Mortenson Pissarides Model

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Mortenson / Pissarides Model

- Search models are popular in many contexts: labor markets, monetary theory, etc.
- They are distinguished by
  1. how agents meet
  2. how the payoffs are determined when agents meet.
- The MP model has
  1. a matching function
  2. Nash bargaining.

Model

- Time is continuous.
- Demographics:
  - There are $\bar{L}$ identical workers.
  - They live forever (or they could die stochastically).
- Preferences:
  - Utility = consumption (one good).
  - Discount rate r.

Technology

- Output is produced from labor only.
- Production can take place only in a worker-job match.
- Each match consists of exactly one job / one worker.
- When matched, a match produces a flow output of A.

Model: The logic

- Enter the "period" with
  - U unemployed workers
  - $F = \bar{L} - U$ job matches.
  - $E = F$ employed workers
- bE matches break up (exogenously)
- Firms post V vacancies, paying a cost.

Model: The logic

- Unemployed workers and vacancies meet at random.
- Workers who don't meet a firm stay unemployed, consume 0.
- **In a match**:
    - Firm and worker **bargain** over the wage (no contracts!).
    - If no agreement is reached, the job becomes vacant and the worker becomes unemployed.
    - If agreement is reached, the pair produces until exogenous breakup occurs.

Workers

- Workers live forever and maximize the expected present value of earnings.
- The discount rate is r (exogenous).
- The only decisions: in wage negotiation.

Firms

- Firms can create jobs (vacancies) at a flow cost of C per unit of time.
- A filled job produces A and pays w (endogenous) to the worker.
- The firm keeps the profit: $A - w - C$.

Matching

- A **matching function** describes how workers are matched to vacancies.
- The number of matches per period is

$$M(U,V) = K\ U^{\beta}\ V^{\gamma} \tag{1}$$

- We take $M(U,V)$ as given.
- Matching functions can be derived from micro-foundations.
- More vacancies or more unemployed workers result in more matches.

Steady state restrictions

- Focus on situations where E, U, V are constant.
- The number of employed workers changes according to

$$\dot{E} = M(U, V) - bE \tag{2}$$

  where b is the exogenous rate of match dissolution.
- In steady state $\dot{E} = 0$:

$$M(U, V) = bE \tag{3}$$

Steady state restrictions

- The number of unemployed follows

$$\dot{U} = bE - M(U,V) \quad (4)$$
$$= -\dot{E} \quad (5)$$

- $\dot{U} = 0$ is implied by $\dot{E} = 0$.

Definitions

- Define the rate of **exit from unemployment**

$$a = \frac{M(U,V)}{U} \tag{6}$$

- Define the rate at which **vacancies are filled**:

$$\alpha = \frac{M(U,V)}{V} \tag{7}$$

Solution method

Assume that all workers receive the same wage w when matched (verify this later).

For a given wage, there is only one decision to be made: **how many vacancies** to create.

- Assume that vacancies are created until they yield zero profit (**free entry**).
- We need to find the value of an open vacancy (V_V).

Then we need to find the bargained **wage**.

For this we need to know the values

- of being employed (V_E) or unemployed (V_U).
- of a filled vacancy (V_F).

Workers: Employed

The value of being employed is

$$rV_E = w + b\left(V_U - V_E\right) \tag{8}$$

Or:

$$V_E = \frac{w + bV_U + (1-b)\,V_E}{1+r}$$

Intuition:

- Receive a flow benefit w.
- With probability b switch to unemployment and lose $V_U - V_E$.

Employed worker: Derivation

- Consider the value of being employed for a short period Δt.
- Receive flow benefit w, discounted at r.
  - Probability of remaining in the match: e^{-bt}.
  - Value: $\int_0^{\Delta t} e^{-(r+b)t} w \; dt = \frac{1-e^{-(r+b)\Delta t}}{r+b} w$.

Employed worker: Derivation

- At the end, at $t+\Delta t$:
  - continue as unemployed with probability $e^{-b\Delta t}$.
  - continue in match with probability $1-e^{-b\Delta t}$.
  - Value: $e^{-r\Delta t}\ \left[e^{-b\Delta t}\ V_E(\Delta t)+\left(1-e^{-b\Delta t}\right)V_U(\Delta t)\right]$.

Employed worker: Derivation

- Value of being employed is then:

$$V_E(\Delta t) = \frac{1-e^{-(r+b)\Delta t}}{r+b} w + e^{-r\Delta t} \left[e^{-b\Delta t} V_E(\Delta t) + \left(1-e^{-b\Delta t}\right) V_U(\Delta t) \right]$$
$$= \frac{w}{r+b} + \frac{\left(1-e^{-b\Delta t}\right) e^{-r\Delta t}}{1-e^{-(r+b)\Delta t}} V_U(\Delta t).$$

- Take the limit as $\Delta t \to 0$.
- Use l'Hopital's rule to evaluate the ratio in front of V_U. It becomes $\frac{b}{r+b}$. Therefore

$$V_E = \frac{w}{r+b} + \frac{b}{r+b} V_U$$

Rearrange. Done.

Unemployed Worker

$$rV_U = 0 + a(V_E - V_U)$$

Or

$$V_U = \frac{0 + aV_E + (1-a)\,V_U}{1+r}$$

Receive nothing right now.

With probability a switch to "employed."

Unfilled Vacancies

$$rV_V = -C + \alpha\,(V_F - V_V)$$

Or

$$V_V = \frac{-C + \alpha V_F + (1-\alpha)\,V_V}{1+r}$$

Pay the vacancy cost C.

With probability α fill it and receive V_F.

Filled vacancies

$$rV_F = A - w - C + b(V_V - V_F)$$

Or

$$V_F = \frac{A - w - C + bV_V + (1 - b)\,V_F}{1 + r}$$

Receive the profit $A - w - C$.

With probability b lose the match and receive V_V.

Stationary equilibrium

A stationary equilibrium determines $(V_U, V_E, V_V, V_F, E, U, V, w)$ such that:

- the values V_x are determined as above.
- the labor market "clears:" $\bar{L} = E + U$.
- the number of employed is constant: $M(U, V) = bE$.
- creating new vacancies yields zero profit: $V_V = 0$
- wages are somehow determined (this is where V_U, V_E come in).
- In addition: a, α are defined above as functions of U, V.

Wage determination

- What happens when firms and workers meet?
- The worker accepts any wage such that $V_E \geq V_U$.
- The firm accepts any wage such that $V_F \geq V_V$.
- Bargaining pins down the exact distribution of the surplus.
- We make an assumption: the surplus is evenly divided:

$$V_E - V_U = V_F - V_V \tag{9}$$

- Note: there is no good theory that would pin down how the surplus is split.

Model summary I

Objects: $(V_U, V_E, V_V, V_F, E, U, V, w)$.

Flow equations:

$$\bar{L} = E + U \tag{10}$$

$$M(U,V) = bE \tag{11}$$

Values:

$$rV_E = w + b(V_U - V_E) \tag{12}$$

$$rV_U = a(V_E - V_U) \tag{13}$$

$$rV_V = -C + \alpha V_F = 0 \tag{14}$$

$$rV_F = A - w - C - bV_F \tag{15}$$

Model summary II

Bargaining:

$$V_E - V_U = V_F - V_V \tag{16}$$

Definitions:

$$a = \frac{M(U,V)}{U} \tag{17}$$

$$\alpha = \frac{M(U,V)}{V} \tag{18}$$

Solving the model

This is just algebra: solve the 8 equations for the 8 unknowns.

From the definitions:

$$V_E - V_U = \frac{w}{a+b+r} \tag{19}$$

$$V_F - V_V = \frac{A-w}{\alpha+b+r} \tag{20}$$

Solve for the wage:

$$w = \frac{(a+b+r)A}{a+\alpha+2b+2r} \tag{21}$$

Intuition:

- the surplus (A) is equally divided when $\alpha = a$.
- if workers have a harder time finding jobs (low a), their surplus share shrinks.

Solving the model

Next: find one equation in E.

Start from free entry

$$rV_V = -C + \alpha\left(V_F - V_V\right) = 0$$

Substitute (19) and the solution for w:

$$\begin{aligned} rV_V &= -C + \alpha \frac{A - \frac{(a+b+r)A}{a+\alpha+2b+2r}}{\alpha+b+r} \\ rV_V &= -C + \frac{\alpha A}{a+\alpha+2b+2r} = 0 \end{aligned} \tag{22}$$

a and α are still endogenous.

Solving the model

Find a in terms of E.

$$
\begin{aligned}
a(E) &= \frac{M(U,V)}{U} \\
&= \frac{bE}{\bar{L}-E}
\end{aligned}
$$

a is increasing in E.

- Higher employment $\rightarrow$ faster exit from unemployment.

Solving the model

Find α in terms of E.

$$\begin{aligned} \alpha &= \frac{M(U,V)}{V} \\ &= \frac{bE}{V} \end{aligned}$$

α is increasing in E, but only for given V.

Solving the model

Solve the matching function for $V(E)$:

$$
\begin{aligned}
V &= \left(\frac{bE}{KU^{\beta}}\right)^{1/\gamma} \\
&= \left(\frac{bE}{K[\bar{L}-E]^{\beta}}\right)^{1/\gamma}
\end{aligned}
$$

Therefore

$$\alpha(E) = K^{1/\gamma}\ (bE)^{(\gamma-1)/\gamma}\ \left(\bar{L}-E\right)^{\beta/\gamma} \tag{23}$$

α is decreasing in E.

Higher employment $\rightarrow$ vacancies are filled more slowly.

Solving the model

Write free entry as

$$rV_V = -C + \frac{\alpha(E)A}{a(E) + \alpha(E) + 2b + 2r} = 0 \tag{24}$$

Recall $a'(E) > 0$ and $\alpha'(E) < 0$.

The fraction term is falling in E.

There is a unique solution E with zero profits.

Equilibrium Illustration

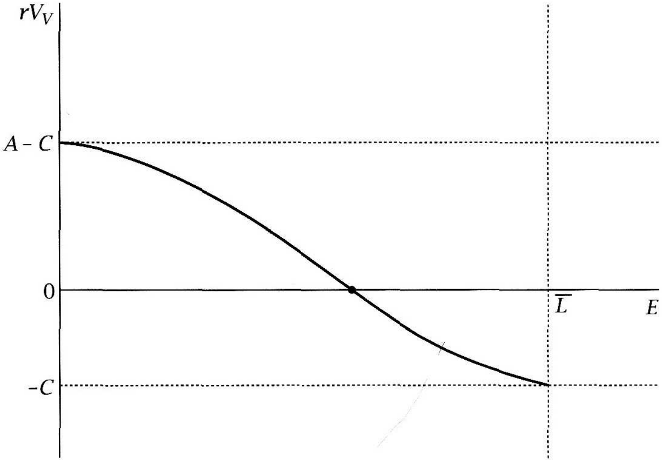


FIGURE 9.6 The determination of equilibrium employment in the search and matching model

Source: Romer, Advanced Macroeconomics

Model summary

The model determines w, E, a, α.

Free entry:

$$rV_V = -C + \frac{\alpha(E)A}{a(E)+\alpha(E)+2b+2r} = 0 \tag{25}$$

Higher employment means faster job finding

$$a'(E) > 0 \tag{26}$$

and slower filling of vacancies

$$\alpha'(E) < 0 \tag{27}$$

Wages are determined from

$$w = \frac{(a(E)+b+r)}{a(E)+\alpha(E)+2b+2r}A \tag{28}$$

Implications

Long-run productivity growth

The model generates a sensible **balanced growth path** with wage growth and no trend in unemployment.

- Assume: productivity A and the cost of vacancies C rise in proportion.
- Then: no effect on employment (E).
- Therefore α, a unchanged.
- Wages rise in proportion with A.

Fluctuations in productivity

Example: Recession. A/C drops.

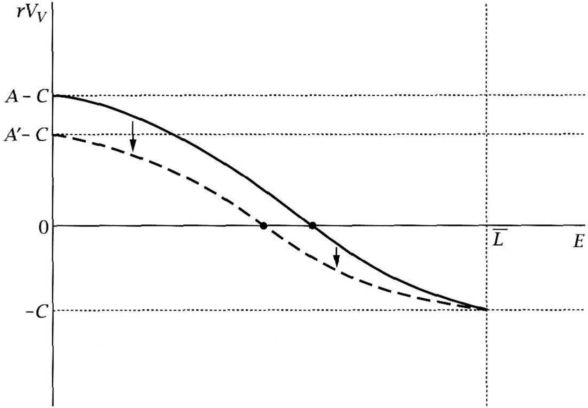


FIGURE 9.7 The effects of a fall in labor demand in the search and matching model

Source: Romer, Advanced Macroeconomics

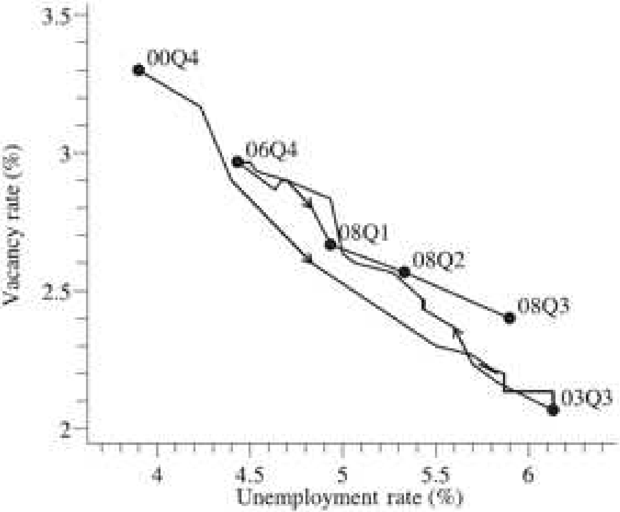
Fluctuations in productivity

Intuition: Think of higher C.

- Post fewer vacancies.
- It also turns out that equilibrium vacancies drop.
- Employment declines.
- The comovement of vacancies and unemployment is observed in the data (the **Beveridge curve**).

Beveridge curve

Figure 1
U.S. Beveridge curve (quarterly, 2000:Q4–2008:Q3)



Fluctuations in productivity

The model does not imply wage rigidity:

- A/C drops $\rightarrow$ E drops.
- $a(E)\downarrow$ and $\alpha(E)\uparrow$.
- Wages are given by (21):

$$w = \frac{(a(E)+b+r)A}{a(E)+\alpha(E)+2b+2r}$$

- Wages fall more than A.

Strongly procyclical wages

Intuition:

- The current surplus from matching $(A - C)$ drops by more than C.
- Firm surplus shrinks even more because vacancies are easily filled.
- Worker surplus, however, shrinks less because jobs are hard to find.

Caveat:

- Cyclical behavior of wages depends on bargaining solution.
- If bargaining weights vary over the cycle, wages could be less cyclical.

Propagation of Shocks

The model implies that **transitory shocks have persistent effects**:

- When A drops, employment does not jump: firms have no incentive to fire workers (unless the shock is large enough).
- Unemployment only rises b/c vacancies decline and dissolved matches are filled more slowly.
- When A returns to normal, it will take time to fill the new vacancies.

This is perhaps the main contribution of the matching model: a propagation mechanism for shocks that is lacking in Walrasian models.

Efficiency

- The equilibrium is generally not efficient.
- There are pecuniary externalities:
    - Posting a new vacancy raises the surplus for workers / reduces it for other firms.
- Under somewhat general conditions, the **Hosios condition** is necessary and sufficient for efficiency:
    - The worker's share of the surplus must equal the elasticity of the matching function with respect to unemployment.

Is unemployment mostly frictional?

In the matching model, there is unemployment even without shocks.

This is useful unemployment: it produces matches.

Even separations can be useful:

- imagine that workers are heterogeneous.
- when a worker finds a job, she does not know whether it is a good match.
- it may be optimal to quit after some time b/c a better match comes along.

How large is frictional unemployment?

The data suggest it may be large.

- 3% of workers leave their jobs each month in U.S. manufacturing.
- 10% of jobs are destroyed each year.

But there is also long-term unemployment which is most likely not frictional.

Summary

- Search models capture the idea that findings jobs takes time.
- They are useful for studying labor market regulation.
- A key shortcoming: Assumptions about bargaining determine the equilibrium.

Reading

- Romer, "Advanced Macroeconomics," ch. 9.8.
- Ljunqvist & Sargent, "Recursive Methods," ch. 26.3. [Their model is easier b/c it has constant returns in the matching function.]
- Williamson (2006), "Notes on macroeconomic theory," ch. 7.
- Rogerson, Richard; Robert Shimer; Randall Wright (2005). "Search-Theoretic Models of the Labor Market: A Survey." *Journal of Economic Literature* 43: 959-988. [A survey of search models.]
- Yashif, Eran (2007). "Labor Search and Matching in Macroeconomics." http://ideas.repec.org/p/cep/cepdps/dp0803.html [A survey]
- Shimer, Robert (2005). "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." *American Economic Review* 95(1): 25-49. [The MP model has problems accounting for labor market fluctuations.]
- Hall, Robert E. (2005). "Employment Fluctuations with Equilibrium Wage Stickiness." *American Economic Review* 95(1), pp. 50-65.