

# Search Models

Prof. Lutz Hendricks

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- Matching models have one drawback:
  - the matching function is a black box
- Search models model explicitly how matches come about.
- Their drawback: hard to take to the data.
- Applications:
  - Money - the main / original motivation for search
  - Housing
  - Labor market

- Most models of money either have no notion of **liquidity** (OLG model) or they assume liquidity (CIA).
- Search models offer a micro-foundation for liquidity.
- **Caveats:**
  - The institutional arrangement is medieval.
  - Money is literally currency, not credit.

# The Idea

- People produce and consume many goods.
- To consume what they want, people need to search for sellers.
- Without money: trade is hard.
  - the double coincidence of wants problem.
- One solution: designate one good to be "money."
- Money means: the good is accepted by all traders.
  - a social convention.

# Kiyotaki-Wright Model

- There is a unit mass of ex ante identical, infinitely lived agents.
- There is a unit mass of goods types.
- Goods are indivisible.
  - The model gets harder when goods are divisible.

# Kiyotaki-Wright Model

## Preferences

- Each agent can consume fraction  $x \in (0, 1)$  of the goods.
- Each good can be consumed by fraction  $x$  of households.
- Consumption yields utility  $U$ .
- Not consuming yields 0.
- Agents cannot consume goods they produce themselves.

# Kiyotaki-Wright Model

## Technology

- An agent can produce a good at utility cost  $0$ .
- The type of the good is random.
- An agent can only hold one unit of the good at a time.
- This is a shortcut for a model with
  - costly production
  - costly storage of inventory

# Non-monetary Equilibrium

## Timing

- Agents start holding one good each.
- Each agent meets another agent with probability  $\theta$ .
- If two agents meet, they can exchange goods, if both agree.
- Trade costs  $\varepsilon$ .
  - Ensures that trade only occurs if both parties can consume.

# Symmetric Equilibrium

- A strategy is a decision whether or not to trade in a meeting.
- Obviously, agents trade when the opponent holds the good they can consume.
- Agents do not trade otherwise.
- There is no gain from changing the type of good held.
- There is no way to compensate the party who cannot consume.
  - This changes when goods are divisible.

- Value before meeting an agent

$$V_c^n = \theta x^2 (U - \varepsilon) + \beta V_c^n \quad (1)$$

$$= \frac{\theta x^2 (U - \varepsilon)}{1 - \beta} \quad (2)$$

- This is a measure of equilibrium welfare.

# Monetary Equilibria

- At the beginning of time,  $\bar{M}$  agents are offered a unit of "money."
- If they accept, they have to discard the good they carry.
- Agents can only carry 1 unit of a good or 1 unit of money.

# Decision to hold money

- Money is a bubble: its value derives from the expectation that money will be valued tomorrow.
- There is always a non-monetary equilibrium.
- $\Pi$  is the probability that agents will accept money tomorrow.
- The state of the economy is  $(\Pi, \bar{M})$ .

# Decision to hold money I

Value function for an agent holding a good:

$$\begin{aligned} V_c &= (1 - \theta) \beta V_c \\ &\quad + \theta M x \max_{\pi} [\pi \beta V_m + (1 - \pi) \beta V_c] \\ &\quad + \theta M (1 - x) \beta V_c \\ &\quad + \theta (1 - M) x^2 (U - \varepsilon + \beta V_c) \\ &\quad + \theta (1 - M) (1 - x^2) \beta V_c \end{aligned}$$

With prob  $(1 - \theta)$ : meet nobody and get  $\beta V_c$ .

With prob.  $\theta$  meet somebody.

- With prob.  $\theta M$  he holds money
  - With prob  $\theta M x$  he likes my good.
    - Then trade with prob.  $\pi$  and get  $\beta V_m$ .

# Decision to hold money II

- If he does not like my good:  $\beta V_c$ .
- With prob.  $\theta(1 - M)$  he holds a good.
  - With prob  $\theta(1 - M)x^2$ : trade and get  $U - \varepsilon + \beta V_c$ .
  - Otherwise, no trade and get  $\beta V_c$ .

Simplify

$$\begin{aligned} V_c &= \theta (1 - M) x^2 (U - \varepsilon + \beta V_c) \\ &\quad + \theta M x \max_{\pi} [\pi \beta V_m + (1 - \pi) \beta V_c] \\ &\quad + [1 - \theta [1 - M] x^2 - \theta M x] \beta V_c \end{aligned}$$

# Decision to hold money I

Value function for an agent holding money:

$$\begin{aligned} V_m &= (1 - \theta) \beta V_m \\ &\quad + \theta M \beta V_m \\ &\quad + \theta (1 - M) x \Pi (U - \varepsilon + \beta V_c) \\ &\quad + \theta (1 - M) (1 - x) \beta V_m \end{aligned}$$

With prob  $(1 - \theta)$ : meet nobody and get  $\beta V_m$ .

With prob  $\theta$  meet someone.

- With prob  $\theta M$  he holds money. No trade. Get  $\beta V_m$ .
- With prob  $\theta (1 - M)$  he holds a good.
  - With prob  $\theta (1 - M) x$  I like the good. Trade with prob.  $\Pi$ .
  - With prob  $\theta (1 - M) (1 - x)$  I don't like the good. No trade and get  $\beta V_m$ .

# Decision to hold money

- Benefits of money:
  - Trade with person holding a good I like who does not like my good.
- Drawback of money:
  - Trade with person who likes my good only with prob.  $\Pi$ .
- No change when:
  - meet somebody who holds money or a good I don't like.

# Decision to hold money

- If  $\Pi < x$ :
  - Money is less liquid than goods.
  - $V_m < V_c$ .
  - Set  $\pi = 0$  and never accept money.
- If  $\Pi > x$ :
  - Money is liquid.
  - $V_m > V_c$ .
  - Set  $\pi = 1$  and always accept money.
- If  $\Pi = x$ :
  - Indifferent between money and goods.
  - Any  $\pi$  is optimal.

- Equilibrium requires  $\pi = \Pi$ .
- Three equilibria:
  - 1  $\Pi = 0$ . Back to the non-monetary equilibrium.
  - 2  $\Pi = 1$  and  $M = \bar{M}$ .
  - 3 Mixed strategy equilibrium:  $\Pi = x$  and any  $M \in [0, \bar{M}]$ .
    - Now  $M$  is indeterminate because agents are indifferent between money and goods.

- Money is purely a way of **breaking a symmetric equilibrium**.
- Any good could take the role of money.
- The question is: how to coordinate agents' expectations about the future acceptability of "money"?

- To find welfare, impose  $\pi = \Pi$  on the value functions and solve.
- Result: The mixed monetary equilibrium ( $\Pi = x$ ) has lower welfare than the non-monetary equilibrium.
- Intuition: Money does not facilitate transactions.
- The welfare cost stems from the fact that some goods were lost in order to hold money.
- Equivalent to an economy in which some goods can no longer be consumed.

- Result: The monetary equilibrium raises welfare iff  $x < 0.5$ .
- Intuition: Trading is always easier.
- But some "goods" cannot be consumed (those held by monetary traders).
- For the liquidity benefit to win, barter trade must be sufficiently hard.

- Goods are divisible:
  - Alberto Trejos and Randall Wright. "Search, Bargaining, Money, and Prices." *The Journal of Political Economy* Vol. 103, No. 1 (Feb., 1995), pp. 118-141.
- Money and goods are divisible / policy analysis:
  - Ricardo Lagos and Randall Wright. "A Unified Framework for Monetary Theory and Policy Analysis." *The Journal of Political Economy* Vol. 113, No. 3 (Jun., 2005), pp. 463-484.

# How Useful Is This?

- People disagree...
- The big question:
  - How does it fit a modern economy where most money is credit?

- Ljunqvist & Sargent, "Recursive Methods," ch. 26.8.