

Bewley Models

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- For many applications we need models with **heterogeneous agents**.
- In Bewley models, agents are ex ante identical.
- They are ex post heterogeneous because they are hit by idiosyncratic shocks.
- Incomplete markets prevents sharing these risks.

Endowment Economy

An Endowment Economy

- Demographics:
 - There is a unit measure of households.
 - Each lives forever.
- Preferences:

$$E \sum_{t=0}^{\infty} \beta^t u(c_t) \quad (1)$$

- Technology:
 - Households receive random endowments $y_t \in Y$ (finite).
 - Transition matrix: $\pi(y'|y)$.

No aggregate uncertainty

- Assume a "law of large numbers."
- Let $\Pi(y)$ be the stationary distribution of y .
- Assume that the fraction of households with endowment y is $\Pi(y)$.
- The aggregate endowment \bar{y} is constant over time.
- With complete markets, households would not face any uncertainty.

- Flow budget constraint:

$$a' = y + (1 + r)a - c \quad (2)$$

- Borrowing constraint:

$$a' \geq -b \quad (3)$$

- Focus on a **stationary equilibrium**.
 - Meaning: Aggregates & prices are constant over time.
- State vector: (a, y) .
- Given: r .
- Bellman equation:

$$v(a, y) = \max u(c) + \beta \sum_{y'} \pi(y'|y) v(a', y') \quad (4)$$

s.t. budget constraint and borrowing constraint.

Household problem I

- The borrowing constraint may bind. We need Kuhn-Tucker.
- Bellman equation

$$v(a, y) = \max u(y + (1+r)a - a') \quad (5)$$

$$+ \beta \sum_{y'} \pi(y'|y) v(a', y') + \mu(a' + b) \quad (6)$$

- First-order conditions:

$$u'(c) = \beta \sum \pi(y'|y) \frac{\partial v(a', y')}{\partial a'} + \mu \quad (7)$$

$$\partial v / \partial a = u'(c) (1+r) \quad (8)$$

$$\mu(a' + b) = 0 \quad (9)$$

- Euler equation

$$u'(c) \geq E\beta(1+r)u'(c') \quad (10)$$

with equality if $a' > -b$.

- Solution: $v(a, y), c(a, y), a'(a, y)$ that satisfy the usual conditions.

How does the household behave?

- For a given y , if "cash on hand"

$$x = y + (1 + r)a \quad (11)$$

is sufficiently high: Choose $a' > -b$ and satisfy standard Euler equation.

- If "cash on hand" is below a cutoff, set $a' = -b$ and "violate" the Euler equation.
 - The borrowing constraint depresses current consumption.

Stationary Recursive Competitive Equilibrium

- Aggregate state: The joint distribution of assets and endowments: $\Phi(a, y)$.
 - This is needed to compute aggregates.
- Objects:
 - Household: $v(a, y), c(a, y), a'(a, y)$.
 - $\Phi(a, y)$.
 - Price function: $r(\Phi)$.
- Equilibrium conditions:
 - Household: see above.
 - Market clearing.
 - Time invariance of Φ .

Stationary Recursive Competitive Equilibrium

Market clearing

- Goods:

$$\int \int c(a, y) \Phi(da, dy) = \int y \Pi(dy) \quad (12)$$

- Bonds:

$$\int \int a'(a, y) \Phi(da, dy) = 0 \quad (13)$$

Time invariance of the distribution

- Informally, household choices determine tomorrow's distribution Φ' .
- The policy function $a'(a, y)$ implies a law of motion for Φ .
- In stationary equilibrium, the law of motion must imply $\Phi' = \Phi$.

- Define a transition function $Q((a, y), (A, Y))$.
- Its value is the probability (mass) of households in state (a, y) today that end up in $(a', y') \in (A, Y)$ tomorrow.

$$Q((a, y), (A, Y)) = \begin{cases} \sum_{y' \in Y} \pi(y'|y) & \text{if } a'(a, y) \in A \\ 0 & \text{otherwise} \end{cases}$$

- This is because a' is deterministic.

- Law of motion

$$\Phi'(A, Y) = \int \int Q((a, y), (A, Y)) \Phi(da, dy) \quad (14)$$

- In words:
 - $\Phi'(A, Y)$: What is the mass of households in the set of states (A, Y) tomorrow?
 - For any (a, y) , this is given by $Q((a, y), (A, Y))$.
 - Sum over all (a, y) to get the total mass.
- Stationarity then means: $\Phi'(A, Y) = \Phi(A, Y)$ for all (A, Y) .

Non-stationary Equilibrium

- The equilibrium concept generalizes easily to economies where Φ evolves over time.
- Household:
 - Add the aggregate state Φ to the household's state: $v(a, y, \Phi)$ and $a'(a, y, \Phi)$.
 - The household takes prices as functions of the aggregate state: $r(\Phi)$.
 - The household knows the law of motion for Φ : $\Phi' = H(\Phi)$.
- Equilibrium:
 - Drop stationarity of Φ .

- Models of the wealth distribution:
 - Krusell, Per; Anthony A. Smith (1998). "Income and wealth heterogeneity in the macroeconomy." *Journal of Political Economy* 5: 867-96.
- Models of business cycles with heterogeneous agents:
 - Rios-Rull, Jose-Victor (1996). "Life-Cycle Economies and Aggregate Fluctuations." *Review of Economic Studies* 63(3): 465-89.

- Acemoglu, "Introduction to Modern Economic Growth," ch. 17.4.
- Krueger, "Macroeconomic Theory," ch. 10.