fully repay its loan, it defaults and the bank recovers the entrepreneur's capital project minus an auditing cost. Since the ability of an entrepreneur to repay its loan depends on the ex post realization of the aggregate state, the ex post real return on the bank's portfolio of loans is open to aggregate risk.

Bernanke et al. study a different loan contracting environment that permits the financial intermediary to escape aggregate risk. The intermediary in their model writes state-contingent loan contracts that shift all aggregate risk onto entrepreneurs. Allowing for state-contingent contracts simplifies the solution to the loan contracting problem but abstracts from an important characteristic of the practical banking environment. Banks cannot perfectly shield their asset returns from fluctuations in the aggregate cycle and I have adopted a modeling strategy to emphasize this.

The bank that I model acquires funds by accepting deposits from the household. In practice, banks do not choose the quantity of deposits that they accept. Instead, they set a deposit rate and receive any deposits that are forthcoming. Banks indirectly influence the amount of deposits they receive by manipulating the deposit rate. Even so, the practical banker cannot claim to control the quantity of deposits on its balance sheet.

In contrast to the practical banking environment, the bank that I model never confronts unexpected intra-period deposit withdrawals. Deposits have the same maturity as bank loans and so the bank easily adjusts its asset holdings in response to fluctuations in deposit availability. Furthermore, the bank understands the structure of the model economy and knows with certainty the quantity of deposits it will attract at each deposit rate. Because of this, I solve the bank's problem as if it chooses deposits directly, but the results would be unchanged if I gave the bank control over the deposit rate instead.

Finally, I introduce a market for interbank loans. Each period, the bank chooses a net position on the interbank market. Clearing requires that the net positions of all banks sum to zero: the banking system cannot be a net borrower or lender to itself. Since I model the decisions of a representative bank, the interbank market clears only if interbank lending does not occur in equilibrium. Nonetheless, I can still define and characterize an equilibrium interbank lending rate[3].

The next three subsections below describe the novel components of my model environment. The remaining subsections contain components of the model that are either identical or similar to components of the Bernanke et al. model. I describe the Bernanke et al. model in Appendix B for comparison.

[3] In the present paper, I will follow Bernanke et al. and adopt the conventional assumption that the central bank's policy instrument is the one-period nominal risk-free rate determined by the household's consumption Euler equation. But historically, the instrument of monetary policy in the U.S. has been the overnight interbank rate. In the future, it may be worthwhile to compare the effectiveness of monetary policy in the model under alternative assumptions about which rate is the policy instrument.

2.1 Banking

At the end of period t, a representative bank chooses a nominal amount B_{t+1} to lend to entrepreneurs, a quantity of deposits D_{t+1} to accept from households, a net position on the interbank lending market B^{ff}_{t+1}, and total reserve holdings M_{t+1}. The bank's balance sheet constraint going into period $t+1$ is:

$$B_{t+1} + B^{ff}_{t+1} + M_{t+1} = D_{t+1}. \tag{1}$$

The bank is required to hold a minimum amount of reserves M^{req}_{t+1} on account with the central bank:

$$M^{req}_{t+1} = \rho D_{t+1}, \tag{2}$$

where ρ is the required reserve ratio. The supply of central bank reserves M_{t+1} must satisfy:

$$M_{t+1} = M^{req}_{t+1} + M^{ex}_{t+1}, \tag{3}$$

where M^{ex}_{t+1} is the bank's excess reserve holdings.

The bank employs household labor and uses excess reserves to originate a loan portfolio. Let $H_{\text{B},t}$ be the labor employed by the bank. For a given level of excess reserve holdings, the amount of labor required to originate the real portfolio B_{t+1}/P_t is determined by:

$$\frac{B_{t+1}}{P_t} = \mathbb{Z}_{\text{B}} \times (H_{\text{B},t})^{\alpha_{\text{B}}} \left[\frac{M^{ex}_{t+1}}{P_t} \right]^{\gamma_{\text{B}}}, \tag{4}$$

where $\mathbb{Z}_{\text{B}} > 0$ is a scalar, $\alpha_{\text{B}}, \gamma_{\text{B}} \in [0, 1)$, and P_t is the nominal price level of the final output good Y^f_t defined below. In the present model, all financial instruments have a one-period maturity. Equation (4) captures the bank's motive for holding excess reserves to buffer against deposit withdrawals. By including household labor in the production function, I ensure that the bank's excess reserve holdings are chosen optimally and are not simply set proportionally to the size of the loan portfolio.

For a given real wage W_t, (4) implies a cost function for producing loans B_{t+1}/P_t:

$$\mathbb{C}\left(\frac{B_{t+1}}{P_t}, \frac{M^{ex}_{t+1}}{P_t}, W_t\right) = W_t \left[\frac{1}{\mathbb{Z}_{\mathrm{B}}} \frac{B_{t+1}}{P_t}\right]^{\frac{1}{\alpha_{\mathrm{B}}}} \times \left[\frac{M^{ex}_{t+1}}{P_t}\right]^{-\frac{\gamma_{\mathrm{B}}}{\alpha_{\mathrm{B}}}}. \tag{5}$$

As the cost function indicates, increasing excess reserve holdings reduces both the total cost and marginal cost of lending. The cost of lending is increasing in the real wage.

Let Φ_t denote the bank's real period t profit:

$$\Phi_t \equiv \frac{1}{1+\pi_t}\left[R^B_t \frac{B_t}{P_{t-1}} + R^{ff}_t \frac{B^{ff}_t}{P_{t-1}} + \frac{M_t}{P_{t-1}} - R^D_t \frac{D_t}{P_{t-1}}\right] - \mathbb{C}\left(\frac{B_{t+1}}{P_t}, \frac{M^{ex}_{t+1}}{P_t}, W_t\right), \tag{6}$$

where R^B_t is the stochastic gross nominal return on the loan portfolio with face value B_t, R^D_t is gross the nominal deposit rate and R^{ff}_t is the gross rate on interbank loans.

The bank is operated by a risk-averse member of the household called a banker[4]. The banker has concave preferences over real period profits denoted by $\tilde{u}(\cdot)$. The banker maximizes the present discounted value of its expected utility choosing B_{t+1}/P_t, B^{ff}_{t+1}/P_t, M_{t+1}/P_t, M^{ex}_{t+1}/P_t, and D_{t+1}/P_t to solve:

$$\max E_t \sum_{s=0}^{\infty} \beta^s \tilde{u}\left(\Phi_{t+s}\right). \tag{7}$$

For convenience, I suppose that the banker's subjective discount factor β is the same as the rest of the household's. I assume the following functional form for the banker's preferences: $\tilde{u}(\Phi) = -\exp(\xi_1 - \xi_0 \Phi)$ where ξ_0 is a positive constant and ξ_1 is a real-valued constant. Under this specification, the banker's preferences are defined over all real profit realizations. The bank may receive negative profits in a period because I assume that the household absorbs any loss the bank incurs in a period where the return on its asset portfolio is inadequate to meet the bank's deposit liabilities and loan origination costs. This allows me abstract

[4]This assumption is appropriate if the banker's consumption is correlated with the performance of the bank. This might arise in practice if the banker's period compensation is tied to the bank's period profits.

from the details of bank failure and specify the return on deposits as being risk-free without explicitly modeling intermediated deposit insurance.

The bank solves (7) subject to (1), (2), and (3). After substituting the constraints into (6), I define $\bar{\Phi}_t$ as the bank's profit in terms of B_{t+1}/P_t, D_{t+1}/P_t, and M^{ex}_{t+1}/P_t only:

$$\bar{\Phi}_t \equiv \frac{1}{1+\pi_t}\left[\left(R^B_t - R^{ff}_t\right)\frac{B_t}{P_{t-1}} + \left(1 - R^{ff}_t\right)\frac{M^{ex}_t}{P_{t-1}} + \left(R^{ff}_t(1-\rho) - R^D_t + \rho\right)\frac{D_t}{P_{t-1}}\right] - \mathbb{C}\left(\frac{B_{t+1}}{P_t}, \frac{M^{ex}_{t+1}}{P_t}, W_t\right). \quad (8)$$

Given this, the bank's first-order conditions for M^{ex}_{t+1}/P_t, B_{t+1}/P_t, and D_{t+1}/P_t are:

$$\beta E_t\left\{\frac{1 - R^{ff}_{t+1}}{1+\pi_{t+1}}\tilde{u}'\left(\bar{\Phi}_{t+1}\right)\right\} = \tilde{u}'\left(\bar{\Phi}_t\right)\mathbb{C}_M\left(\frac{B_{t+1}}{P_t}, \frac{M^{ex}_{t+1}}{P_t}, W_t\right), \quad (9)$$

$$\beta E_t\left\{\frac{R^B_{t+1} - R^{ff}_{t+1}}{1+\pi_{t+1}}\tilde{u}'\left(\bar{\Phi}_{t+1}\right)\right\} = \tilde{u}'\left(\bar{\Phi}_t\right)\mathbb{C}_B\left(\frac{B_{t+1}}{P_t}, \frac{M^{ex}_{t+1}}{P_t}, W_t\right), \quad (10)$$

$$R^{ff}_{t+1}(1-\rho) = R^D_{t+1} - \rho, \quad (11)$$

where $\mathbb{C}_M(\cdot)$ denotes the first partial derivative of $\mathbb{C}(\cdot)$ with respect to M^{ex}/P and $\mathbb{C}_B(\cdot)$ denotes the first partial derivative of $\mathbb{C}(\cdot)$ with respect to B/P.

Equation (9) is the banker's first-order condition for holding excess reserves. On the left side is the expected marginal cost to the banker in period $t+1$ of holding excess reserves discounted back to period t. Each excess reserve earns the bank a gross nominal return of 1, so $(1 - R^{ff}_{t+1})$ is the net nominal return on each additional reserve. The right-hand side reflects the marginal benefit in period t of carrying excess reserves into $t+1$. Holding excess reserves reduces the cost of producing a given quantity of loans so $\mathbb{C}_M(\cdot) < 0$ for all $M^{ex}/P < 0$.

Equation (10) equates the banker's discounted expected net marginal benefit in $t+1$ from lending in period t with the marginal cost in period t of originating loans. The loan production function (4) implies that the bank faces a rising marginal cost of loan production so that $\mathbb{C}_B(\cdot) > 0$ for all $B/P < 0$. Equation (11) requires that the return on interbank loans be proportional to the deposit rate.

The risk averse banker can use excess reserves as a tool for managing risk. Equations (9) and (10) prescribe the optimal allocation of the bank's non-required reserve assets between excess reserves and loans. If sufficiently negative returns on loans are possible or if negative returns on loans are sufficiently probable, then holding excess reserves would allow the risk averse banker to reduce the risk on its asset portfolio. This is a potentially important consequence of the model. It suggests that banks may respond to increased volatility on loan returns by increasing their excess reserve holdings. However, when I simulate the model below, I use linear approximations of the model's equilibrium conditions and this particular mechanism drops out[5].

Equations (9) and (10) capture an additional consequence of the banker's concave utility. The bank incurs a cost in period t to originate a loan portfolio that matures in period $t+1$. The banker faces an intertemporal substitution problem compelling it to smooth the path of its marginal utility. This is particularly important if the bank receives a negative shock to the return on its loan portfolio maturing in period t. The banker perceives this as an adverse shock to its period t profits and shifts its portfolio going into period $t+1$ away from loans and into excess reserves. This raises period t profits by reducing the real cost of lending. This mechanism is preserved in the linear approximation below.

Now, denote the banker's marginal utility as: $\Lambda_t^{\mathrm{B}} \equiv \tilde{u}'\left(\bar{\Phi}_t\right)$. Conditions (10) and (11) can be combined to yield:

$$E_t\left\{\frac{R_{t+1}^B \Lambda_{t+1}^{\mathrm{B}}}{1+\pi_{t+1}}\right\} = E_t\left\{\frac{\Lambda_{t+1}^{\mathrm{B}}}{1+\pi_{t+1}}\right\}\left(\frac{R_{t+1}^D-\rho}{1-\rho}\right) + \beta^{-1}\Lambda_t^{\mathrm{B}} \cdot \mathbb{C}_B\left(\frac{B_{t+1}}{P_t}, \frac{M_{t+1}^{ex}}{P_t}, W_t\right). \quad (12)$$

[5]This particular motive for holding excess reserves relies on the concavity of the banker's utility function. As (9) and (10) suggest, the banker uses excess reserves as one way to manipulate the expected values of two nonlinear functions of random variables. The presence of these nonlinear functions means that second and higher-order moments of the return to lending matter in principle for the bank's decision. But the linear approximation replaces the nonlinear expressions in (9) and (10) with linear functions of model variables so that only the first moment of loan returns affects dynamics in the linear model. In my future work, it will be worthwhile to use a higher-order approximation method to simulate the model so that the risk aversion mechanism remains in tact.

The right-hand side of (12) is the expected cost in period $t+1$ of originating a loan portfolio in period t and the left-hand side is the expected return. In the next subsection, I use this equation to form the constraint on the expected return that the bank will require on each loan contract that it originates.

Together, (9), (10), and (11) reflect the bank's optimality conditions for lending, excess reserve-holding, and deposit-taking. It is instructive to examine the implications of these equations for the non-stochastic steady state. Letting letters without subscripts denote the steady state values of the respective variables, (10), (9), and (11) imply:

$$\beta(R^B - R^{ff}) = \mathbb{C}_B\left(\frac{B}{P}, \frac{M^{ex}}{P}, W\right), \tag{13}$$

$$\beta(1 - R^{ff}) = \mathbb{C}_M\left(\frac{B}{P}, \frac{M^{ex}}{P}, W\right), \tag{14}$$

$$R^{ff}(1-\rho) = R^D - \rho. \tag{15}$$

Holding more excess reserves reduces the real cost of producing a given amount of loans. From equation (14), the steady state return on interbank loans is linked to the steady state marginal reduction in lending costs that excess reserves provide. Without this, the bank would only hold excess reserves in the steady state if $R^{ff} = 1$. But in practice, banks hold a small amount of excess reserves on average and the average gross interbank rate exceeds one. Therefore, I require that:

$$\mathbb{C}_M < 0, \tag{16}$$

in the steady state since I am particularly interested in studying how excess reserve holdings fluctuate endogenously with the business cycle.

2.2 The demand for capital

The loan contracting environment that I study is different from the environment in Bernanke et al. in two important ways. First, I assume that a loan contract specifies the nominal repayment rate in advance. This forces the bank to bear aggregate risk associated with unpredictable fluctuations in inflation and the aggregate return on capital. Second, I model stochastic volatility in the distribution of capital returns across borrowers. This is a tool for introducing an exogenous shock to the share of loans in the bank's portfolio that default and allows me to examine how a shock to the return on assets in the banking system is transmitted to the aggregate economy.

At the end of period t, an entrepreneur j has accumulated real net worth N_{t+1}^j that it uses to purchase capital K_{t+1}^j at a real price of Q_t. Both N_{t+1}^j and Q_t are measured in terms of the final output good. Capital purchases in excess of net worth are financed by a nominal bank loan B_{t+1}^j:

$$B_{t+1}^j = P_t \left(Q_t K_{t+1}^j - N_{t+1}^j \right). \tag{17}$$

The ex post gross return to the entrepreneur's capital is $\omega_{t+1}^j R_{t+1}^K$, where R_{t+1}^K is the aggregate return to capital and ω_{t+1}^j is an idiosyncratic disturbance that scales jth entrepreneur's capital return relative to the aggregate return. I describe how R_{t+1}^K is determined in the next subsection. ω_{t+1}^j is i.i.d. across entrepreneurs and time with a log-normal distribution:

$$\omega_{t+1}^j \sim \text{log-}\mathcal{N}\left(-\frac{\sigma_{\omega,t+1}^2}{2}, \sigma_{\omega,t+1}^2 \right), \tag{18}$$

where $\sigma_{\omega,t+1}^2$ is a stationary, strictly positive stochastic process with mean σ_ω^2. Under this specification, $E_t(\omega_{t+1}^j | \sigma_{\omega,t+1}^2) = 1$[6]. I find it useful to anchor the conditional mean of ω_{t+1}^j at 1 so that – as in Bernanke et al. – the conditional distribution of idiosyncratic returns to capital is always centered around the mean $E_t R_{t+1}^K$.

The entrepreneur's demand for capital is determined by its net worth and the terms of its loan contract. A loan contract is characterized by a non-default nominal gross repayment

[6] A random variable X that follows a log-normal distribution with parameters τ and σ^2 has mean $E(X) = \exp(\tau)$ and variance $V(X) = \exp(\sigma^2 - 1) \cdot \exp(2\tau + \sigma^2)$. It follows that if $\tau = -\sigma^2/2$, then $E(X) = 1$ and $V(X) = \exp(\sigma^2 - 1)$.

rate $\bar{R}^j_{t+1}$ and a nominal loan amount B^j_{t+1}. Given ex post realizations of inflation and the aggregate return to capital, a stochastic default threshold can be defined as:

$$\bar{\omega}^j_{t+1} \equiv \frac{\bar{R}^j_{t+1}\left(Q_t K^j_{t+1} - N^j_{t+1}\right)}{(1+\pi_{t+1})R^K_{t+1} Q_t K^j_{t+1}}, \tag{19}$$

such that if $\omega^j_{t+1} \geqslant \bar{\omega}^j_{t+1}$, then the real return on the entrepreneur's capital project is sufficient for the entrepreneur to be able to repay its loan.

The variance of the idiosyncratic disturbance is realized after the loan contract is made. When I simulate the model, I suppose its square root evolves according to:

$$\log\left(\sigma_{\omega,t+1}/\sigma_\omega\right) = \rho_\sigma \log\left(\sigma_{\omega,t}/\sigma_\omega\right) + \varepsilon^\sigma_{t+1}, \tag{20}$$

where ε^σ_{t+1} is zero-mean i.i.d process. A positive realization of ε^σ_{t+1} increases the probability that ω^j_{t+1} will be realized less than the threshold $\bar{\omega}^j_{t+1}$ and so increases the likelihood that any entrepreneur j will default in period $t+1$. Accordingly, a positive shock to $\sigma^2_{\omega,t+1}$ causes an exogenous rise in the proportion of entrepreneurs that default on their loans. For this reason, I interpret a shock to ε^σ_{t+1} as an unanticipated shock to the return on lending or, more concisely, a financial shock[7][8].

A borrower in default surrenders the realized value of its investment project to the bank, but the bank incurs an auditing cost when it takes over the project. This cost is a fixed proportion μ of the realized value of the project in $t+1$. Therefore, the bank receives $(1-\mu)\omega^j_{t+1}R^K_{t+1}Q_t K^j_{t+1}$ from a project in default. The parameter μ reflects a deadweight loss

[7] Christiano, Motto and Rostagno (2003) also incorporate stochastic volatility in the distribution of the idiosyncratic shock to entrepreneurial returns. However, they assume that the volatility shock is realized *before* loan contracts are written and so the financial shock is not a source of unanticipated defaults.

[8] In the present model, the bank and entrepreneurs observe ex post realizations of the aggregate state with certainty so a positive shock to $\sigma^2_{\omega,t+1}$ only causes unanticipated defaults for a single period. In practice, banks do not necessarily observe what has caused a sudden shift in the proportion of their loans that default. As I continue to develop this work, I plan to restrict the bank from directly observing ex post realizations of $\sigma^2_{\omega,t+1}$ or R^K_{t+1} directly. Instead, it will instead receive a noisy signal that combines information about the two variables. The bank will then have to solve a simple signal extraction to uncover what component of the underlying state is driving the fluctuation in loan defaults. Presumably this modification will cause the aggregate effects of shocks to $\sigma^2_{\omega,t+1}$ more persistent as the bank gradually uncovers the true state of the economy.

associated with debt default and is an important source of financial friction in the model. In the special case where $\mu = 0$, the bank incurs no auditing cost and recovers the full realized value of all projects in default.

Now, for a given ex post realization of the aggregate state, the bank expects to receive in period $t+1$ from an entrepreneur j:

$$\left\{ \left[1 - F(\bar{\omega}^j_{t+1}|\sigma^2_{\omega,t+1})\right] \bar{\omega}^j_{t+1} + (1-\mu) \int_0^{\bar{\omega}^j_{t+1}} \omega dF(\omega|\sigma^2_{\omega,t+1}) \right\} R^K_{t+1} Q_t K^j_{t+1}. \tag{21}$$

Now, define:

$$\Gamma(\bar{\omega}^j_{t+1}|\sigma^2_{\omega,t+1}) \equiv \left[1 - F(\bar{\omega}^j_{t+1}|\sigma^2_{\omega,t+1})\right] \bar{\omega}^j_{t+1} + \int_0^{\bar{\omega}^j_{t+1}} \omega dF(\omega|\sigma^2_{\omega,t+1}), \tag{22}$$

and:

$$\mu G(\bar{\omega}^j_{t+1}|\sigma^2_{\omega,t+1}) \equiv \mu \int_0^{\bar{\omega}^j_{t+1}} \omega dF(\omega|\sigma^2_{\omega,t+1}). \tag{23}$$

Note that $\Gamma(\cdot|\cdot)$ is the expected share of the entrepreneur's capital project going to the bank and $G(\cdot|\cdot)$ is the expected cost of monitoring one unit of the entrepreneur's capital project. Observe that (21) can now be written concisely as:

$$\left[\Gamma(\bar{\omega}^j_{t+1}|\sigma^2_{\omega,t+1}) - \mu G(\bar{\omega}^j_{t+1}|\sigma^2_{\omega,t+1})\right] R^K_{t+1} Q_t K^j_{t+1}. \tag{24}$$

Now, I use (24) to define $R^{B,j}_{t+1}$ as the expected nominal return from lending to entrepreneur j conditional on aggregate realizations of R^K_{t+1}, π_{t+1}, and $\sigma^2_{\omega,t+1}$:

$$\frac{R^{B,j}_{t+1}}{1+\pi_{t+1}} = \left[\Gamma(\bar{\omega}^j_{t+1}|\sigma^2_{\omega,t+1}) - \mu G(\bar{\omega}^j_{t+1}|\sigma^2_{\omega,t+1})\right] R^K_{t+1} \frac{Q_t K^j_{t+1}}{Q_t K^j_{t+1} - N^j_{t+1}}. \tag{25}$$

The optimal contract with entrepreneur j must satisfy:

$$E_t \left\{ \Lambda^{\mathrm{B}}_{t+1} \frac{R^{B,j}_{t+1}}{1+\pi_{t+1}} \right\} = \Xi_t, \tag{26}$$

where Ξ_t is the cost of lending determined by the right-hand side of (12):

$$\Xi_t \equiv E_t \left\{ \frac{\Lambda^{\mathrm{B}}_{t+1}}{1+\pi_{t+1}} \right\} \left(\frac{R^D_{t+1} - \rho}{1-\rho} \right) + \beta^{-1} \Lambda^{\mathrm{B}}_t \cdot \mathbb{C}_B \left(\frac{B_{t+1}}{P_t}, \frac{M^{ex}_{t+1}}{P_t}, W_t \right). \tag{27}$$

Note that Ξ_t is treated as a constant in the contracting problem. Equation (26) is important because it shows how the banker's marginal utility is used to price risk in the loan contract. By combining (25) and (26), I obtain the appropriate constraint on the bank's return in the optimal loan contract:

$$E_t \left\{ \Lambda^{\mathrm{B}}_{t+1} \left[\Gamma(\bar{\omega}^j_{t+1} | \sigma^2_{\omega,t+1}) - \mu G(\bar{\omega}^j_{t+1} | \sigma^2_{\omega,t+1}) \right] R^k_{t+1} \right\} Q_t K^j_{t+1} = \Xi_t (Q_t K^j_{t+1} - N^j_{t+1}), \tag{28}$$

where the expectation is over R^K_{t+1}, π_{t+1}, and $\bar{\omega}^j_{t+1}$ given the information available at time t.

The optimal debt contract maximizes the expected return to the entrepreneur under the constraint that the expected return to the bank is determined by (26). While the bank chooses a loan amount B^j_{t+1} and a non-default repayment rate $\bar{R}^j_{t+1}$, I can solve the problem as if it directly chooses the entrepreneur's capital purchase K^j_{t+1}. Therefore, the contracting problem is:

$$\max_{K^j_{t+1}, \bar{R}^j_{t+1}} E_t \left\{ \left[1 - \Gamma(\bar{\omega}^j_{t+1} | \sigma^2_{\omega,t+1}) \right] R^K_{t+1} \right\} Q_t K^j_{t+1}, \tag{29}$$

subject to (19) and (28). Let λ^C_{t+1} be the multiplier on (28). It turns out that the solution to (29) implies that each entrepreneur receives the same loan rate $\bar{R}_{t+1}$ and a loan amount such that $Q_t K_{t+1}/N_{t+1}$ is identical across all entrepreneurs. So I can drop the entrepreneur-specific index j and write the first order conditions for (29) with respect to K_{t+1} and $\bar{R}_{t+1}$

as:

$$
\begin{aligned}
&E_t\Big\{ \left[1-\Gamma(\bar{\omega}_{t+1}|\sigma^2_{\omega,t+1})\right] R^K_{t+1}\Big\} - E_t\left\{\frac{\Gamma_\omega(\bar{\omega}_{t+1}|\sigma^2_{\omega,t+1})}{1+\pi_{t+1}}\right\}\bar{R}_{t+1}\frac{N_{t+1}}{Q_tK_{t+1}} \\
&\quad + \lambda^C_t E_t\left\{\Lambda^{\mathrm{B}}_{t+1}\left[\Gamma_\omega(\bar{\omega}_{t+1}|\sigma^2_{\omega,t+1}) - \mu G_\omega(\bar{\omega}_{t+1}|\sigma^2_{\omega,t+1})\right]\frac{1}{1+\pi_{t+1}}\right\}\bar{R}_{t+1}\frac{N_{t+1}}{Q_tK_{t+1}} \\
&\quad\quad + \lambda^C_t E_t\Big\{\Lambda^{\mathrm{B}}_{t+1}\left[\Gamma(\bar{\omega}_{t+1}|\sigma^2_{\omega,t+1}) - \mu G(\bar{\omega}_{t+1}|\sigma^2_{\omega,t+1})\right] R^K_{t+1}\Big\} \\
&\quad\quad\quad = \lambda^C_t \Xi_t,
\end{aligned} \tag{30}
$$

and:

$$
E_t\left\{\Gamma_\omega(\bar{\omega}_{t+1}|\sigma^2_{\omega,t+1})\frac{1}{1+\pi_{t+1}}\right\} = \lambda^C_t E_t\left\{\Lambda^{\mathrm{B}}_{t+1}\left[\Gamma_\omega(\bar{\omega}_{t+1}|\sigma^2_{\omega,t+1}) - \mu G_\omega(\bar{\omega}_{t+1}|\sigma^2_{\omega,t+1})\right]\frac{1}{1+\pi_{t+1}}\right\}. \tag{31}
$$

As condition (30) suggests, the quantity K_{t+1} in the loan contract has direct and indirect effects on the returns to the entrepreneur and lender. The first term on the left-hand side reflects the direct expected marginal benefit to the entrepreneur from being allocated an additional unit of capital while the fourth term captures the direct marginal benefit going to the bank.

Allocating more capital to the entrepreneur increases the threshold $\bar{\omega}_{t+1}$ by:

$$
\frac{\partial \bar{\omega}_{t+1}}{\partial K_{t+1}} = \frac{\bar{R}_{t+1}}{R^K_{t+1}(1+\pi_{t+1})K_{t+1}}\frac{N_{t+1}}{Q_tK_{t+1}}. \tag{32}
$$

The proportion of borrowers that default is increasing in $\bar{\omega}_{t+1}$ and so the second and third terms on the left-hand side of (30) reflect the indirect marginal cost to the entrepreneur and bank that arises from an increase in K_{t+1}.

Next, (31) reflects how $\bar{R}_{t+1}$ affects the returns of the entrepreneur and bank. The threshold $\bar{\omega}_{t+1}$ is also increasing in $\bar{R}_{t+1}$ and the left-hand side reflects the marginal effect of this on the entrepreneur's return. The right-hand side reflects the marginal effect of the additional loan defaults on the bank's return.

Equations (19), (28), (30), and (31) characterize the entrepreneur's demand for capital given the terms of the optimal loan contract.

2.3 The entrepreneurial sector and net worth

In this section I describe the behavior of the entrepreneurial sector. Most of this section mirrors the exposition in Bernanke, et al. However, I obtain a different expression for the law of motion of entrepreneurial net worth because of the adjustments that I have made to structure of the contracting environment. The entrepreneurial sector enters period t with capital K_t. At the beginning of the period, entrepreneurs hire labor from a competitive labor market to combine with capital to produce the wholesale good Y_t. The aggregate output of the entrepreneurial sector is:

$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha}. \tag{33}$$

where Z_t is an exogenous aggregate technology process[9]. Note that Z_t is a distinct process from the idiosyncratic disturbance to each entrepreneur's return to capital. After production, the entrepreneurial sector sells its wholesale output to the retail sector at a real price of $1/X_t$ per unit. During the production process, a fraction δ of the capital stock depreciates. The remaining capital is sold to the capital-producing sector – discussed in the next section – at Q_t per unit. The aggregate return to capital – after depreciation – is:

$$R_t^K = \frac{\frac{1}{X_t}\frac{\alpha Y_t}{K_t} + Q_t(1-\delta)}{Q_{t-1}}, \tag{34}$$

[9]The production function for an entrepreneur is

$$Y_t^j = Z_t \left(K_t^j\right)^{\alpha} \left(L_t^j\right)^{1-\alpha},$$

where L_t^j and Y_t^j are the labor input and output of the jth entrepreneur. Entrepreneurs pay the same wages to labor and so each chooses the same capital-to-labor ratio. Write the entrepreneur's production function as:

$$Y_t^j = Z_t \left(\frac{K_t}{L_t}\right)^{\alpha} L_t^j,$$

and integrate this expression over j to obtain (33).

where:

$$\frac{1}{X_t}\frac{\alpha Y_t}{K_t}. \tag{35}$$

is the marginal product of capital in terms of the final output good. Together equations (33) and (34) show how exogenous fluctuations in aggregate productivity Z_t drive fluctuations in the aggregate return to capital R_t^K.

In the production function (33), L_t is a composite of household labor H_t and entrepreneurial labor H_t^e:

$$L_t = H_t^{\Omega}\left(H_t^e\right)^{1-\Omega}. \tag{36}$$

The labor inputs are each paid their marginal products. The real household wage W_t must satisfy:

$$(1-\alpha)\Omega\frac{Y_t}{H_t X_t} = W_t, \tag{37}$$

while the entrepreneurial wage W_t^e satisfies:

$$(1-\alpha)(1-\Omega)\frac{Y_t}{H_t^e X_t} = W_t^e. \tag{38}$$

Next, it is essential that entrepreneurs be prevented from accumulating sufficient wealth to become self-financing. To ensure this, I take the same approach as Bernanke et al. Each period, after all business between entrepreneurs and the bank has been settled, an exogenous fraction $1-\gamma$ of randomly selected entrepreneurs close their firms, consume their accumulated wealth, and exit the model. Each departing entrepreneur is replaced by a new entrepreneur with no accumulated wealth.

Let V_t denote the equity accumulated by entrepreneurs immediately after concluding their relationship with the intermediary at the beginning of period t. From (22), V_t can be expressed as:

$$V_t = \left[1 - \Gamma\left(\bar{\omega}_t|\sigma^2_{\omega,t}\right)\right] R^K_t Q_{t-1} K_t, \tag{39}$$

where $\Gamma(\cdot|\cdot)$ is the share entrepreneurial capital income that is transferred to the bank. Then, at the end of period t, the accumulated net worth of the entrepreneurial sector is:

$$N_{t+1} = \gamma V_t + W^e_t, \tag{40}$$

where W^e_t is the wage income under the assumption that each entrepreneur inelastically supplies a single unit of labor for production. Now it is straightforward to characterize the evolution of entrepreneurial net worth by combining equations (38), (39), and (40):

$$N_{t+1} = \gamma\left[1 - \Gamma(\bar{\omega}_t|\sigma^2_{\omega,t})\right] R^K_t Q_{t-1} K_t + (1-\alpha)(1-\Omega) Z_t K^\alpha_t H^{(1-\alpha)\Omega}_t / X_t. \tag{41}$$

Recall that $\Gamma(\cdot|\cdot)$ is the share of entrepreneurial income going to the bank so $1 - \Gamma(\cdot|\cdot)$ is simply the share that the entrepreneurs are able to keep. Bernanke et al. find a value of γ that is greater than 0.9 in their calibration. Since $\bar{\omega}_t$ is a function of N_t, (41) indicates that fluctuates in net are highly persistent.

Finally, entrepreneurial consumption C^e_t is given as:

$$C^e_t = (1-\gamma) V_t. \tag{42}$$

2.4 The household

Each period, the household consumes, supplies labor, and supplies deposits to the bank. The household's preferences are represented by:

$$E_t \sum_{s=0}^{\infty} \beta^s \left\{ \frac{C^{1-\sigma_{\text{C}}}_{t+s}}{1-\sigma_{\text{C}}} + \zeta_{\text{H}} \frac{(1-H_{t+s})^{1-\eta}}{1-\eta} + \zeta_{\text{D}} \frac{(D_{t+1+s}/P_{t+s})^{1-\sigma_{\text{D}}}}{1-\sigma_{\text{D}}} \right\}, \tag{43}$$

where $\beta \in (0,1)$ is the subjective discount factor and $\sigma_{\rm C}$, $\sigma_{\rm D}$, η, $\zeta_{\rm H}$, and $\zeta_{\rm D}$ are positive constants. C_t and H_t represent household consumption and labor. Like Canzoneri et al. (2008), I incorporate real bank deposits D_{t+1}/P_t directly into the utility function to reflect the transaction services that deposits provide.

The household's period budget constraint is essentially the same as the one presented in Bernanke et al.:

$$C_t + \frac{D_{t+1}}{P_t} = W_t H_t - T_t + \Pi_t + \frac{R_t^D D_t}{(1+\pi_t)P_{t-1}}, \tag{44}$$

The household's first-order conditions are:

$$\zeta_{\rm H}(1-H_t)^{-\eta} = C_t^{-\sigma_C} W_t, \tag{45}$$

$$\left(\frac{D_{t+1}}{P_t}\right) = \left[\zeta_{\rm D}^{-1} C_t^{-\sigma_{\rm C}} \left(\frac{R_{t+1}^n - R_{t+1}^D}{R_{t+1}^n}\right)\right]^{-\frac{1}{\sigma_{\rm D}}}, \tag{46}$$

where:

$$\frac{1}{R_{t+1}^n} = \beta E_t \left\{ \frac{C_{t+1}^{-\sigma_{\rm C}}}{C_t^{-\sigma_{\rm C}}} \frac{1}{1+\pi_{t+1}} \right\}. \tag{47}$$

Equation (46) is a deposit supply expression. It is analogous to a conventional money demand equation derived within a money-in-the-utility function model. This is a potentially important component of the bank lending channel. Since deposits provide the household with transaction services, the bank can borrow from the household at a rate below the risk-free nominal rate.

This concludes the exposition of the novel components of the model environment. The remainder of the economic structure is essentially the same as that described in Bernanke, et al.

2.5 Capital production

Capital is produced during the period by a competitive capital-producing firm. Immediately following production in period t, the firm buys the entire capital stock K_t from the entrepreneurs and combines it with some of the final output good I_t to produce new capital K_{t+1} that is sold back to the entrepreneurs. Capital accumulates subject to a convex adjustment cost. Adjustment costs induce variability in the price of capital and entrepreneurial net worth. Assuming that entrepreneurs repurchase the entire capital stock each period allows capital adjustment costs to be considered separately from the entrepreneur's financial problem.

The capital-producer solves

$$\max_{K_t, I_t} Q_t K_{t+1} - I_t - \bar{Q}_t (1-\delta) K_t, \tag{48}$$

subject to:

$$K_{t+1} = \Phi \left(\frac{I_t}{K_t} \right) K_t + (1-\delta) K_t, \tag{49}$$

where $\bar{Q}_t$ is the price of capital in period t after production, but before new capital has been produced. As (49) suggests, investment I_t in period t results in only $\Phi(I_t/K_t)K_t$ units of period $t+1$ capital. $\Phi(\cdot)$ is increasing and concave with $\Phi(0) = 0$. The first-order conditions for I_t and K_t are:

$$Q_t = \Phi' \left(\frac{I_t}{K_t} \right), \tag{50}$$

$$\bar{Q}_t (1-\delta) = Q_t \Phi \left(\frac{I_t}{K_t} \right) + Q_t (1-\delta) - \frac{I_t}{K_t}. \tag{51}$$

Bernanke et al. assume that $Q = 1$ in the steady state. They do not explicitly discuss the functional form of $\Phi(\cdot)$, but for completeness, I assume the following form for $\Phi(\cdot)$:

$$\Phi\left(\frac{I_t}{K_t}\right) \equiv \frac{1}{1+\varphi}\left(\frac{I_t}{K_t}\right)^{1+\varphi}\left(\frac{\bar{K}}{\bar{I}}\right)^{\varphi}, \tag{52}$$

where $\bar{K}$ and $\bar{I}$ are the steady-state values of capital and investment and $\varphi < 0$. Since $Q = 1$ in the steady state, the difference between Q_t and $\bar{Q}_t$ is of second-order consequence and so I follow Bernanke et al. and set $\bar{Q}_t = Q_t$. It is straightforward to show that with the assumed form of $\Phi(\cdot)$, φ is the elasticity of the steady state capital price Q with respect to the steady state investment to capital ratio.

2.6 Retail goods, final output, and price setting

The retail sector comprises a continuum of firms that purchase wholesale goods from the entrepreneurial sector and produce retail goods by costlessly differentiating the wholesale output. Retailers are monopolistically competitive and set the prices of their products according to the familiar Calvo (1983) mechanism. Bernanke et al. introduce retailers into the supply chain specifically to separate the price setting decision from the entrepreneurs' financial problem. The final good producer purchases the retail goods and produces the final output good using a CES aggregation technology.

The final output good Y_t^f is a Dixit-Stiglitz aggregate of retail goods:

$$Y_t^f = \left(\int_0^1 Y_t(i)^{(\epsilon-1)/\epsilon}\, di\right)^{\epsilon/(\epsilon-1)}, \tag{53}$$

where $Y_t(i)$ is the retail output from retailer i in terms of the wholesale good Y_t and $\epsilon > 1$ is the elasticity of substitution among the retail goods. The demand for each retail good is obtained by solving for the minimum cost combination of retail goods to produce a given quantity of the final good:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t^f, \tag{54}$$

where $P_t(i)$ is the price of good i and:

$$P_t = \left(\int_0^1 P_t(i)^{(1-\epsilon)} \, di \right)^{1/(1-\epsilon)}, \tag{55}$$

is the nominal price index of the final good.

Retailers set their prices optimally subject to the familiar Calvo (1983) price-setting mechanism. In period t a fraction $1-\chi$ of retailers are allowed to set the price of their good before any period t shocks are realized and taking the price of wholesale goods P_t^w as given. This means that inflation between period t and $t+1$ is determined by the end of period t. Recall that $1/X_t$ is the real price of a wholesale good so:

$$P_t^w \equiv \frac{P_t}{X_t}. \tag{56}$$

All retailers optimizing in period t choose P_t^* to solve:

$$\sum_{k=0}^{\infty} \chi^k E_t \left\{ \Delta_{t,t+k}^{real} \frac{P_t^* - P_{t+k}^w}{P_{t+k}} Y_{t+k}^*(i) \right\}, \tag{57}$$

where $\Delta_{t,t+k}^{real} \equiv \beta^k C_t / C_{t+k}$ is the relevant discount factor and $Y_{t+k}^*(i)$ is the quantity of retail good i demanded in period $t+k$[10]. The first-order condition for optimizing (57) is:

$$\sum_{k=0}^{\infty} \chi^k E_t \left\{ \Delta_{t,t+k}^{real} \left(\frac{P^*}{P_{t+k}} \right)^{-\epsilon} Y_{t+k}^*(i) \left[\frac{P_t^*}{P_{t+k}} - \left(\frac{\epsilon}{\epsilon - 1} \right) \frac{P_{t+k}^w}{P_{t+k}} \right] \right\} = 0. \tag{58}$$

Finally, from (55) it follows that under the assumed pricing mechanism, the law of motion for the price level is:

$$P_t = \left[\chi P_{t-1}^{1-\epsilon} + (1-\chi) \left(P_t^* \right)^{1-\epsilon} \right]^{1/(1-\epsilon)} \tag{59}$$

Equations (58) and (59) can be log-linearized around a zero-inflation steady state to obtain a forward-looking new-Keynesian Phillips curve.

[10] Bernanke et al. actually assume that the expectation in (57) is given information at $t-1$ so that inflation between t and $t+1$ is predetermined with respect to period t.

2.7 Government

The government comprises a fiscal authority and a central bank. It finances an exogenous stream of real purchases G_t by collecting lump-sum taxes T_t and issuing nominal reserves M_t. The government budget constraint is:

$$G_t = \frac{M_{t+1} - M_t}{P_t} + T_t. \tag{60}$$

I assume that monetary policy is set according to a feedback rule for the nominal interest rate:

$$\hat{r}^n_{t+1} = \rho_r \hat{r}^n_t + \varsigma_\pi \hat{\pi}_t + \varsigma_y \hat{y}_t + v^{ff}_t, \tag{61}$$

where v^{ff}_t is an exogenous monetary policy shock process and $\hat{r}^n_t$, $\hat{y}_t$, and $\hat{\pi}_t$ reflect the log-deviations of R^n_t, Y_t, and $1 + \pi_t$ from their steady state values.

2.8 Market clearing

The aggregate resource constraint is the same as that from Bernanke et al.:

$$Y^f_t = C_t + C^e_t + I_t + G_t + \mu R^K_t Q_{t-1} K_t G(\bar{\omega}_t | \sigma^2_{\omega,t}). \tag{62}$$

And finally, clearing in the interbank market implies:

$$B^{ff}_{t+1} = 0. \tag{63}$$

The last expression follows because the net positions of banks on the interbank market must offset each other.

3 Equilibrium Dynamics in the Lending Channel Model

In this section, I examine the dynamic properties of the lending channel model. First, I log-linearize the model around a deterministic, zero-inflation steady state. I report the linearized equilibrium conditions in Appendix A. Next, I select values for the model parameters and simulate the dynamic responses of model variables to several exogenous shocks. Finally, I compare the proposed bank lending channel model to the Bernanke et al. model and a baseline model with no financial friction.

3.1 Calibration

I use parameter values from Bernanke et al. where my model overlaps with theirs. For the parameters that are unique to the lending channel model that I have proposed, I choose values that I consider plausible. As I move this work forward, it will be essential to rigorously calibrate the model parameters so that I can attach quantitative meaning to the simulated results.

I set the mean of the technology process Z to 1.32 in order to normalize steady state output to 1. I set the share of capital in production α to 0.35 and take Ω to be 0.9846 so that the share of household labor in production is 0.64. I set β to be 0.99 so that the steady state real rate is 1.0101. I choose the elasticity of substitution across retail goods ϵ to be 5 so that monopolistic competition induces a 25 percent steady state markup of retail prices over marginal costs. I set the utility parameters σ_{C}, η, and ξ to 1. I assume that capital depreciates at a quarterly rate of 2.5 percent, that the ratio of government consumption to real GDP is 20 percent, and that φ, the elasticity of the price of capital with respect to the steady state investment to capital ratio, is -0.25. Finally, I set the Calvo parameter χ to 0.75 so that the price of a retail good is fixed for one year on average.

Following Bernanke et al., I choose financial parameters to obtain the following three steady state characteristics: first, a quarterly difference between the steady state return on capital R^K and the borrowing rate R^B to 0.005; second, a 3 percent loan default rate $F(\bar{\omega})$; and third, a capital to net worth ratio of 2. Accordingly, for the entrepreneurial survival rate γ, I select 0.9788. I assume that ω is log-normally distributed with variance 0.0764.

Regarding the novel parameters of the model, I assume a 10 percent reserve requirement, a gross return on deposits equal to 1.005, and I set the difference between the steady state return on the bank's portfolio and the interbank rate to 0.01. I set the household utility parameters $\sigma_{\mathrm{D}} = 2$ and $\zeta_{\mathrm{D}} = 0.0545$. I set M^{ex}/D equal to 0.005 which is close to the average ratio of excess reserves to demand deposits in the U.S. from January 1959 to August 2008. I set the preference parameters in the banker's utility function as $\xi_0 = 1.125$ and $\xi_1 = -0.1079$ And finally, I set the loan production parameters α_{B} to 0.65 and γ_{B} to 0.0025.

3.2 Simulated impulse responses

With the parameter values from above, I solve the linear model using Klein's (2000) method. I compute impulse responses of the model variables to an aggregate productivity shock, a monetary policy shock, and a financial shock. For comparison, I also compute and report impulse responses from two other models. The first of the comparison models is the Bernanke et al. model and I identify the impulse responses by the abbreviation 'BGG' in the figures. The second is a baseline model – abbreviated 'BL' in the figures – that I obtain by 'turning off' the financial frictions in BGG. This is done by setting the loan monitoring cost parameter $\mu = 0$ to remove the financial friction in BGG. The baseline is representative of the New Neoclassical Synthesis (NNS) class of models[11]. The Bernanke et al. model nests the baseline model as a special case.

It is not appropriate to consider the Bernanke et al. model as a special case of the lending channel model. The underlying structure of the lending channel model has several features that make the model distinct from Bernanke et al. First, the lending channel model embeds

[11] Goodfriend and King (1997) define the New Neoclassical Synthesis (NNS) as an approach to modeling the business cycle that draws from both real-business-cycle (RBC) analysis and New Keynesian (NK) macroeconomics. NNS models incorporate imperfect competition and nominal frictions into business cycle models founded on intertemporal optimization and rational expectations.

a model of loan contracting that forces the bank to bear aggregate risk. Second, the bank in the lending channel model solves a more complex intertemporal optimization problem than the nonbank intermediary in Bernanke et al. Third, the bank also faces an increasing real marginal cost for producing loans. And fourth, the bank's deposit liabilities provide the household with direct utility because the household finds deposits useful for facilitating transactions. Therefore, while the two models share common traits, the bank lending channel model does not nest the Bernanke et al. model.

3.2.1 Productivity shock

First I consider a shock to aggregate productivity. The shock is persistent with $\rho_z = 0.95$. I report the simulated responses in Figures C1, C2, and C3. The shock is 10 percent on impact and aggregate productivity is depicted in the northwest panel of Figure C1. The dynamic response within the baseline model is typical of the class of NNS models. On impact, the return to capital increases. Funds flow from the household to the entrepreneurs leading to more investment and a higher price of capital. Entrepreneurial net worth rises on impact. Inflation falls, but by a small amount. All variables return monotonically to the steady state as the shock dissipates.

The differences between the impulse responses from the Bernanke et al. model and the baseline model are due to the financial accelerator mechanism. In the Bernanke et al. model, entrepreneurs face borrowing costs that decrease with their net worth because borrowers with higher net worth, other things equal, borrow less and are less likely to default. The productivity shock pushes up entrepreneurial net worth in two ways. First, the shock raises the return on the aggregate capital stock and increases the net worth of entrepreneurs that do not default. Second, the shock reduces the proportion of entrepreneurs that default so that more entrepreneurs receive the benefit from the higher average capital return. Qualitatively, the effect of the accelerator mechanism in Bernanke et al. is to amplify the response of output, investment, net worth, and the return to capital to the productivity shock. Household consumption is dampened slightly relative to the baseline to accommodate the dramatic rise in investment brought on by reduced borrowing costs for entrepreneurs.

In the lending channel model, the effects of the productivity shock are generally more pronounced than in the Bernanke et al. model. On impact, the productivity shock raises the aggregate return to capital and reduces the proportion of entrepreneurs that default. The default reduction is reflected in the response of the threshold $\bar{\omega}_t$ shown in Figure C3. By reducing the proportion of entrepreneurs that default, the productivity shock is a windfall for entrepreneurs and they see an immediate rise in net worth that is greater than what is generated in the Bernanke et al. The amplified effect on net worth in the lending channel model leads to greater borrowing, investment, and output. Excess reserve holdings fall as the bank expends its loan portfolio.

By the end of the first period after the shock, the bank has had time to observe the productivity shock and write loan contracts taking into account the expected path of future aggregate productivity. The bank raises the non-default loan repayment rate and the deposit rate going into the second period. The bank contracts its loan portfolio and accumulates excess reserves beyond its steady state holdings. By the second period, most of the entrepreneurs' net worth gains from the first period are transferred to the household. The transfer to the household creates the hump-shaped response in household consumption.

From the second period, the impulse responses in the lending channel model return smoothly and monotonically towards the steady state on a path that is qualitatively closer to the baseline, than the Bernanke et al. model.

3.2.2 Monetary policy shock

In Figures C4, C5, and C6, I plot impulse responses obtained from the three models in response to a positive shock to the nominal interest rate. The shock is moderately persistent with $\rho_v = 0.50$ and is 5 percent relative to the steady state. I depict the shock in the northeast panel of Figure C4. As above, the response in the baseline model is consistent with how

NNS-style models respond to nominal interest rate shocks. The monetary contraction raises the real rate and drives down the return to capital, reduces investment, output, inflation, and consumption. The Bernanke et al. model exhibits a response that illustrates the accelerator mechanism clearly: the effects of the monetary contraction are amplified relative to the baseline model.

Relative to the impulse responses from the Bernanke et al. model, the mechanisms in the lending channel model amplify the effects of the monetary policy shock on impact. Through the usual interest rate channel, the positive nominal interest rate shock contracts aggregate activity. But since the loan repayment rate in the lending channel model is fixed in advance, loan defaults rise by about 50 percent more than in the Bernanke et al. model. The additional loan defaults can be seen in the rise in the threshold $\bar{\omega}_t$ in Figure C6. A greater proportion of entrepreneurs in default amplifies the contraction in entrepreneur net worth, investment, output, labor, and consumption. The bank reduces the size of its loan portfolio and accumulates excess reserves.

In the second period after the interest rate shock, the capital price, labor, investment, and output jump back towards the path produced by the baseline model. In the subsequent periods, the model variables return smoothly towards the steady state. As with the productivity shock, the mechanisms in the lending channel model appear to amplify the monetary shock on impact and then dampen the response in subsequent periods as the shock dissipates.

3.2.3 Financial shock

The final shock that I consider is the financial shock or, more precisely, the shock to the standard deviation of the idiosyncratic disturbance ω_t^j visited upon each entrepreneur. I set the autocorrelation of the financial shock to $\rho_\sigma = 0.9$. The financial shock arrives after loan contracts have been set. The idiosyncratic shock ω_t^j follows a log-normal distribution with a unit conditional mean. Since the log-normal distribution is right-skewed, a positive shock to the standard deviation of ω_t^j raises the proportion of entrepreneurs that will receive a draw of ω_t^j less than a given threshold $\bar{\omega}_t$. So, other things equal, the financial shock can be understood as an unanticipated shock to the proportion of borrowers in default.

In the baseline model, the financial shock creates a modest increase in loan defaults. But since there are no default costs in the baseline model, the additional defaults does not impose any real resource cost. The financial shock has essentially no effect in the baseline model beyond the share of entrepreneurs that default.

In the Bernanke et al. model, however, the financial shock does produce real dynamic effects. As the response of the threshold $\bar{\omega}_t$ in Figure C9 indicates, the shock produces a positive jump in equilibrium defaults. This draws down the return to capital and entrepreneurial net worth and reduces investment and output. Consumption rises as households consume some the accumulated capital stock. Most variables return monotonically to the steady state with the exception of consumption. After about four periods, consumption falls below the steady state as the economy replenishes its depleted capital stock.

In the lending channel model, the impact response to the financial shock is once again more pronounced than the response in the Bernanke et al. model. With loan repayment terms set before the shock, the rise in the proportion of entrepreneurs that default is more than twice that observed in the Bernanke et al. model. The amplified response of defaults amplifies the effect of the shock on other variables. Relative to the Bernanke et al. model, the reduction in entrepreneurial net worth is amplified and this leads to an amplified contraction in entrepreneurial borrowing and investment.

The unanticipated rise in loan defaults causes the bank in the lending channel model to experience an unanticipated reduction in the return on its portfolio of loans originated in the previous period. The bank can offset this negative shock by reducing the cost of originating a new portfolio of loans. The bank reduces the cost of lending by making fewer loans and by holding more excess reserves. However, the excess reserve build-up dissipates by the second period after the shock as the bank resets its loan contracts with entrepreneurs to reflect the new conditional distribution of the idiosyncratic disturbance.

4 Conclusion

I have developed a model for monetary policy analysis that emphasizes the role of banking in the shock transmission process. I compared impulse responses computed from the lending channel with ones obtained from the Bernanke et al. financial accelerator model. The mechanisms in the lending channel model have two general implications for shock transmission. First, relative to the Bernanke et al. model, exogenous shocks to the lending channel model are amplified in the period in which they occur. Second, in the periods following the shock impact, the lending channel model exhibits an attenuated response compared to Bernanke et al. To the extent that the model of banking that I have proposed is a reasonable representation of the practical banking environment, then the simulation results suggest that the Bernanke et al. financial accelerator model does not accurately reflect how financial factors influence aggregate dynamics if borrowers are constrained at the margin by the supply of bank-intermediated credit.

In the banking model, the bank chooses to hold excess reserves as a consequence of optimal behavior. The bank uses excess reserves to manage its current cost of lending and the expected return on its future loan portfolio. The bank's motive for holding excess reserves fluctuates with the aggregate cycle and this leads to endogenous variation in excess reserves. A particular result is that the bank chooses to temporarily increase its excess reserve holdings when confronted by an unexpected increase in loan defaults. While this does not explain the recent protracted excess reserve build-up in the U.S. banking system, it does begin to offer an explanation about what might motivate banks to hoard an asset that offers no nominal return[12].

Of course, from the simulation results it might be tempting to argue that I have endogenized excess reserves too successfully. The bank reduces excess reserve holdings in response to a positive productivity shock and the bank responds to a monetary contraction by increas-

[12]Recall that the Federal Reserve did not begin paying interest on reserves until October 2008 – more than a year after the initial excess reserve build-up

ing excess reserves. But this behavior is surely exaggerated by the fact that the bank in the model has only two assets available. If I were to allow the bank to also hold short term government bonds, then access to these interest-bearing assets would reduce the bank's demand for excess reserves. This would be especially true if I were willing to assume that government bonds, like excess reserves, were arguments in the bank's loan production function. Given the highly liquid market for T-bills, this would be a reasonable assumption.

Now, as I acknowledged in the introduction, this paper reflects work that is still in progress. There are two tasks that I need to complete in order to finish this current project. First, I will rigorously calibrate the model parameters by imposing appropriate steady state moment conditions. Second, as I mentioned in a footnote in section 2.2, I will restrict how the bank observes innovations in the aggregate state. In the model, an unanticipated wave of loan defaults can be caused by several things: a monetary contraction, a negative shock to aggregate productivity, or a positive financial shock[13]. While a practical bank might reasonably be able to discern that a monetary policy shock has arrived, it is less plausible that a bank would be able to distinguish between defaults caused by an aggregate productivity shock or the financial shock. I will introduce a simple signal extraction problem that requires the bank to optimally screen information from a noisy signal combining information about the aggregate productivity shock or the financial shock. This modeling innovation is intended to underscore the uncertainty characterizing the banking environment.

The present paper is the first of two components of my dissertation on the bank lending channel and monetary policy. While computing and comparing impulse responses from various models can be instructive, the exercises alone do not produce a prescription for implementing monetary policy; especially since the impulse responses are obtained using an assumed interest rate feedback rule. My objective is to define a welfare criterion for the central bank and then to compute optimized interest rate feedback rules in the spirit of Rotemberg and Woodford (1999) for the three models that I considered in Section 3. Then

[13]Recall that the financial shock is a shock to the distribution of the idiosyncratic productivity disturbance.

I can use the welfare criterion to determine the welfare cost in the lending channel model if the central bank uses one of the other optimized rules. This will allow me to determine if it is important for the central bank consider the lending channel when it implements policy.

Appendix

A Linearized Equilibrium Conditions for the Lending Channel Model

From the lending channel model, I obtain 23 equations representing the linearized equilibrium conditions governing the evolution of 23 endogenous variables. Hatted Greek and Roman letters represent log-deviations of model variables from their steady state values. Capital letters without time subscripts denote steady state values. Eleven conditions are novel to the model I have proposed. The remaining twelve equations have close analogues in Bernanke et al. The novel equations are:

$$\frac{\Gamma_\sigma(\bar{\omega}|\sigma_\omega^2) - \mu G_\sigma(\bar{\omega}|\sigma_\omega^2)}{\Gamma(\bar{\omega}|\sigma_\omega^2) - \mu G(\bar{\omega}|\sigma_\omega^2)} \cdot 2\sqrt{\sigma_\omega^2} \cdot \hat{\sigma}_{\omega,t} = \hat{r}_t^B - \hat{\pi}_t - \hat{r}_t^K + \frac{N}{K-N}\left(\hat{q}_{t-1} + \hat{k}_t - \hat{n}_t\right) - \frac{\Gamma_\omega(\bar{\omega}|\sigma_\omega^2) - \mu G_\omega(\bar{\omega}|\sigma_\omega^2)}{\Gamma(\bar{\omega}|\sigma_\omega^2) - \mu G(\bar{\omega}|\sigma_\omega^2)} \cdot \bar{\omega} \cdot \hat{\bar{\omega}}_t \qquad \text{(A1)}$$

$$\hat{d}_{t+1} - \frac{b}{b+m^{ex}}\hat{b}_{t+1} - \frac{m^{ex}}{b+m^{ex}}\hat{m}_{t+1}^{ex} = 0 \qquad \text{(A2)}$$

$$\sigma_{\mathrm{D}}\hat{d}_{t+1} - \frac{R^D}{R^n - R^D}\hat{r}_{t+1}^D + \frac{R^D}{R^n - R^D}\hat{r}_{t+1}^n = \sigma_{\mathrm{C}}\hat{c}_t \qquad \text{(A3)}$$

$$\beta R^B \Lambda^{\mathrm{B}} E_t \hat{r}_{t+1}^B + \beta \Lambda^{\mathrm{B}}\left[R^B - \frac{R^D - \rho}{1-\rho}\right]\left(E_t\hat{\Lambda}_{t+1}^{\mathrm{B}} - E_t\hat{\pi}_{t+1}\right) - \beta\Lambda^{\mathrm{B}}\frac{R^D}{1-\rho}\hat{r}_{t+1}^D - \mathbb{C}_{BB}\Lambda^{\mathrm{B}} \cdot b \cdot \hat{b}_{t+1} - \mathbb{C}_{BM}\Lambda^{\mathrm{B}} \cdot m^{ex} \cdot \hat{m}_{t+1}^{ex} = \mathbb{C}_{BW}\Lambda^{\mathrm{B}} W\hat{w}_t + \mathbb{C}_B\hat{\Lambda}_t^{\mathrm{B}} \qquad \text{(A4)}$$

$$\beta\Lambda^{\mathrm{B}}\left[R^B-\frac{R^D-\rho}{1-\rho}\right]\left(E_t\hat{\Lambda}^{\mathrm{B}}_{t+1}-E_t\hat{\pi}_{t+1}\right)-\beta\Lambda^{\mathrm{B}}\frac{R^D}{1-\rho}\hat{r}^D_{t+1}-\mathbb{C}_{MB}\Lambda^{\mathrm{B}}\cdot b\cdot\hat{b}_{t+1}$$
$$-\mathbb{C}_{MM}\Lambda^{\mathrm{B}}\cdot m^{ex}\cdot\hat{m}^{ex}_{t+1}=\mathbb{C}_{MW}\Lambda^{\mathrm{B}}W\hat{w}_t+\mathbb{C}_M\hat{\Lambda}^{\mathrm{B}}_t \quad \text{(A5)}$$

$$\mathbb{C}_B\cdot b\cdot\hat{b}_{t+1}+\mathbb{C}_M\cdot m^{ex}\cdot\hat{m}^{ex}_{t+1}=R^B\cdot b\cdot\left(\hat{r}^B_t+\hat{b}_t\right)+m^{ex}\cdot\hat{m}^{ex}_t-\left(R^D-\rho\right)\cdot d\cdot\hat{d}_t-R^D\cdot d\cdot\hat{r}^D_t$$
$$-\left(R^Bb+m^{ex}-(R^D-\rho)d\right)\hat{\pi}_t-\mathbb{C}_W\cdot W\cdot\hat{w}_t-\frac{\Lambda^{\mathrm{B}}}{\tilde{u}''(\bar{\Phi})}\hat{\Lambda}^{\mathrm{B}}_t \quad \text{(A6)}$$

$$\Xi E_t\hat{r}^K_{t+1}+\left[\left(\Gamma(\bar{\omega}|\sigma^2_\omega)-\mu G(\bar{\omega}|\sigma^2_\omega)\right)R^K+\left(\Gamma_\omega(\bar{\omega}|\sigma^2_\omega)-\mu G_\omega(\bar{\omega}|\sigma^2_\omega)\right)\bar{R}\frac{N}{K}\right]\lambda^c\Lambda^{\mathrm{B}}E_t\hat{\Lambda}^{\mathrm{B}}_{t+1}$$
$$+\left\{\lambda^c\Lambda^{\mathrm{B}}\left[\left(\Gamma_{\omega\sigma}(\bar{\omega}|\sigma^2_\omega)-\mu G_{\omega\sigma}(\bar{\omega}|\sigma^2_\omega)\right)\bar{R}\frac{N}{K}+\left(\Gamma_\sigma(\bar{\omega}|\sigma^2_\omega)-\mu G_\sigma(\bar{\omega}|\sigma^2_\omega)\right)R^K\right]\right.$$
$$\left.-\ \Gamma_\sigma(\bar{\omega}|\sigma^2_\omega)R^K-\Gamma_{\omega\sigma}(\bar{\omega}|\sigma^2_\omega)\frac{N}{K}\bar{R}\right\}\cdot 2\cdot\sqrt{\sigma^2_\omega}\cdot\rho_\sigma\hat{\sigma}_{\omega,t}$$
$$+\left\{\lambda^c\Lambda^{\mathrm{B}}\left(\Gamma_{\omega\omega}(\bar{\omega}|\sigma^2_\omega)-\mu G_{\omega\omega}(\bar{\omega}|\sigma^2_\omega)\right)\bar{R}\frac{N}{K}-\Gamma_{\omega\omega}(\bar{\omega}|\sigma^2_\omega)\frac{N}{K}\bar{R}\right\}\bar{\omega}E_t\hat{\bar{\omega}}_{t+1}$$
$$=\ \Xi\cdot\hat{\Xi}_t-\left[\left(\Gamma(\bar{\omega}|\sigma^2_\omega)-\mu G(\bar{\omega}|\sigma^2_\omega)\right)R^K\right.$$
$$\left.+\ \left(\Gamma_\omega(\bar{\omega}|\sigma^2_\omega)-\mu G_\omega(\bar{\omega}|\sigma^2_\omega)\right)\bar{R}\frac{N}{K}\right]\lambda^c\Lambda^{\mathrm{B}}\hat{\lambda}_t \quad \text{(A7)}$$

$$E_t\hat{\Lambda}^{\mathrm{B}}_{t+1}+\frac{\Gamma_\omega(\bar{\omega}|\sigma^2_\omega)-\mu G_\omega(\bar{\omega}|\sigma^2_\omega)}{\Gamma(\bar{\omega}|\sigma^2_\omega)-\mu G(\bar{\omega}|\sigma^2_\omega)}\bar{\omega}E_t\hat{\bar{\omega}}_{t+1}+\frac{\Gamma_\sigma(\bar{\omega}|\sigma^2_\omega)-\mu G_\sigma(\bar{\omega}|\sigma^2_\omega)}{\Gamma(\bar{\omega}|\sigma^2_\omega)-\mu G(\bar{\omega}|\sigma^2_\omega)}\cdot 2\cdot\sqrt{\sigma^2_\omega}\cdot\rho_\sigma\hat{\sigma}_{\omega,t}$$
$$+E_t\hat{r}^K_{t+1}+\frac{N}{K-N}\hat{n}_{t+1}-\frac{N}{K-N}\left(\hat{q}_t+\hat{k}_t\right)=\hat{\Xi}_t \quad \text{(A8)}$$

$$0=\frac{N}{K-N}\left[\hat{q}_{t-1}+\hat{k}_t-\hat{n}_t\right]+\hat{\bar{r}}_t-\hat{\bar{\omega}}_t-\hat{\pi}_t-\hat{r}^K_t \quad \text{(A9)}$$

$$\beta\left[\frac{R^D-\rho}{1-\rho}\right]\left(E_t\hat{\Lambda}^{\mathrm{B}}_{t+1}-E_t\hat{\pi}_{t+1}\right)+\beta\frac{R^D}{1-\rho}\hat{r}^D_{t+1}+\mathbb{C}_{BB}\cdot b\cdot\hat{b}_{t+1}+\mathbb{C}_{BM}\cdot m^{ex}\cdot\hat{m}^{ex}_{t+1}$$
$$=\ \Xi\cdot\hat{\Xi}_t-\mathbb{C}_{BW}\cdot W\cdot\hat{w}_t-\mathbb{C}_B\cdot\hat{\Lambda}^{\mathrm{B}}_t \quad \text{(A10)}$$

$$\left[\frac{\Gamma_{\omega\omega}(\bar{\omega}|\sigma_\omega^2) - \mu G_{\omega\omega}(\bar{\omega}|\sigma_\omega^2)}{\Gamma_\omega(\bar{\omega}|\sigma_\omega^2) - \mu G_\omega(\bar{\omega}|\sigma_\omega^2)} - \frac{\Gamma_{\omega\omega}(\bar{\omega}|\sigma_\omega^2)}{\Gamma_\omega(\bar{\omega}|\sigma_\omega^2)}\right] \bar{\omega} \cdot E_t \hat{\bar{\omega}}_{t+1}$$
$$+ \left[\frac{\Gamma_{\omega\sigma}(\bar{\omega}|\sigma_\omega^2) - \mu G_{\omega\sigma}(\bar{\omega}|\sigma_\omega^2)}{\Gamma_\omega(\bar{\omega}|\sigma_\omega^2) - \mu G_\omega(\bar{\omega}|\sigma_\omega^2)} - \frac{\Gamma_{\omega\sigma}(\bar{\omega}|\sigma_\omega^2)}{\Gamma_\omega(\bar{\omega}|\sigma_\omega^2)}\right] \cdot 2 \cdot \sqrt{\sigma_\omega^2} \cdot \rho_\sigma \cdot \hat{\sigma}_{\omega,t}$$
$$+ E_t \hat{\Lambda}^{\text{B}}_{t+1} = -\hat{\lambda}^c_t \tag{A11}$$

And the remaining twelve:

$$-z_t = \alpha \hat{k}_t + \left[(1-\alpha)\Omega - 1\right] \hat{h}_t - \hat{x}_t - \hat{w}_t \tag{A12}$$

$$\hat{k}_{t+1} = (1+\varphi)\delta \hat{\imath}_t + (1 - (1+\varphi)\delta)\, \hat{k}_t \tag{A13}$$

$$\hat{n}_{t+1} - \mathbf{N}^1_{zhx} \hat{z}_t - \mathbf{N}^1_\sigma \hat{\sigma}_{\omega,t} = \mathbf{N}^1_{rkq} \left(\hat{r}^K_t + \hat{q}_{t-1}\right) + \mathbf{N}^1_k \hat{k}_t + \mathbf{N}^1_\omega \hat{\bar{\omega}}_t$$
$$+ \mathbf{N}^1_{zhx} \left[(1-\alpha)\Omega \hat{h}_t - \hat{x}_t\right] \tag{A14}$$

$$\hat{q}_t = \varphi \hat{k}_t - \varphi \hat{\imath}_t \tag{A15}$$

$$r^n_{t+1} - v^{ff}_t = \varsigma_\pi \hat{\pi}_t + \varsigma_y \hat{y}_t \tag{A16}$$

$$\vartheta \hat{q}_t = \hat{r}^K_t + (1-\vartheta)\hat{x}_t + (1-\vartheta)\hat{k}_t - (1-\vartheta)\hat{y}_t + \hat{q}_{t-1} \tag{A17}$$

$$\beta E_t \hat{\pi}_{t+1} = \hat{\pi}_t + \kappa \hat{x}_t \tag{A18}$$

$$\sigma_{\text{C}} E_t \hat{c}_{t+1} + E_t \hat{\pi}_{t+1} - \hat{r}^n_{t+1} = \sigma_{\text{C}} \hat{c}_t \tag{A19}$$

$$0 = \hat{y}_t - (1 + (\eta - 1)H)(1-H)^{-1} \hat{h}_t - \sigma_{\text{C}} \hat{c}_t - \hat{x}_t \tag{A20}$$

$$-\hat{z}_t = \alpha\hat{k}_t + (1-\alpha)\Omega\hat{h}_t - \hat{y}_t \tag{A21}$$

$$-\frac{G}{Y}\hat{g}_t - \mathbf{Y}^1_\sigma\hat{\sigma}_{\omega,t} = \frac{C}{Y}\hat{c}_t + \frac{I}{Y}\hat{\imath}_t + \frac{C^e}{Y}\hat{c}^e_t - \hat{y}_t + \mathbf{Y}^1_{rkqk}\left(\hat{r}^K_t + \hat{q}_{t-1} + \hat{k}_t\right) + \mathbf{Y}^1_\omega\hat{\bar{\omega}}_t \tag{A22}$$

$$\mathbf{C}^{e1}_n\hat{n}_{t+1} + \mathbf{C}^{e1}_{zkhx}\hat{z}_t = \hat{c}^e_t - \mathbf{C}^{e1}_{zkhx}\left(\alpha\hat{k}_t + (1-\alpha)\Omega\hat{h}_t - \hat{x}_t\right) \tag{A23}$$

where:

$$\vartheta \equiv \frac{1-\delta}{\alpha Y/XK + 1 - \delta} \tag{A24}$$

$$\varphi \equiv \frac{\Phi''(I/K)}{\Phi'(I/K)}\frac{I}{K} \tag{A25}$$

$$\kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \tag{A26}$$

and:

$$\mathbf{N}^1_{rkq} \equiv \frac{\gamma R^K K}{N}\left[1 - \Gamma(\bar{\omega}|\sigma^2_\omega)\right] \tag{A27}$$

$$\mathbf{N}^1_{zhx} \equiv \frac{(1-\alpha)(1-\Omega)Y}{NX} \tag{A28}$$

$$\mathbf{N}^1_\omega \equiv -\frac{\gamma R^K K}{N}\left[\Gamma_\omega(\bar{\omega}|\sigma^2_\omega)\right]\bar{\omega} \tag{A29}$$

$$\mathbf{N}^1_\sigma \equiv -\frac{\gamma R^K K}{N}\left[\Gamma_\sigma(\bar{\omega}|\sigma^2_\omega)\right]\cdot 2\cdot\sigma^2_\omega \tag{A30}$$

$$\mathbf{N}^1_k \equiv \mathbf{N}^1_{rkq} + \alpha\mathbf{N}^1_{zhx} \tag{A31}$$

$$\mathbf{Y}^1_{rkqk} \equiv \mu R^K K G(\bar{\omega}|\sigma^2_\omega)/Y \tag{A32}$$

$$\mathbf{Y}^1_\omega \equiv \mu R^K K G_\omega(\bar{\omega}|\sigma^2_\omega)\bar{\omega}/Y \tag{A33}$$

$$\mathbf{Y}^1_\sigma \equiv \mu R^K K G_\sigma(\bar{\omega}|\sigma^2_\omega)\cdot 2\cdot\sigma^2_\omega/Y \tag{A34}$$

$$\mathbf{C}_n^{e1} \equiv \frac{(1-\gamma)N}{\gamma} \tag{A35}$$

$$\mathbf{C}_{zkhx}^{e1} \equiv C^e - \mathbf{C}_n^{e1} \tag{A36}$$

Equation (A1) defines nominal return on the bank's loan portfolio. Naturally, this is increasing in the aggregate return to capital and inflation. The presence of equilibrium loan defaults causes the nominal return on loans to also be decreasing in the size of the loan portfolio at the margin. Finally, the return on loans is decreasing in $\bar{\omega}_t$ – the proportion of entrepreneurs in default – and $\sigma_{\omega,t}$ – the standard deviation of the idiosyncratic shock to entrepreneurial returns. This last observation shows an important mechanism by which fluctuations in loan default probabilities enter into the bank's optimization problem.

Equation (A2) is the bank's balance sheet identity. Equation (A3) is the household's supply of real deposits. Deposit supply is increasing in household consumption and decreasing in the cost of holding deposits. This cost is measured by the spread between the deposit and nominal rate. Since the household finds deposits useful for saving wealth and for completing transactions, banks can borrow funds from the household at a cost below the nominal rate.

Next, equation (A4) combines the bank's first-order conditions for receiving deposits and making loans. Equation (A5) is a combination of the bank's first order conditions for holding excess reserves and making loans. These equations, together with (A6) – the definition of the banker's marginal utility – describe the bank's optimal behavior. The banker's marginal utility is determined by the net return on is financial portfolio originated in the previous period and the cost of originating a new portfolio. As (A4) and (A5) indicate, the bank's problem is intertemporal and optimal behavior depends on the curvature of the banker's utility function. The banker cares about smoothing its marginal utility across time and, as (A6) suggests, it uses excess reserves to do so when confronted with unanticipated losses.

Finally, equations (A7), (A8), (A9), (A10), and (A11) are equilibrium conditions from the bank's optimal loan contracting problem. First, (A7) is the first-order condition for the amount of capital the entrepreneurs purchase under the optimal contract. Second, (A8) is the constraint that bank's expected return from lending equal its opportunity cost of funds.

Next, (A9) is the definition of the stochastic default threshold $\hat{\bar{\omega}}_t$. This determines the proportion of entrepreneurs that default on their loans each period. This proportion is decreasing in the price of capital, the capital stock, and the borrowing rate. Equation (A9) because it shows how the debt-deflation story of Fisher (1933) enters into the present model. Falling inflation raises the real debt burden of entrepreneurs and, other things equal, pushes up the proportion of entrepreneurs in default. This in turn drives down entrepreneur net worth and drives up defaults in the following period.

Next, (A10) defines the bank's cost of funds $\hat{\Xi}_t$. And finally, (A11) is the first-order condition for the non-default borrowing rate in the contracting problem. The remaining twelve equations either have direct ancestors in Bernanke et al. or, like the equation defining the real wage (A12), are common features in business cycle models. I discuss them in Appendix B.

Now, I characterize the evolution of the 4 exogenous variables: the aggregate productivity process $\hat{z}_t$, the government consumption process $\hat{g}_t$, the monetary policy shock $\hat{v}_t$, and the volatility of the idiosyncratic productivity shock $\hat{\sigma}_{\omega,t}$:

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_t^z \tag{A37}$$

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_t^g \tag{A38}$$

$$\hat{v}_t^{ff} = \rho_v \hat{v}_{t-1}^{ff} + \varepsilon_t^v \tag{A39}$$

$$\hat{\sigma}_{\omega,t+1} = \rho_\sigma \hat{\sigma}_{\omega,t} + \varepsilon_t^\sigma \tag{A40}$$

B Bernanke, Gertler, and Gilchrist (1999) Model

The model described in Bernanke et al. is the foundation of my work. In this section I describe the components of their model and characterize its equilibrium. The model economy is occupied by entrepreneurs, a representative household, and a government. The household owns a non-bank financial intermediary, a capital good producer, retail firms, and a final good producer. Entrepreneurs are a group of agents distinct from the household that use capital and labor to produce a homogenous output called the wholesale good. Monopolistically competitive retailers purchase the wholesale goods, differentiate them, and then sell the differentiated output to the competitive final good producer. The final good producer combines all the retail goods to produce a composite output good that satisfies aggregate demand.

At the end of each period, entrepreneurs purchase capital to use for production at the beginning of the next period. Entrepreneurs finance capital purchases using their accumulated wealth plus funds borrowed from the intermediary. The return on an entrepreneur's capital is subject to systematic and idiosyncratic risk. To create a role for intermediated finance, Bernanke et al. introduce a costly state verification (CSV) problem similar to the problem studied by Townsend (1979). The intermediary observes the return on the aggregate capital stock, but only observes the return on a specific entrepreneur's capital by incurring an auditing cost. Under the optimal loan contract, the intermediary only audits entrepreneurs in default and the expected auditing costs are passed on to the entrepreneur as a premium on the non-default loan repayment rate.

The Bernanke et al. model is designed so that the asymmetric information between borrower and lender is the only source of financial market friction. To ensure this, they make two important assumptions about financial structure in their model. First, they assume that the aggregate price level is set one period in advance so financial arrangements can be written without inflation risk. Second, they assume that the financial intermediary in their model is a non-bank lending institution. The intermediary is not subject to a reserve requirement and so monetary policy cannot directly affect the liability side of the intermediary's balance sheet. With these assumptions in place, their credit channel transmission mechanism is easy to isolate and, as I show below, also easy to turn off for counterfactual simulation exercises.

B1 Financial intermediation and the demand for capital

The financial intermediary is a competitive firm that specializes in originating loans, processing repayments, and recovering assets from borrowers in default. The intermediary funds a nominal loan portfolio B_{t+1} with one-period nominal debt liabilities $\bar{A}_{t+1}$ purchased by the household.

At the end of period t, an entrepreneur j has accumulated real net worth N^j_{t+1} that it uses to purchase capital K^j_{t+1} at a real price of Q_t. Net worth N^j_{t+1} is measured in terms of the final output good. Capital purchases in excess of net worth are financed by a nominal bank loan B^j_{t+1}:

$$B^j_{t+1} = P_t \left(Q_t K^j_{t+1} - N^j_{t+1} \right). \tag{B1}$$

The ex post gross return to the entrepreneurs capital is $\omega^j R^k_{t+1}$, where ω^j is a disturbance to the jth entrepreneur's capital return. ω^j is i.i.d. across time and entrepreneurs with c.d.f. $F(\cdot)$ over a non-negative support such that $E(\omega^j) = 1$[14]. While the ex post realization of ω^j is privately observed by the entrepreneur, the realization of R^k_{t+1} is perfectly observed by the intermediary without cost.

The entrepreneur's demand for capital is determined by its net worth and the terms of its loan contract for B^j_{t+1}. A loan contract is characterized by a non-default nominal gross repayment rate $\bar{R}^j_{t+1}$ and a threshold $\bar{\omega}^j$ such that the entrepreneur is able to repay its loan when $\omega^j \geqslant \bar{\omega}^j$. The threshold satisfies:

$$\bar{\omega}^j R^k_{t+1} Q_t K^j_{t+1} = \frac{\bar{R}^j_{t+1}}{1+\pi_{t+1}} \frac{B^j_{t+1}}{P_t}, \tag{B2}$$

[14] Additionally, assume that $F(\cdot)$ is continuous and at least once differentiable satisfying:

$$\frac{\partial h(\omega)}{\partial \omega} > 0,$$

where $h(\omega) \equiv \frac{dF(\omega)}{1-F(\omega)}$ is the hazard rate. As Bernanke et al. (1999) discuss, this regularity condition is important for excluding credit rationing from the debt contracting equilibria described below.

where $1+\pi_{t+1}$ is the gross inflation between t and $t+1$. By assumption, π_{t+1} is determined in period t. A borrower in default surrenders the realized value of its investment project to the intermediary, but the intermediary incurs an auditing cost when it takes over the investment project. This cost is a fixed proportion μ of the realized value of the project in $t+1$. Therefore, the intermediary receives $(1-\mu)\omega^j R^k_{t+1} Q_t K^j_{t+1}$ from a project in default. The parameter μ reflects a deadweight loss associated with debt default and is the source of financial friction. When $\mu = 0$, the intermediary incurs no auditing cost and recovers the full realized of a project in default.

In practice, loan arrangements typically specify a state-invariant repayment rate. Bernanke et al., however, assume that the repayment terms of intermediated loans are contingent on the observable realization of the aggregate return to capital R^k_{t+1}. The intermediary writes contracts that shift aggregate risk onto the risk-neutral entrepreneurs so that the lender bears only idiosyncratic risk on individual loans. Then, by holding a fully diversified loan portfolio, the intermediary is guaranteed a risk-free return. Modeling the lending environment this way allows Bernanke et al. to show clearly how borrower-specific default risk affects the financial market equilibrium by insulating the intermediary's return from uncertainty in the aggregate return to capital.

For a realization of R^k_{t+1} and a threshold $\bar{\omega}^j$, a loan B^j_{t+1} provides the intermediary with an expected real gross return of:

$$\left[1 - F(\bar{\omega}^j)\right] \frac{\bar{R}^j_{t+1}}{1+\pi_{t+1}} \frac{B^j_{t+1}}{P_t} + (1-\mu) R^k_{t+1} Q_t K^j_{t+1} \int_0^{\bar{\omega}^j} \omega dF(\omega). \tag{B3}$$

After substituting (B2) into (B3), collecting terms, and requiring that the intermediary earn no expected profits, it follows that the optimal loan contract must satisfy:

$$\left(\left[1 - F(\bar{\omega}^j)\right] \bar{\omega}^j + (1-\mu) \int_0^{\bar{\omega}^j} \omega dF(\omega) \right) R^k_{t+1} Q_t K^j_{t+1} = \frac{\bar{R}_{t+1} \left(Q_t K^j_{t+1} - N^j_{t+1} \right)}{1+\pi_{t+1}}, \tag{B4}$$

where the right-hand side of (B4) is the real cost of lending for the intermediary facing a nominal cost of funds $\bar{R}_{t+1}$. The only variables in equation (B4) that are not determined by the end of period t are R^k_{t+1} and $\bar{\omega}^j$. The threshold $\bar{\omega}^j$ adjusts so (B4) holds with certainty in all states.

Now, the entrepreneur's expected real return from entering into a loan agreement is:

$$E_t \left\{ R^k_{t+1} Q_t K^j_{t+1} \int_{\bar{\omega}^j}^{\infty} \omega \, dF(\omega) - \left[1 - F(\bar{\omega}^j)\right] \frac{\bar{R}^j_{t+1}}{1+\pi_{t+1}} \frac{B^j_{t+1}}{P_t} \right\}, \tag{B5}$$

where the expectation is over R^k_{t+1}; understanding that $\bar{\omega}^j$ is contingent on the realization of R^k_{t+1}. Using (B2) and reorganizing the previous expression, the entrepreneur's expected return is expressed as:

$$E_t \left\{ R^k_{t+1} Q_t K^j_{t+1} \left(1 - \int_0^{\bar{\omega}^j} \omega \, dF(\omega) - \bar{\omega}^j \left[1 - F(\omega)\right] \right) \right\}. \tag{B6}$$

An optimal loan contract then is a pair $(\bar{\omega}^j, K^j_{t+1})$ that maximizes (B6) subject to constraint (B4). As Bernanke et al. show, the optimal contract implies a capital demand function that relates capital expenditures to entrepreneur net worth, the expected return to capital, and the intermediary's cost of funds:

$$Q_t K^j_{t+1} = \psi(s_t) N^j_{t+1}, \tag{B7}$$

where

$$s_t \equiv E_t \left\{ R^k_{t+1} \right\} \frac{1+\pi_{t+1}}{\bar{R}_{t+1}}. \tag{B8}$$

Here, s_t is the expected real present discounted value of the aggregate return to capital and is unit-less[15]. The function $\psi(\cdot)$ is strictly increasing in s_t and satisfies $\psi(1) = 1$. It can also be shown that for a given s, $\psi(s)$ is decreasing in the auditing cost parameter μ. Greater auditing costs raise borrowing costs and suppress the demand for capital. The aggregate

[15] This is because R^k_{t+1} and $\left[\bar{R}_{t+1}/(1+\pi_{t+1})\right]$ are both in terms of period $t+1$ final output per unit of period t final output.

demand for capital is obtained by aggregating (B7) over j:

$$Q_t K_{t+1} = \psi(s_t) N_{t+1}. \tag{B9}$$

Equation (B9) reflects how financial market frictions interfere with equilibrium in the market for physical capital. It is the first of two equations in the Bernanke et al. model that produces the financial accelerator mechanism. In the absence of financial friction, $\mu = 0$ and $E_t R^k_{t+1} = \bar{R}_{t+1}/(1+\pi_{t+1})$ so (B9) would collapse to:

$$s_t = 1. \tag{B10}$$

B2 The entrepreneurial sector and the evolution of net worth

The entrepreneurial sector enters period t with capital K_t. At the beginning of the period, entrepreneurs hire labor from a competitive labor market to combine with capital to produce the wholesale good Y_t. The aggregate output of the entrepreneurial sector is:

$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha}. \tag{B11}$$

where Z_t is an exogenous aggregate technology process. Note that Z_t is a distinct process from the idiosyncratic shocks to an entrepreneur's return to capital. L_t is a composite of household labor H_t and entrepreneurial labor H_t^e:

$$L_t = H_t^{\Omega} \left(H_t^e\right)^{1-\Omega}. \tag{B12}$$

The entrepreneurial sector sells its wholesale output to the retail sector at a real price of $1/X_t$ per unit. During the production process, a fraction δ of the capital stock is destroyed. The remaining capital is sold to the capital-producing sector – discussed in the next section – at Q_t per unit.

The labor inputs are paid their marginal product. The real household wage W_t must satisfy:

$$(1-\alpha)\Omega \frac{Y_t}{H_t X_t} = W_t, \tag{B13}$$

while the entrepreneurial wage W_t^e satisfies:

$$(1-\alpha)(1-\Omega)\frac{Y_t}{H_t^e X_t} = W_t^e. \tag{B14}$$

The aggregate return to capital is:

$$R_t^k = \frac{\frac{1}{X_t}\frac{\alpha Y_t}{K_t} + Q_t(1-\delta)}{Q_{t-1}}, \tag{B15}$$

where:

$$\frac{1}{X_t}\frac{\alpha Y_t}{K_t}. \tag{B16}$$

is the rent paid to the aggregate capital stock.

The entrepreneurial sector receives $R_t^k Q_{t-1} K_t$ from its capital holdings at the beginning of the period. Under the contractual arrangements with the intermediary, a fraction $1 - F(\bar{\omega})$ of entrepreneurs transfer a share $\bar{\omega}$ of their earnings to the intermediary while the remaining entrepreneurs surrender all of their earnings. The real amount transferred from the entrepreneurial sector to the intermediary in period t is therefore:

$$\left([1 - F(\bar{\omega})]\,\bar{\omega} + \int_0^{\bar{\omega}} \omega dF(\omega) \right) R_t^k Q_{t-1} K_t. \tag{B17}$$

Using (B4), equation (B17) can be rewritten as:

$$\left(\frac{\bar{R}_t}{1+\pi_t} + \frac{\mu \int_0^{\bar{\omega}} \omega \; dF(\omega) R_t^k Q_{t-1} K_t}{Q_{t-1}K_t - N_t} \right) (Q_{t-1}K_t - N_t)\,. \tag{B18}$$

Equation (B18) reflects the aggregate real cost of funds for the entrepreneurial sector and the term,

$$\frac{\mu \int_0^{\bar{\omega}} \omega \, dF(\omega) R_t^k Q_{t-1} K_t}{Q_{t-1} K_t - N_t}, \tag{B19}$$

represents the external finance premium on uncollateralized debt. The external finance premium is strictly increasing in the auditing cost parameter μ.

The entrepreneurial sector must consume enough of its wealth each period so that it never accumulates enough wealth to become self-financing. Bernanke et al. do not confront this with a model of the entrepreneurs' choice between consumption and saving. Rather, they assume that after settling their business with the intermediary, an exogenous fraction $1-\gamma$ of randomly selected entrepreneurs close their firms, consume their accumulated wealth, and exit the model. Each departing entrepreneur is replaced by a new entrepreneur with no accumulated wealth.

Let V_t denote the equity accumulated by entrepreneurs immediately after concluding their relationship with the intermediary at the beginning of period t. From (B18), V_t can be expressed as:

$$V_t = R_t^k Q_{t-1} K_t - \left(\frac{\bar{R}_t}{1+\pi_t} + \frac{\mu \int_0^{\bar{\omega}_t} \omega dF(\omega) R_t^k Q_{t-1} K_t}{Q_{t-1} K_t - N_t} \right) (Q_{t-1} K_t - N_t) \, . \tag{B20}$$

Then, at the end of period t, the accumulated net worth of the entrepreneurial sector is:

$$N_{t+1} = \gamma V_t + W_t^e, \tag{B21}$$

where W_t^e is the wage income under the assumption that each entrepreneur inelastically supplies a single unit of labor for production. Entrepreneurial consumption C_t^e is given as:

$$C_t^e = (1-\gamma) V_t. \tag{B22}$$

Now it is straightforward to write down an equation for the evolution of entrepreneurial net worth using equations (B14), (B20) and (B21):

$$N_{t+1} = \gamma \left[R_t^k Q_{t-1} K_t - \left(\frac{\bar{R}_t}{1+\pi_{t+1}} + \frac{\mu \int_0^{\bar{\omega}_t} \omega dF(\omega) R_t^k Q_{t-1} K_t}{Q_{t-1}K_t - N_t} \right) (Q_{t-1}K_t - N_t) \right] + (1-\alpha)(1-\Omega) Z_t K_t^{\alpha} H_t^{(1-\alpha)\Omega} / X_t. \tag{B23}$$

This is the second component of the financial accelerator mechanism. Bernanke et al. find γ that is greater than 0.9 in their calibration so net worth is highly persistent. Together, equations (B9) and (B23) show how financial frictions distort the market for physical capital.

B3 Capital production

Capital is produced during the period by a competitive capital-producing firm. Immediately following production in period t, the firm buys the entire capital stock K_t from the entrepreneurs and combines it with some of the final output good I_t to produce new capital K_{t+1} that is sold back to the entrepreneurs. Capital accumulates subject to a convex adjustment cost. Adjustment costs induce variability in the price of capital and entrepreneurial net worth. Assuming that entrepreneurs repurchase the entire capital stock each period allows capital adjustment costs to be considered separately from the entrepreneur's financial problem.

The capital-producer solves

$$\max_{K_t, I_t} Q_t K_{t+1} - I_t - \bar{Q}_t(1-\delta)K_t, \tag{B24}$$

subject to:

$$K_{t+1} = \Phi\left(\frac{I_t}{K_t}\right) K_t + (1-\delta)K_t, \tag{B25}$$

where $\bar{Q}_t$ is the price of capital in period t after production, but before new capital has been produced. As (B25) suggests, investment I_t in period t results in only $\Phi(I_t/K_t)K_t$ units of period $t+1$ capital. $\Phi(\cdot)$ is increasing and concave with $\Phi(0) = 0$. The first-order conditions for I_t and K_t are:

$$Q_t = \Phi'\left(\frac{I_t}{K_t}\right), \tag{B26}$$

$$\bar{Q}_t(1-\delta) = Q_t\Phi\left(\frac{I_t}{K_t}\right) + Q_t(1-\delta) - \frac{I_t}{K_t}. \tag{B27}$$

Bernanke et al. assume that $Q = 1$ in the steady state. They do not explicitly discuss the functional form of $\Phi(\cdot)$, but for completeness, I assume the following form for $\Phi(\cdot)$:

$$\Phi\left(\frac{I_t}{K_t}\right) \equiv \frac{1}{1+\varphi}\left(\frac{I_t}{K_t}\right)^{1+\varphi}\left(\frac{\bar{K}}{\bar{I}}\right)^{\varphi}, \tag{B28}$$

where $\bar{K}$ and $\bar{I}$ are the steady-state values of capital and investment and $\varphi < 0$. I assume this form for $\Phi(\cdot)$ because it's first two moments coincide with the moments Bernanke et al. state in the text. Since $Q = 1$ in the steady state, the difference between Q_t and $\bar{Q}_t$ is of second-order consequence and so I follow Bernanke et al. and set $\bar{Q}_t = Q_t$. It is straightforward to show that with the assumed form of $\Phi(\cdot)$, φ is the elasticity of the steady state capital price Q with respect to the steady state investment to capital ratio.

B4 Retail goods, final output, and price setting

The retail sector comprises a continuum of firms that purchase wholesale goods from the entrepreneurial sector and produce retail goods by costlessly differentiating the wholesale output. Retailers are monopolistically competitive and set the prices of their products according to the familiar Calvo (1983) mechanism. Bernanke et al. introduce retailers into the supply chain specifically to separate the price setting decision from the entrepreneurs' financial problem. The final good producer purchases the retail goods and produces the final output good using a CES aggregation technology.

The final output good Y_t^f is a Dixit-Stiglitz aggregate of retail goods:

$$Y_t^f = \left(\int_0^1 Y_t(i)^{(\epsilon-1)/\epsilon} \, di \right)^{\epsilon/(\epsilon-1)}, \tag{B29}$$

where $Y_t(i)$ is the retail output from retailer i in terms of the wholesale good Y_t and $\epsilon > 1$ is the elasticity of substitution among the retail goods. The demand for each retail good is obtained by solving for the minimum cost combination of retail goods to produce a given quantity of the final good:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t^f, \tag{B30}$$

where $P_t(i)$ is the price of good i and:

$$P_t = \left(\int_0^1 P_t(i)^{(1-\epsilon)} \, di \right)^{1/(1-\epsilon)}, \tag{B31}$$

is the nominal price index of the final good.

Retailers set their prices optimally subject to the familiar Calvo (1983) price-setting mechanism. In period t a fraction $1-\chi$ of retailers are allowed to set the price of their good before any period t shocks are realized and taking the price of wholesale goods P_t^w as given. This means that inflation between period t and $t+1$ is determined by the end of period t. Recall that $1/X_t$ is the real price of a wholesale good so:

$$P_t^w \equiv \frac{P_t}{X_t}. \tag{B32}$$

All retailers optimizing in period t choose P_t^* to solve:

$$\sum_{k=0}^{\infty} \chi^k E_t \left\{ \Delta_{t,t+k}^{real} \frac{P_t^* - P_{t+k}^w}{P_{t+k}} Y_{t+k}^*(i) \right\}, \tag{B33}$$

Where $\Delta_{t,t+k}^{real} \equiv \beta^k C_t / C_{t+k}$ is the relevant discount factor and $Y_{t+k}^*(i)$ is the quantity of retail good i demanded in period $t+k$. The first-order condition for optimizing (B33) is:

$$\sum_{k=0}^{\infty}\chi^k E_t\left\{\Delta^{real}_{t,t+k}\left(\frac{P^*}{P_{t+k}}\right)^{-\epsilon}Y^*_{t+k}(i)\left[\frac{P^*_t}{P_{t+k}}-\left(\frac{\epsilon}{\epsilon-1}\right)\frac{P^w_{t+k}}{P_{t+k}}\right]\right\}=0. \tag{B34}$$

Finally, from (B31) it follows that under the assumed pricing mechanism, the law of motion for the price level is:

$$P_t=\left[\chi P^{1-\epsilon}_{t-1}+(1-\chi)\left(P^*_t\right)^{1-\epsilon}\right]^{1/(1-\epsilon)} \tag{B35}$$

B5 Government

The government is a simple operation finances an exogenous stream of real purchases G_t by collecting lump-sum taxes T_t and issuing base money M_t. The government budget constraint is:

$$G_t=\frac{\bar{M}_{t+1}-\bar{M}_t}{P_t}+T_t. \tag{B36}$$

Bernanke et al. assume that monetary policy is set according to a feedback rule for the nominal interest rate:

$$\bar{r}_{t+1}=\rho_r\bar{r}^n_t+\varsigma_\pi\pi_t+v^r_t, \tag{B37}$$

where ε^r_t is an exogenous monetary policy shock process.

B6 The household

The representative household has preferences over a stochastic stream of consumption, leisure and real money holdings that are represented by:

$$E_t\sum_{s=0}^{\infty}\beta^s\left\{\frac{C^{1-\sigma_{\text{C}}}_{t+s}}{1-\sigma_{\text{C}}}+\zeta_{\text{H}}\frac{(1-H_{t+s})^{1-\eta}}{1-\eta}+\zeta_{\text{D}}\frac{(\bar{M}_{t+1+s}/P_{t+s})^{1-\sigma_{\text{D}}}}{1-\sigma_{\text{D}}}\right\}, \tag{B38}$$

where $\beta \in (0,1)$ is the subjective discount factor and σ_{C}, σ_{D}, η, ζ_{H}, and ζ_{D} are positive constants[16]. In period t, the household consumes C_t units of the final output good and supplies H_t units of labor to the labor market. The household uses real money $\bar{M}_{t+1}/P_t$ held from period t to $t+1$ to facilitate transactions in period t.

The household maximizes (B38) subject to an infinite sequence of period budget constraints of the form:

$$C_t + \frac{M_{t+1}}{P_t} + \frac{\bar{A}_{t+1}}{P_t} \leqslant W_t H_t - T_t + \Pi_t + \frac{M_t}{(1+\pi_t)P_{t-1}} + \frac{\bar{R}_t \bar{A}_t}{(1+\pi_t)P_{t-1}}, \tag{B39}$$

where W_t is the real wage, T_t is net government transfers from the household, and Π_t denotes the real profit receipts from retail firm ownership. The gross inflation from period $t-1$ to period t is denoted $1+\pi_t$. The household uses two nominal assets to transfer its wealth intertemporally: money $\bar{M}_{t+1}$ and the asset $\bar{A}_{t+1}$. Money is a liability of the government and pays no interest. The liability of the intermediary – $\bar{A}_{t+1}$ – pays nominal interest $\bar{R}_{t+1}$.

By letting λ_t denote the multiplier on the budget constraint, the first order conditions for the maximization of (B38) subject to (B39) are:

$$C_t^{-\sigma_{\mathrm{C}}} = \lambda_t, \tag{B40}$$

$$\lambda_t = \zeta_{\mathrm{D}}(M_{t+1}/P_t)^{-\sigma_{\mathrm{D}}} + \beta \frac{E_t(\lambda_{t+1})}{1+\pi_{t+1}}, \tag{B41}$$

$$\zeta_{\mathrm{H}}(1-H_t)^{-\eta} = \lambda_t w_t, \tag{B42}$$

$$\lambda_t = \beta \frac{E_t(\lambda_{t+1}\bar{R}_{t+1})}{1+\pi_{t+1}}. \tag{B43}$$

[16]Bernanke et al. actually assume the period utility function is logarithmic in its arguments; i.e. $\sigma_{\mathrm{C}} = \eta = \sigma_{\mathrm{D}} = 1$

These conditions can be rewritten to eliminate λ_t:

$$\zeta_{\text{H}}(1-H_t)^{-\eta} = C_t^{-\sigma_C} W_t, \tag{B44}$$

$$\left(\frac{M_{t+1}}{P_t}\right)^{\sigma_{\text{D}}} = \zeta_{\text{D}} C_t^{\sigma_{\text{C}}} \left(\frac{\bar{R}_{t+1}-1}{\bar{R}_{t+1}}\right)^{-1}, \tag{B45}$$

$$C_t^{-\sigma_{\text{C}}} = \beta \frac{\bar{R}_{t+1}}{1+\pi_{t+1}} E_t(C_t^{-\sigma_{\text{C}}}). \tag{B46}$$

Finally, the aggregate goods market must clear:

$$Y_t^f = C_t + C_t^e + I_t + G_t + \mu R_t^k Q_{t-1} K_t \int_0^{\bar{\omega}} \omega \, dF(\omega). \tag{B47}$$

and the intermediary's balance sheet must balance:

$$B_{t+1} = \bar{A}_{t+1}, \tag{B48}$$

so that the amount of the asset $\bar{A}_{t+1}$ held by the household equals borrowing B_{t+1} by the entrepreneurs.

B7 Linearized equilibrium conditions

Here I describe equilibrium in the Bernanke et al. framework. I present 13 linearized equilibrium conditions describing the evolution of 13 endogenous variables. Lowercase variables are in log-deviations from the steady state and capital letters denote steady state values. The variables ϕ_t^y, $\phi_t^{c^e}$, and ϕ_t^n collect terms that, according to Bernanke et al., do not affect the dynamics under a reasonably general set of parameterizations.

$$0 = \frac{C}{Y}c_t + \frac{I}{Y}i_t + \frac{G}{Y}g_t + \frac{C^e}{Y}c_t^e - y_t + \phi_t^y \tag{B49}$$

$$\sigma_{\text{C}} E_t c_{t+1} - (\bar{r}_{t+1} - \pi_{t+1}) = \sigma_{\text{C}} c_t \tag{B50}$$

$$-\frac{1}{\bar{R}-1}\bar{r}_{t+1} = \sigma_{\text{D}} m_{t+1} - \sigma_{\text{C}} c_t \tag{B51}$$

$$\frac{\gamma^{-1}(1-\gamma)N}{C^e} n_{t+1} = c_t^e - \phi_t^{c^e} \tag{B52}$$

$$\bar{r}_{t+1} = \rho_r \bar{r}_t + \varsigma_\pi \pi_t + v_t^r \tag{B53}$$

$$E_t r_{t+1}^k - (\bar{r}_{t+1} - \pi_{t+1}) + \nu^{-1}(n_{t+1} - q_t) = \nu^{-1} k_{t+1} \tag{B54}$$

$$\vartheta q_t = r_t^k + (1-\vartheta)(x_t + k_t - y_t) + q_{t-1} \tag{B55}$$

$$q_t = \varphi(k_t - i_t) \tag{B56}$$

$$0 = z_t + \alpha k_t - y_t + (1-\alpha)\Omega h_t \tag{B57}$$

$$0 = y_t - (1 + (\eta - 1)H)(1-H)^{-1} h_t - \sigma_{\text{C}} c_t - x_t \tag{B58}$$

$$k_{t+1} = (1+\varphi)\delta i_t + (1 - \delta(1+\varphi)) k_t \tag{B59}$$

$$-\kappa E_t x_{t+1} + \beta E_t \pi_{t+2} = \pi_{t+1} \tag{B60}$$

$$n_{t+1} = \frac{\gamma \bar{R} K}{N} r_t^k + \left(\gamma \bar{R} - \frac{\gamma \bar{R} K}{N}\right)(\bar{r}_t - \pi_t) + \gamma \bar{R} n_t + \phi_t^n \tag{B61}$$

where:

$$\phi_t^y \equiv \frac{DK}{Y} \log\left(\mu R_t^k Q_{t-1} K_t E(\omega | \omega \leqslant \bar{\omega}_t)/DK\right)$$

$$\phi_t^{c^e} \equiv -\frac{\gamma^{-1}(1-\gamma)(1-\alpha)(1-\Omega)Y}{C^e X}(y_t - x_t)$$

$$\begin{aligned}\phi_t^n \equiv & \frac{\gamma K}{N}(R^k - \bar{R})(q_{t-1} + k_t) + (1-\alpha)(1-\Omega)\frac{Y}{X}(y_t - x_t) \\ & + \gamma DK \log\left(\mu R_t^k Q_{t-1} K_t E(\omega | \omega \leqslant \bar{\omega}_t)/DK\right)\end{aligned}$$

and:

$$
\begin{aligned}
D &\equiv \mu E(\omega | \omega \leqslant \bar{\omega}) \\
\nu &\equiv \frac{\psi'(R^k/\bar{R})}{\psi(R^k/\bar{R})} \frac{R^k}{\bar{R}} \\
\vartheta &\equiv \frac{1-\delta}{\alpha Y/XK + 1 - \delta} \\
\varphi &\equiv \frac{\Phi''(I/K)}{\Phi'(I/K)} \frac{I}{K} \\
\kappa &\equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}
\end{aligned}
$$

Equation (B49) is the linearized aggregate resource constraint and (B50) is the household's Euler equation. Notice that since inflation and the nominal rate are predetermined at date t, the household saves at the real risk-free rate. Equation (B51) is the household's demand for real money. Equation (B52) links entrepreneurial consumption to net worth. With the parameter values that Bernanke et al. use for their computational analysis, the coefficient on net worth is close 1 so that movements in entrepreneurial net worth are closely matched by movements in entrepreneurial consumption. Equation (B53) is the policy rule for the nominal interest rate.

Equation (B54) is a linearized version of the entrepreneur's demand for capital. Financial market frictions create a spread between the expected return to capital $E_t r^k_{t+1}$ and the real cost of funds for the intermediary $\bar{r}_{t+1} - \pi_{t+1}$. Other things equal, this spread narrows with higher entrepreneurial net worth and lower capital prices. When credit frictions are removed, (B54) becomes:

$$
E_t r^k_{t+1} = \bar{r}_{t+1} - \pi_{t+1}. \tag{B62}
$$

Next, equation (B55) is the linearized return to capital. Equation (B56) is the first order condition of the capital producer and it links the price of capital to movement in investment and the capital stock. Equation (B57) is the linearized production function and

(B58) combines the household's first-order condition for supplying labor with the marginal product of labor. The evolution of capital is represented by (B59). This equation differs slightly from what appears in Bernanke et al. because of the assumption I made about the explicit from of the capital production function. And equation (B60) is a version of the new Keynesian Phillips curve. This is the same equation that appears in Bernanke et al. except that I have iterated it forward one period. Notice that while the Philips curve is forward-looking, inflation is determined one period in advance.

Equation (B61) describes how entrepreneurial net worth evolves. Net worth is increasing in the previous period's return to capital r_t^k and decreasing in the real cost of loanable funds $\bar{r}_t - \pi_t$. In plausible parameterizations, the product $\gamma \bar{R}$ is close to – but less then – 1 so fluctuations in net worth are highly persistent. This equation appears different from what is given in Bernanke et al. because I have taken care to write out the full linearization of the net worth equation.

Finally, the 3 exogenous variables – government consumption g_t, aggregate productivity z_t, and the monetary policy shock v_t^r – are assumed to evolve according to AR(1) processes:

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g \tag{B63}$$

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z \tag{B64}$$

$$v_t^r = \rho_v v_{t-1}^r + \varepsilon_t^r \tag{B65}$$

where ε_t^g, ε_t^z, and ε_t^r are i.i.d. disturbances. This concludes the description of the basic Bernanke et al. model.

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C Figures

Figure C1: Impulse responses to a productivity shock.

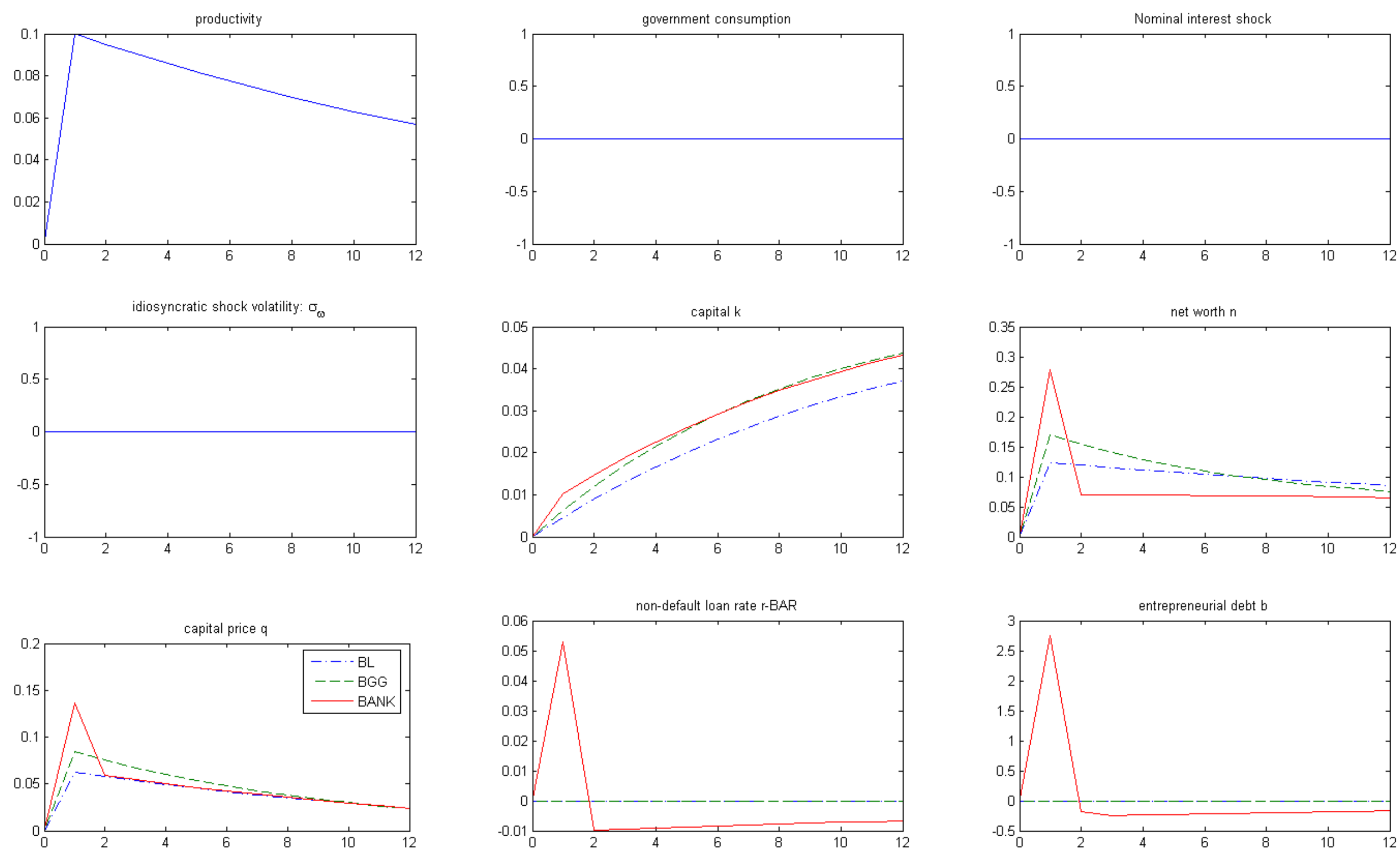

Figure C2: Impulse responses to a productivity shock (continued).

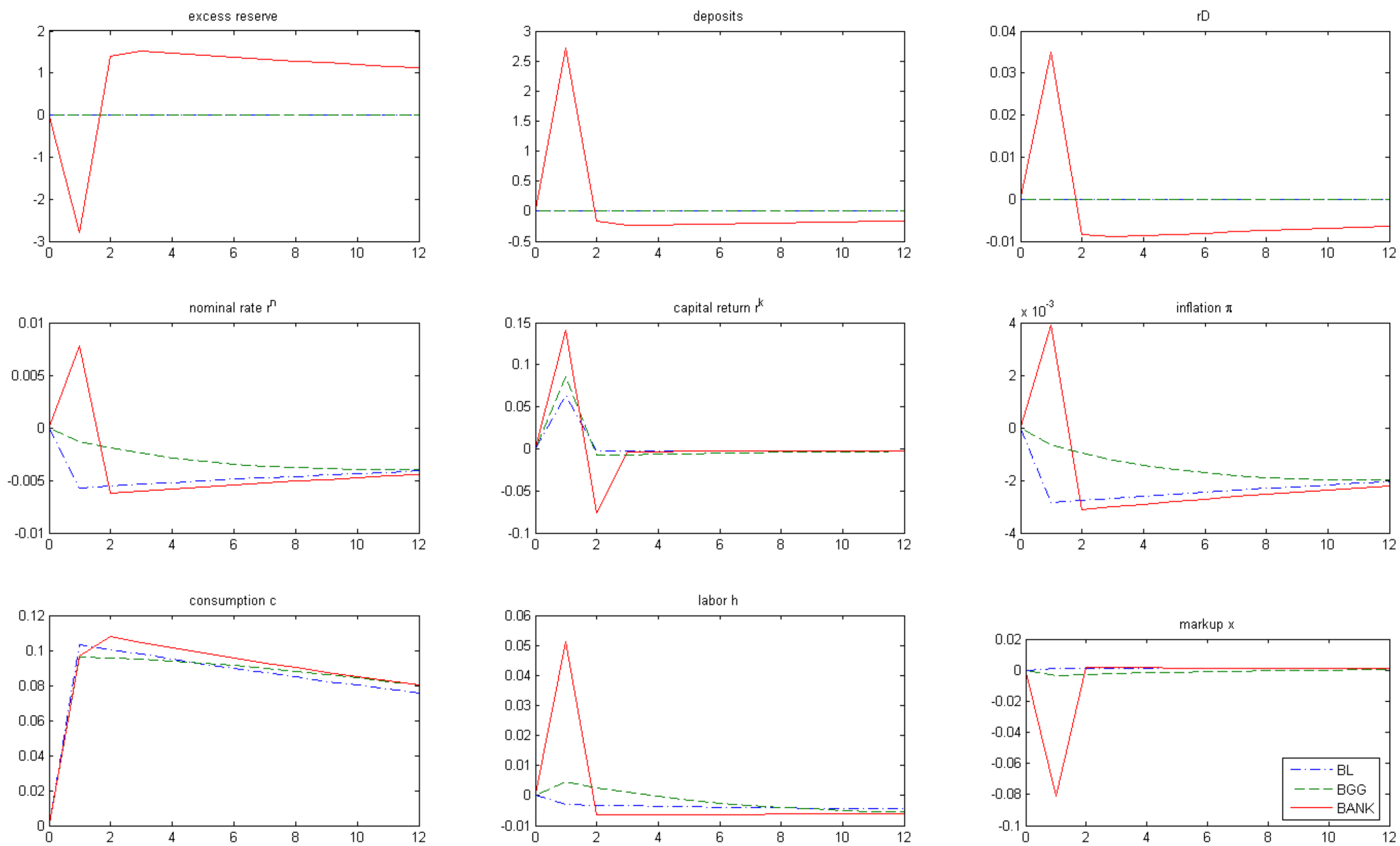

Figure C3: Impulse responses to a productivity shock (continued).

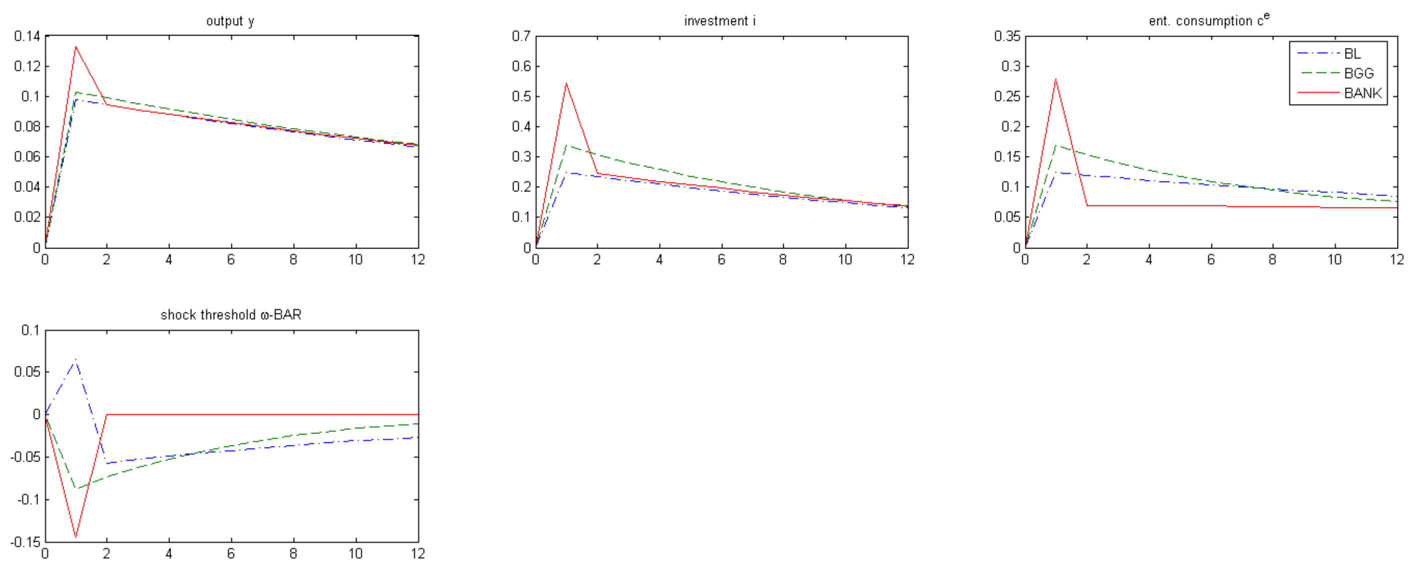

Figure C4: Impulse responses to a monetary policy shock.

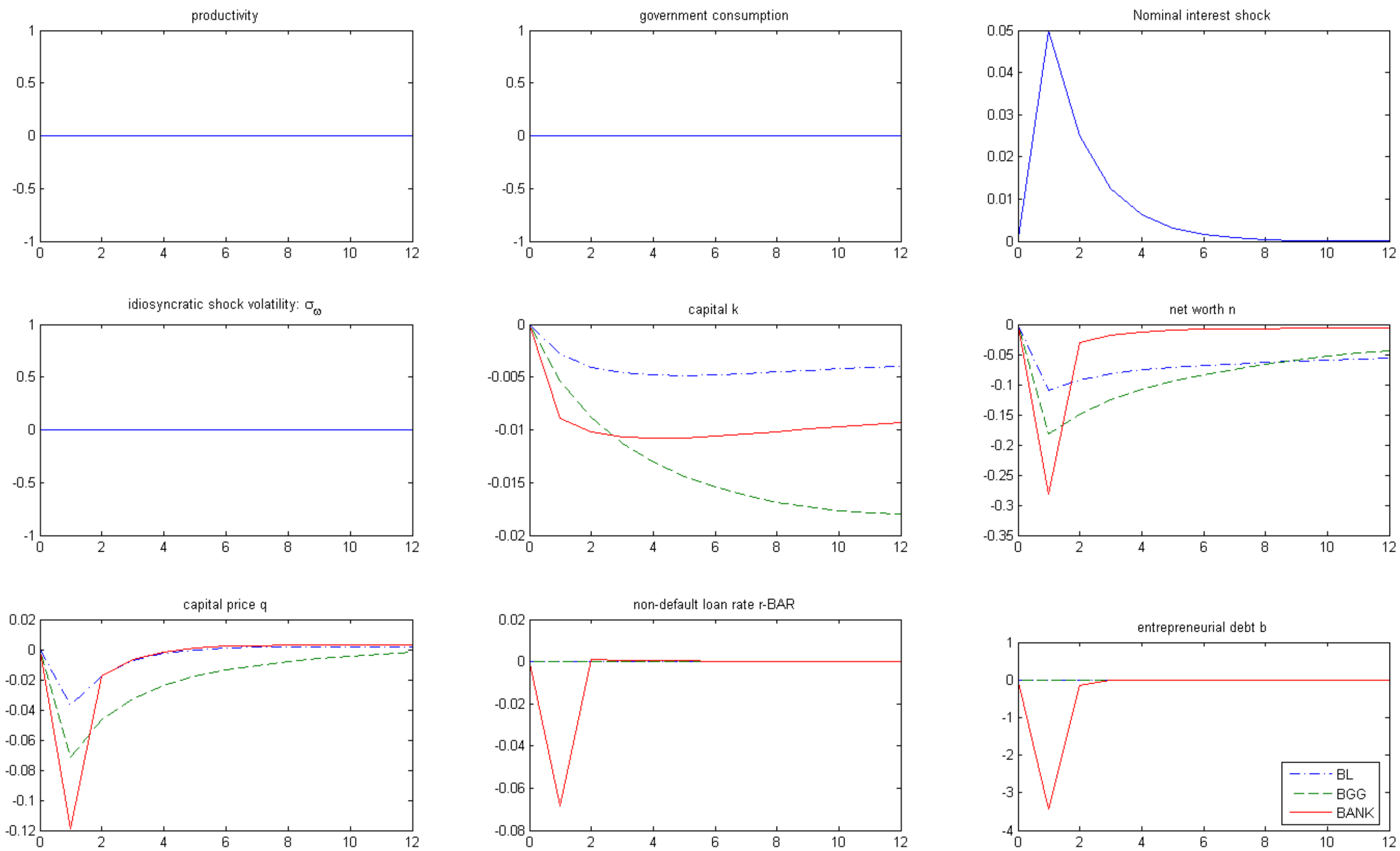

Figure C5: Impulse responses to a monetary policy shock (continued).

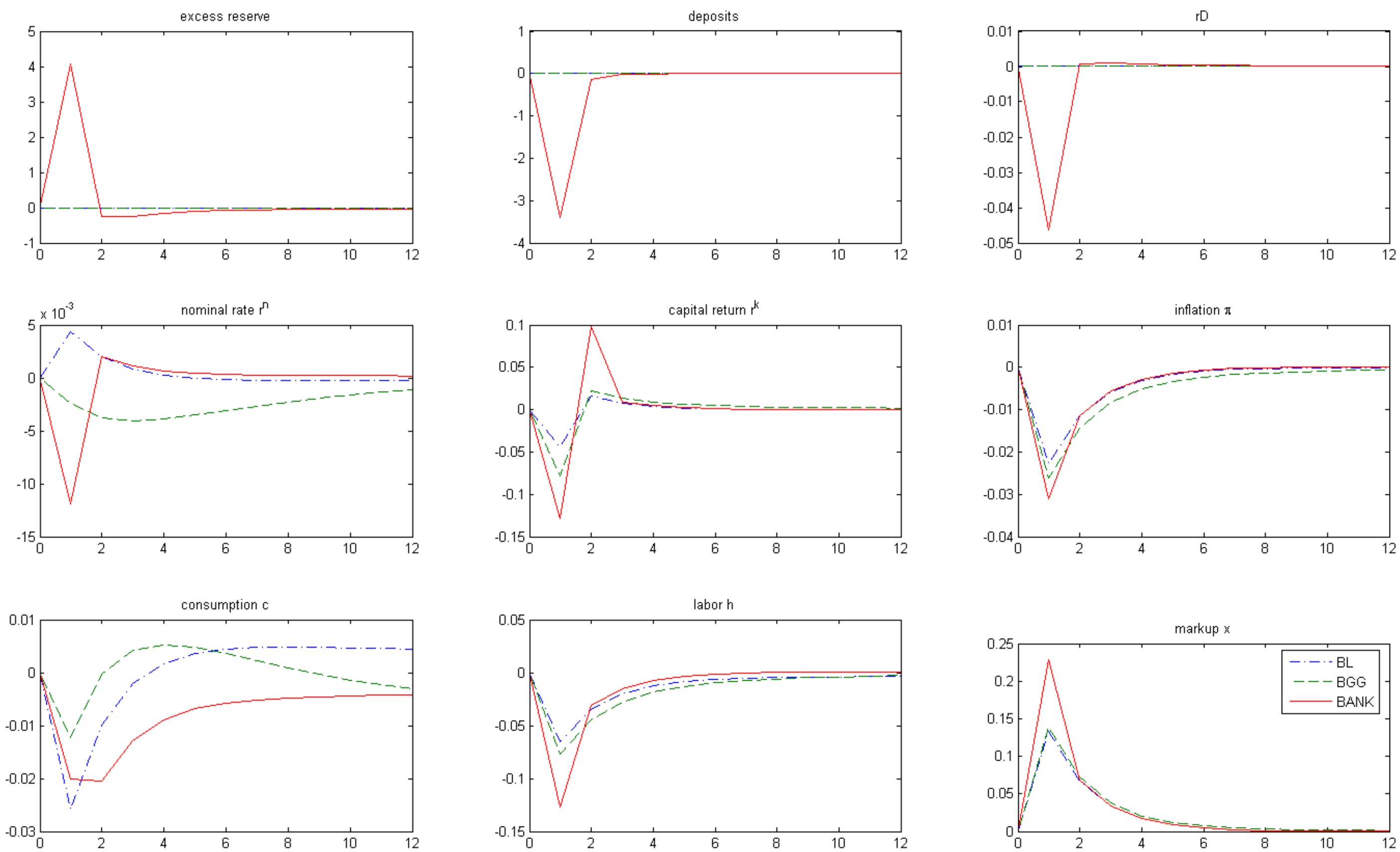

Figure C6: Impulse responses to a monetary policy shock (continued).

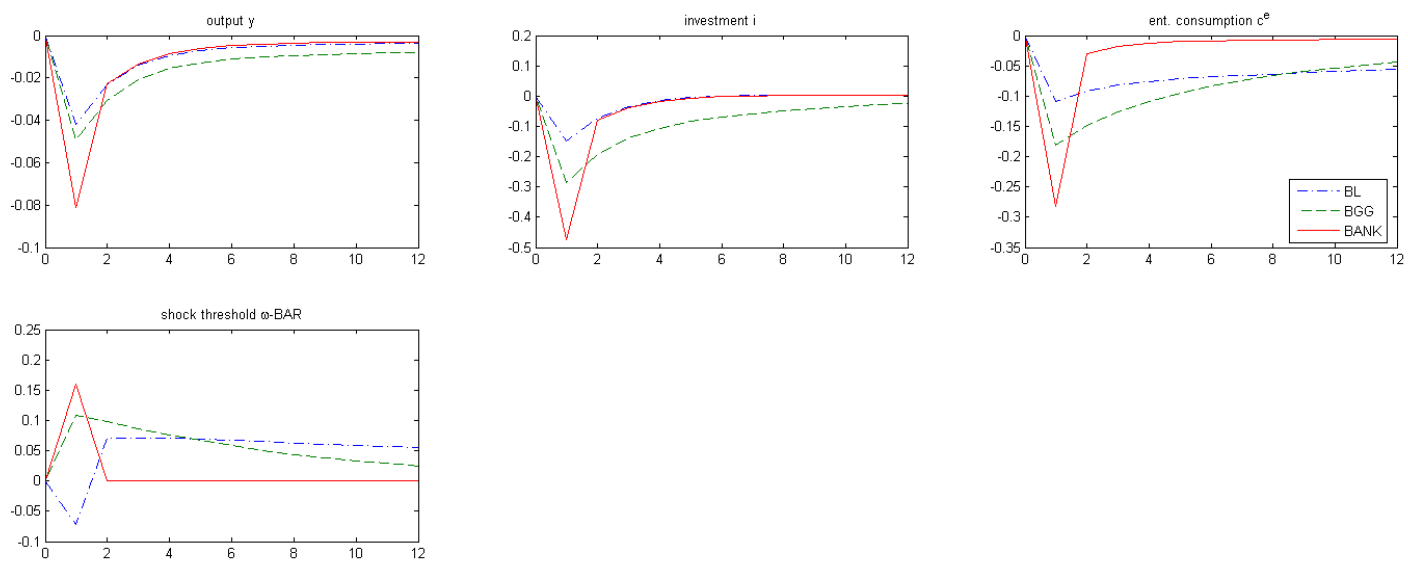

Figure C7: Impulse responses to the financial shock.

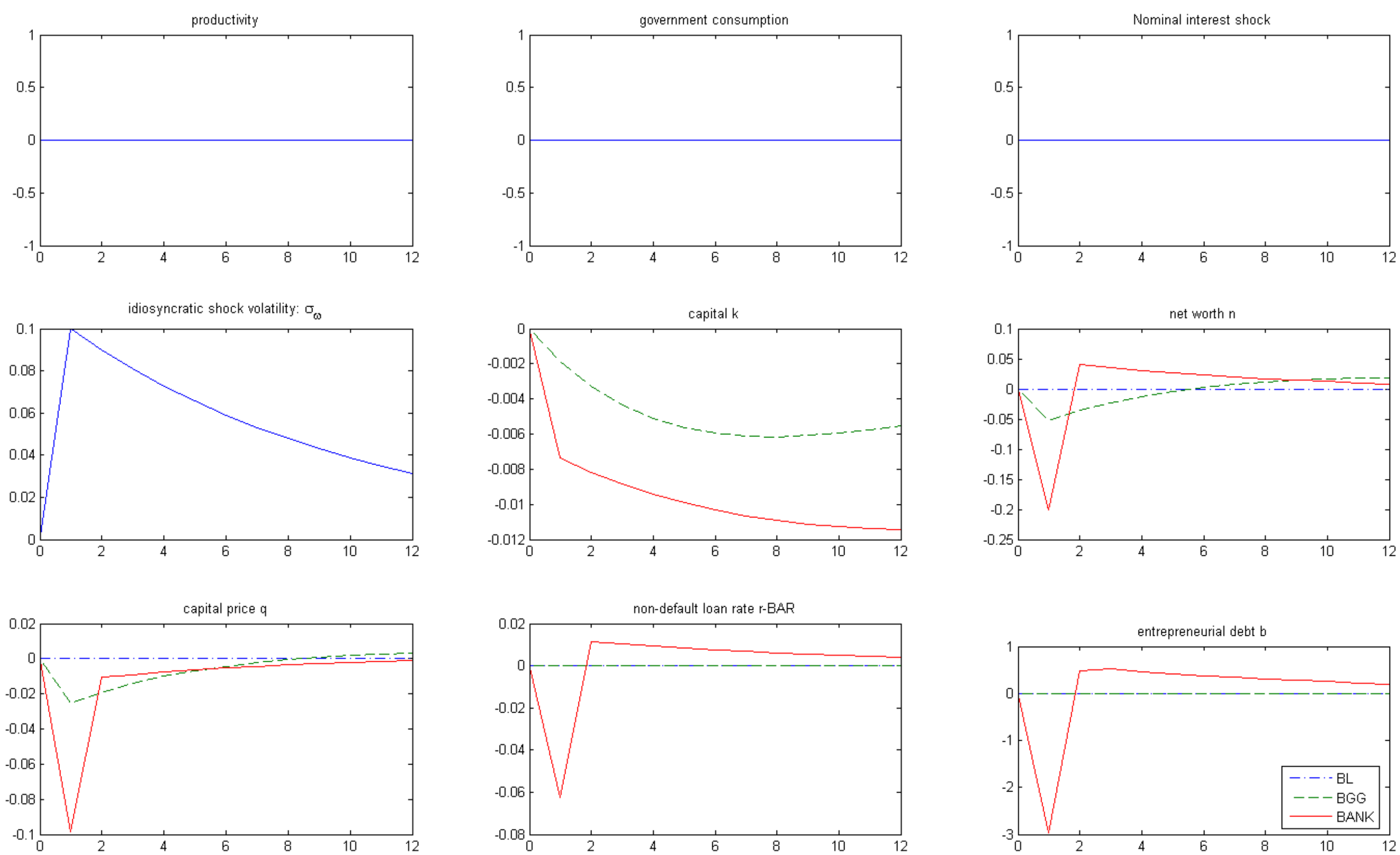

Figure C8: Impulse responses to the financial shock (continued).

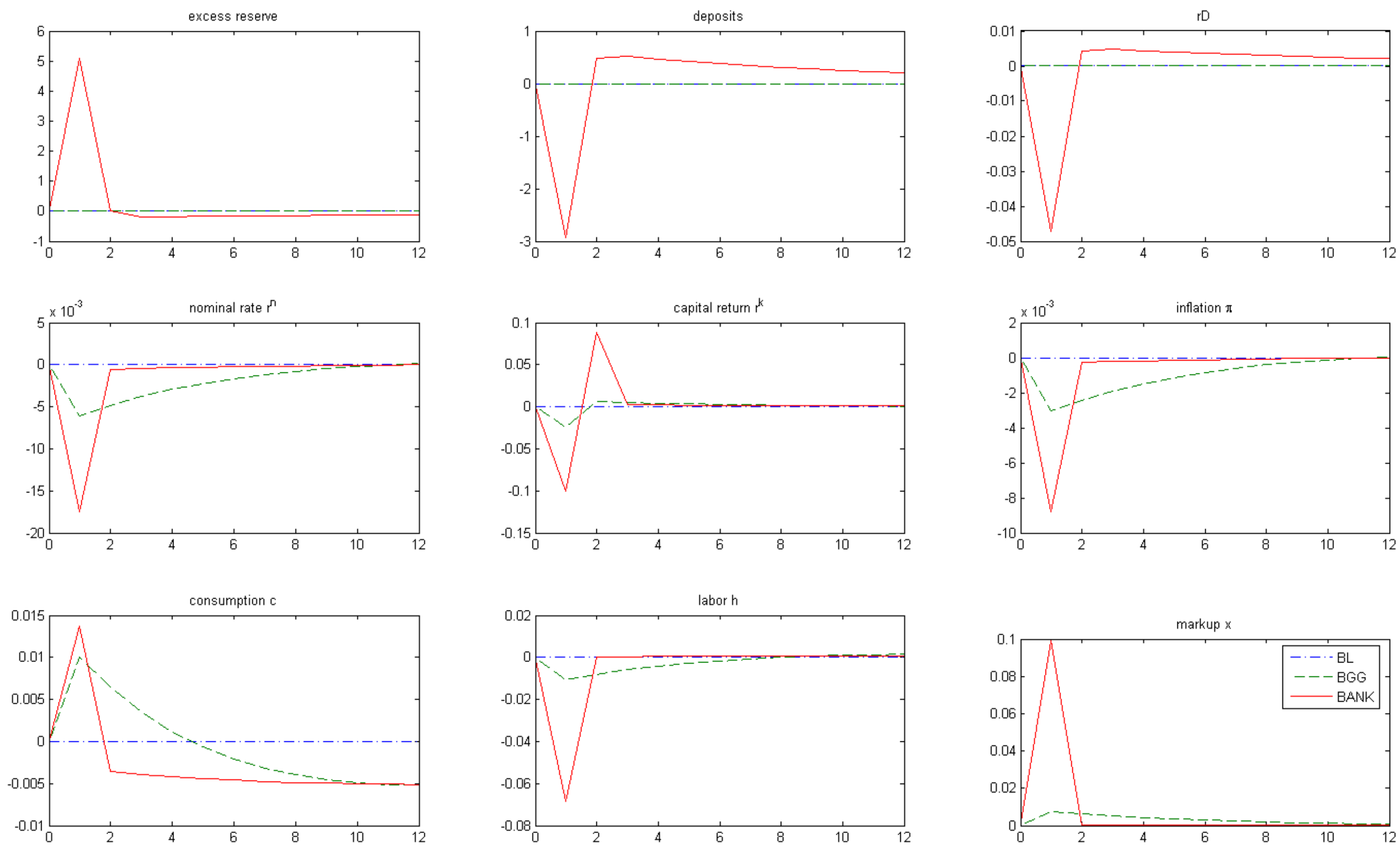

Figure C9: Impulse responses to the financial shock (continued).