

Aggregate Uncertainty: Krusell and Smith

Econ720

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A Bewley Model of the Wealth Distribution

- We study Krusell and Smith (1998 JPE).
- The problem: In models with aggregate uncertainty, the entire (wealth) distribution is a state variable.
- KS propose an important method for solving models with aggregate uncertainty and heterogeneity.
- It contains an important finding: **approximate aggregation**
 - First moments are often enough to approximate the entire distribution of the state vector.
 - The aggregate law of motion for K looks a lot like an individual's decision rule.

The Question

- Economists commonly use models with representative households.
- Are these **good approximations** for models with heterogeneous agents?

- 1 In the standard RBC framework, heterogeneity does not seem to matter.
- 2 A method for computing models with heterogeneity and aggregate uncertainty.

The approach:

- Compute a standard RBC model.
- Add uninsured employment risk.
- Compare: how good is the representative agent approximation?

Approximate Aggregation

- Aggregate consumption and saving resemble those of a representative agent.
- Therefore: it is enough to keep track of mean wealth, instead of keeping track of the wealth distribution, in order to forecast future prices.

- The distribution of wealth is unimportant, if most agents have the same marginal propensity to consume out of aggregate shocks.
- This is true in the model because agents achieve good self-insurance (consumption policy functions are roughly linear).
- Only for the very poor does self-insurance fail. But the very poor account for only a small fraction of aggregate consumption.
- A point made in passing: **preference heterogeneity** permits the model to match the U.S. wealth distribution.

- Output is produced from a Cobb-Douglas production function

$$\bar{y} = z \bar{k}^\alpha \bar{l}^{1-\alpha}$$

where z is a two-state Markov process.

- \bar{k}, \bar{l} : aggregate capital and labor inputs.
- The resource constraint is

$$\bar{k}' = (1 - \delta) \bar{k} + \bar{y} - \bar{c}$$

- There is a single representative agent who maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t)$$

- The agent has a random labor endowment: $\varepsilon = 0$ or $\varepsilon = 1$.
 - no serial correlation
- The fraction of employed people ($\varepsilon = 1$) is u_g in good times (high z) and u_b in bad times.
- Households hold capital, but cannot borrow: $k \geq 0$.

Household problem

- Individual state: k, ε .
- Aggregate state: z, Γ .
- Γ is the distribution of households over (k, ε) .
- Bellman equation:

$$v(k, \varepsilon, z, \Gamma) = \max u(c) + \beta E v(k', \varepsilon', z', \Gamma')$$

subject to

$$\begin{aligned}k' &= r(\bar{k}, \bar{l}, z) k + w(\bar{k}, \bar{l}, z) \varepsilon + (1 - \delta) k - c \\ \Gamma' &= H(\Gamma, z, z')\end{aligned}$$

- H is the law of motion for the distribution, given z (basically due to household saving decisions).

Objects:

- Household value function v and decision rule $k' = f(k, \varepsilon, z, \Gamma)$.
- Price functions $r(\cdot)$ and $w(\cdot)$.
- Law of motion for the distribution of (k, ε) : H .

These satisfy:

- v, f solve the household problem.
- r, w are consistent with firm profit maximization.
- H is "consistent with" household decision rules f .
 - see Bewley slides

- Problem: The distribution Γ cannot be described with a finite number of parameters.
- KS's idea: Only keep track of a small number of moments of the distribution: ℓ .
- Guess a law of motion for ℓ : $\ell' = h(z, \ell)$.
- Solve the household problem, given h . Same method as before.

- Bellman equation:

$$v(k, \varepsilon, z, \ell) = \max u(c) + \beta E v(k', \varepsilon', z', \ell')$$

subject to

$$\begin{aligned}k' &= r(z, \ell) k + w(z, \ell) \varepsilon + (1 - \delta) k - c \\ \ell' &= h(z, \ell)\end{aligned}$$

- Update the guess for h from the household solution: Simulate many households and "find" laws of motion for ℓ (how?).
- Iterate until the guesses for h converge.

The key problem:

- How to represent the distribution using a small vector ℓ ?
- How to find the law of motion h from simulated household histories?

Krusell and Smith choose to approximate Γ using the first J moments: mean, standard deviation, etc.

To check the accuracy of the approximation:

- Verify that the forecast errors are "small."
- Verify that increasing J has little effect on the equilibrium properties.

This is a form of **bounded rationality**: Households only use the first J moments and forecast them using only today's moments.

Updating the laws of motion

- Simulate a large number of households from their decision rules.
- Compute the summary statistics ℓ for each date.
- Postulate a functional form to approximate h .
 - E.g.: Postulate that ℓ'_j is a linear function of ℓ .
- Estimate the coefficients by running a regression of ℓ' on ℓ .
- To check the accuracy of the approximation: Try alternative functional forms for h .

Example

- Approximate the distribution by keeping track only of aggregate capital: $\ell = \bar{k}$, $J = 1$.
- Impose a log-linear law of motion. The function H is of the form:
 - If $z = z_g$: $\ln(\bar{k}') = a_0 + a_1 \ln(\bar{k})$.
 - If $z = z_b$: $\ln(\bar{k}') = b_0 + b_1 \ln(\bar{k})$.
- Solve and simulate the household problem, given (a_0, a_1, b_0, b_1) . Obtain a sequence $\{\bar{k}_t\}$.
- Estimate new values for (a_0, a_1, b_0, b_1) using OLS.
- Calculate forecasting errors from the regression. If they are small: stop.
- Otherwise, resolve household problem with estimated values for (a_0, a_1, b_0, b_1) and iterate.

- Choose standard RBC parameters:
- Preferences: $\beta = 0.99$, $\sigma = 1$.
- Technology: $\alpha = 0.36$. $z_g = 1.01$ and $z_b = 0.99$ based on size of aggregate output fluctuations.
- Unemployment rates: $u_g = 0.04$ and $u_b = 0.1$.
- P_z : match length of business cycles.
- P_ε : Labor endowments match length of unemployment spells.

- Solve for $J = 1$ and $\ell = \bar{k}$.
- Forecasting equations are of the form: $\ln(\bar{k}') = a_0 + a_1 \ln(\bar{k})$.
- Goodness of fit:
 - $R^2 = 0.999998$.
 - Variance of error term: $\sigma^2 = 0.00003$.
- The log-linear forecasting equation is nearly perfect.
- The welfare gains from better forecasts are negligible.

Why approximate aggregation?

- Individual decision rules are nearly linear.
- All agents have nearly identical marginal propensities to consume.
- Redistributing wealth has essentially no effect on aggregate consumption.

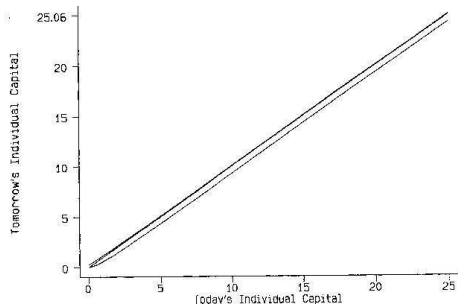


FIG. 2.—An individual agent's decision rules (benchmark model, aggregate capital = 11.7, good aggregate state).

Why approximate aggregation?

- Why are decision rules nearly linear?
- Most agents are rich enough to almost completely smooth shocks.
- One reason: aggregate capital is (by construction) 3 times larger than output.
- Another reason: agents live forever.
- Only a small number of poor agents cannot self-insure. But they account for a tiny fraction of aggregate wealth.

How important is heterogeneity for business cycles?

- The experiment: Compare two identical economies, except that one has complete markets (therefore no heterogeneity).
- Finding: heterogeneity has little effect on the model's business cycle properties.

AGGREGATE TIME SERIES

Model	Mean(k_t)	Corr(c_t, y_t)	Standard Deviation (i_t)	Corr(y_t, y_{t-4})
Benchmark:				
Complete markets	11.54	.691	.031	.486
Incomplete markets	11.61	.701	.030	.481
$\sigma = 5$:				
Complete markets	11.55	.725	.034	.551
Incomplete markets	12.32	.741	.033	.524
Real business cycle:				
Complete markets	11.56	.639	.027	.342
Incomplete markets	11.58	.669	.027	.339
Stochastic- β :				
Incomplete markets	11.78	.825	.027	.459

Preference heterogeneity

- The question:
 - Does approximate aggregation still hold when there is a realistic amount of wealth heterogeneity?
- The approach:
 - Add enough preference heterogeneity to the model to roughly replicate the observed distribution of wealth.
 - Check that the mean is enough to forecast prices very accurately.
- Model
 - Allow for 3 arbitrary values of β : 0.986, 0.989, 0.993.
 - Agents switch β values stochastically, on average every 50 years (once per generation).

Approximate aggregation is still very good:

- $R^2 = 0.99999$
- $\sigma^2 = 0.00006$.

- The main contribution of Krusell and Smith is the **method** for computing economies with heterogeneity and aggregate uncertainty.
- The finding that **approximate aggregation** holds seems robust for frictionless business cycle models (the RBC type), but we don't know whether it holds more generally.

Krusell, Per; Anthony A. Smith (1998). "Income and wealth heterogeneity in the macroeconomy." *Journal of Political Economy* 5: 867-96.