

Problem Set 8: Search and Matching

Econ720. Fall 2011. Prof. Lutz Hendricks

1 Business Cycles and Search

Consider the following version of a McCall model.

Time is discrete: $t = 0, 1, 2, \dots$. There is a continuum of workers that are either employed or unemployed. Each worker dies with probability μ in each period.

An employed worker enters the period with the current wage offer w . She can choose to remain employed at that wage or to draw a new offer from a distribution $F(w)$ with support $[0, M]$. If she chooses to draw a new offer, her previous job is lost and she becomes unemployed. Then she earns benefits b in the current period and enters the next period as an unemployed worker with offer w' in hand.

An unemployed worker faces essentially the same problem. She can also choose to work at last period's offered wage, or she can draw a new offer. If she draws a new offer, the previous one is lost and she receives benefits b in the current period.

Each worker seeks to maximize $E \sum_{t=0}^{\infty} (1 - \mu)^t \beta^t c_t$ where c_t equals the worker's income.

Questions:

1. Characterize the decision rule of an employed / unemployed worker. Hint: How do the two differ?
2. How does μ affect the decision rule of an unemployed worker? What is the intuition?
3. Now consider an environment where the wage fluctuates in an i.i.d. fashion between high and low values (booms and recessions). To implement this easily, assume that the worker is paid $w + z_s$ where the wage is drawn from $F(w)$ as before and z_s is high in a boom ($s = B$) and low in a recession ($s = R$). Show that the reservation wage is lower in a boom than in a recession. So the model can generate voluntary quits.

1.1 Answer: Business Cycles and Search

[Based on a question due to Ljungqvist and Sargent 2007]

1. Decision rule: Note that the employed and unemployed are identical once they enter the period. They only differ in last period's income. So they have the same value functions.

The worker has the past wage offer w in hand. If he accepts it, he earns w today and $V(w)$ tomorrow. If he rejects, w is lost and he must draw a new offer w' .

$$V(w) = \max \left\{ w + \beta(1 - \mu)V(w), b + \beta(1 - \mu) \int V(w')dF(w') \right\} \quad (1)$$

Now we show that this problem implies a reservation wage property. Assume that $V(w)$ is weakly increasing. Then $V(w)$ is the max of a constant (reject) and an increasing function of w . This implies the reservation wage property as in the McCall model.

For wages above the reservation wage: $V(w) = \frac{w}{1 - \beta(1 - \mu)}$. Now we solve for the reservation wage. The steps are analogous to the McCall model and so is the result:

$$\bar{w} - b = \frac{\beta(1 - \mu)}{1 - \beta(1 - \mu)} \int_{\bar{w}}^M (w' - \bar{w})dF(w') \quad (2)$$

2. Effect of μ : Note that the LHS of (2) is increasing in \bar{w} while the RHS is decreasing. Therefore an increase in μ results in a decrease in \bar{w} . Intuition: the RHS of (2) is the option value of searching for a better offer. If one does not expect to live very long, the option is less valuable and one should be less picky when an offer arrives.

3. Cycles: The Bellman equation is now

$$V(w, s) = \max \left\{ w + z_s + \sum_{s'} \frac{\beta}{2}(1 - \mu)V(w, s'), b + \sum_{s'} \frac{\beta}{2}(1 - \mu) \int V(w', s')dF(w') \right\} \quad (3)$$

This implies a reservation wage as before. The reservation wages solve

$$\bar{w}_s + \sum_{s'} \frac{\beta}{2}(1 - \mu)V(\bar{w}_{s'}, s') = Q - z_s \quad (4)$$

so that the reservation wage is decreasing in z_s . Workers may accept a job in a boom and quit it in a recession.