Search Models of Money

Prof. Lutz Hendricks

Econ720

November 22, 2017

Search Models

- Matching models have one drawback:
  - the matching function is a black box
- Search models model explicitly how matches come about.
- Their drawback: hard to take to the data.
- Applications:
  - Money - the main / original motivation for search
  - Housing
  - Labor market

Search Models of Money

- Most models of money either have no notion of **liquidity** (OLG model) or they assume liquidity (CIA).
- Search models offer a micro-foundation for liquidity.
- **Caveats**:
  - The institutional arrangement is medieval.
  - Money is literally currency, not credit.
  - Some recent work tries to relax this.

The Idea

- People produce and consume many goods.
- To consume what they want, people need to search for sellers.
- Without money: trade is hard.
  - the double coincidence of wants problem.
- One solution: designate one good to be "money."
- Money means: the good is accepted by all traders.
  - a social convention.

Kiyotaki-Wright Model

- There is a unit mass of ex ante identical, infinitely lived agents.
- There is a unit mass of goods types.
- Goods are indivisible.
  - The model gets harder when goods are divisible.

Preferences

- Each agent can consume fraction $x \in (0, 1)$ of the goods.
- Each good can be consumed by fraction x of households.
- Consumption yields utility U.
- Not consuming yields 0.
- Agents cannot consume goods they produce themselves.

Technology

- An agent can produce a good at utility cost 0.
- The type of the good is random.
- An agent can only hold one unit of the good at a time.
- This is a shortcut for a model with
  - costly production
  - costly storage of inventory

Non-monetary Equilibrium

Timing

- Agents start holding one good each.
- Each agent meets another agent with probability θ.
- If two agents meet, they can exchange goods, if both agree.
- Trade costs ε.
  - Ensures that trade only occurs if both parties can consume.

Symmetric Equilibrium

- A strategy is a decision whether or not to trade in a meeting.
- Obviously, agents trade when the opponent holds the good they can consume.
- Agents do not trade otherwise.
- There is no gain from changing the type of good held.
- There is no way to compensate the party who cannot consume.
  - This changes when goods are divisible.

Symmetric Equilibrium

- Value before meeting an agent

$$V_c^n = \theta\ x^2\ (U-\varepsilon)+\beta V_c^n \tag{1}$$

$$= \frac{\theta\ x^2\ (U-\varepsilon)}{1-\beta} \tag{2}$$

- This is a measure of equilibrium welfare.
- With probability θ: meet an agent.
- With probability x^2: both hold goods that the other can eat.
- Otherwise: get 0 this period.

Monetary Equilibria

- At the beginning of time, $\bar{M}$ agents are offered a unit of "money."
- If they accept, they have to discard the good they carry.
- Agents can only carry 1 unit of a good or 1 unit of money.

Decision to hold money

- Money is a bubble: its value derives from the expectation that money will be valued tomorrow.
- There is always a non-monetary equilibrium.
- Π is the probability that agents will accept money tomorrow.
- The state of the economy is $\left(\Pi, \bar{M}\right)$.

Decision to hold money

Value function for an agent holding a good:

$$\begin{aligned} V_c = &\left([1-\theta]+\theta M[1-x]+\theta[1-M]\left[1-x^2\right]\right)(0+\beta V_c) \\ &+\theta(1-M)x^2(U-\varepsilon+\beta V_c) \\ &+\theta M x \max_{\pi}\left[\pi\beta V_m+(1-\pi)\beta V_c\right] \end{aligned}$$

π: the agent can play mixed strategies

Agent holding a good

With prob $(1-\theta)$: meet nobody and get βV_c.

With prob. θ meet somebody.

- With prob. θM he holds money
  - With prob $\theta M x$ he likes my good.
    - Then trade with prob. π and get βV_m.
  - If he does not like my good: βV_c.
- With prob. $\theta(1-M)$ he holds a good.
  - With prob $\theta(1-M)x^2$: trade and get $U-\varepsilon+\beta V_c$.
  - Otherwise, no trade and get βV_c.

Simplify

$$
\begin{aligned}
V_c &= \theta(1-M)x^2(U-\varepsilon+\beta V_c) \\
&\quad +\theta M\ x\ \max_{\pi}\left[\pi\beta V_m+(1-\pi)\beta V_c\right] \\
&\quad +\left[1-\theta\left[1-M\right]x^2-\theta Mx\right]\beta V_c
\end{aligned}
$$

Agent holding money

$$\begin{aligned} V_m &= [(1-\theta)+\theta M+\theta(1-M)(1-x)](0+\beta V_m) \\ &\quad +\theta(1-M)x\Pi(U-\varepsilon+\beta V_c) \end{aligned}$$

With prob $(1-\theta)$: meet nobody and get βV_m.

With prob θ meet someone.

- With prob θM he holds money. No trade. Get βV_m.
- With prob $\theta(1-M)$ he holds a good.
  - With prob $\theta(1-M)x$ I like the good. Trade with prob. Π.
  - With prob $\theta(1-M)(1-x)$ I don't like the good. No trade and get βV_m.

Decision to hold money

- Benefits of money:
  - Trade with person holding a good I like who does not like my good.
- Drawback of money:
  - Trade with person who likes my good only with prob. Π.
- No change when:
  - meet somebody who holds money or a good I don't like.

Decision to hold money

- If $\Pi < x$:
  - Money is less liquid than goods.
  - $V_m < V_c$.
  - Set $\pi = 0$ and never accept money.
- If $\Pi > x$:
  - Money is liquid.
  - $V_m > V_c$.
  - Set $\pi = 1$ and always accept money.
- If $\Pi = x$:
  - Indifferent between money and goods.
  - Any π is optimal.

Equilibrium

- Equilibrium requires $\pi = \Pi$.
- Three equilibria:
  1. $\Pi = 0$. Back to the non-monetary equilibrium.
  2. $\Pi = 1$ and $M = \bar{M}$.
  3. Mixed strategy equilibrium: $\Pi = x$ and any $M \in \left[0, \bar{M}\right]$.
     - Now M is indeterminate because agents are indifferent between money and goods.

Remarks

- Money is purely a way of **breaking a symmetric equilibrium**.
- Any good could take the role of money.
- The question is: how to coordinate agents' expectations about the future acceptability of "money"?

Welfare

- To find welfare, impose $\pi = \Pi$ on the value functions and solve.
- Result: The mixed monetary equilibrium $(\Pi = x)$ has lower welfare than the non-monetary equilibrium.
- Intuition: Money does not facilitate transactions.
- The welfare cost stems from the fact that some goods were lost in order to hold money.
- Equivalent to an economy in which some goods can no longer be consumed.

Monetary equilibrium

- Result: The monetary equilibrium raises welfare iff $x < 0.5$.
- Intuition: Trading is always easier.
- But some "goods" cannot be consumed (those held by monetary traders).
- For the liquidity benefit to win, barter trade must be sufficiently hard.

Extensions

- Goods are divisible:
  - Alberto Trejos and Randall Wright. "Search, Bargaining, Money, and Prices." The Journal of Political Economy Vol. 103, No. 1 (Feb., 1995), pp. 118-141.
- Money and goods are divisible / policy analysis:
  - Ricardo Lagos and Randall Wright. "A Unified Framework for Monetary Theory and Policy Analysis." The Journal of Political Economy Vol. 113, No. 3 (Jun., 2005), pp. 463-484.

How Useful Is This?

- People disagree...
- The big question:
  - How does it fit a modern economy where most money is credit?

Reading

- Ljunqvist & Sargent, "Recursive Methods," ch. 26.8.