

# McCall Model

Prof. Lutz Hendricks

Econ720

November 29, 2011

- We would like to study basic labor market data:
  - unemployment and its duration
  - wage heterogeneity among seemingly identical workers
  - job to job transitions
  - how do policies affect those variables?
- Frictionless models of the labor market cannot talk about these issues.
- We need models in which workers must search for jobs.

- Unemployment is a productive activity: search for a new job.
- Types of models:
  - 1 Decision theoretic (McCall model).
  - 2 Matching: A matching function creates new jobs.
  - 3 Search: Random encounters and bargaining.

- A partial equilibrium model of a worker searching for a job.
- The worker lives forever, in discrete time.
- Preferences:

$$\sum_{t=0}^{\infty} \beta^t y_t$$

- $y_t$  is income.
- When employed:  $y = w$ . When unemployed:  $y = c$ .

- Enter the period as unemployed worker.
- Draw a wage offer  $w$  from the distribution  $F(W) = \Pr(w \leq W)$ .
- Support:  $[0, B]$ .
- Choose whether to accept or reject.
- If accept: work forever at wage  $w$  with lifetime income  $\frac{w}{1-\beta}$ .
- If reject: start over next period.

# Bellman equation

- State: current wage offer  $w$ .
- Control: accept / reject.
- Bellman:

$$v(w) = \max \left\{ \frac{w}{1-\beta}, Q \right\}$$

- $Q$  is the expected continuation value:

$$Q = c + \beta \int_0^B v(w') dF(w')$$

# Reservation wage property

- If reject: get the same continuation value regardless of  $w$ .
- If accept: get the present value of earnings:  $\frac{w}{1-\beta}$
- The reservation wage makes the worker indifferent between accepting and rejecting:

$$v(\bar{w}) = \frac{\bar{w}}{1-\beta} = Q \quad (1)$$

- Note: For  $w < \bar{w}$  the worker still gets  $v(\bar{w})$ .

- Write the reservation wage as (proof below):

$$\begin{aligned}\bar{w} - c &= \beta \int_{\bar{w}}^B \frac{w' - \bar{w}}{1 - \beta} dF(w') \\ &= \beta E \left\{ \frac{w' - \bar{w}}{1 - \beta} \mid w' \geq \bar{w} \right\} \Pr(w' \geq \bar{w})\end{aligned}$$

- In words:
  - the surplus from working now ( $\bar{w} - c$ ) equals
  - the surplus from searching: the expected lifetime wage gain from perhaps finding a better job

Write the indifference condition as

$$\frac{\bar{w}}{1-\beta} = c + \beta \int_0^{\bar{w}} \frac{\bar{w}}{1-\beta} dF(w') + \beta \int_{\bar{w}}^B \frac{w'}{1-\beta} dF(w')$$

Simplify:

$$\begin{aligned} & \int_0^{\bar{w}} \frac{\bar{w}}{1-\beta} dF(w') + \int_{\bar{w}}^B \frac{w'}{1-\beta} dF(w') \\ = & \int_0^B \frac{\bar{w}}{1-\beta} dF(w') + \int_{\bar{w}}^B \frac{w' - \bar{w}}{1-\beta} dF(w') \\ = & \frac{\bar{w}}{1-\beta} + \int_{\bar{w}}^B \frac{w' - \bar{w}}{1-\beta} dF(w') \end{aligned}$$

Therefore:

$$\frac{\bar{w}}{1-\beta} - c = \beta \frac{\bar{w}}{1-\beta} + \beta \int_{\bar{w}}^B \frac{w' - \bar{w}}{1-\beta} dF(w')$$

# Implications: Unemployment Benefits

- What is the effect of more generous unemployment benefits? (higher  $c$ ).
- Expected surplus shrinks when  $\bar{w}$  rises.
- Optimality:  $c = \bar{w} - \text{expected surplus}$
- RHS increases in  $\bar{w}$ .
- Higher  $c \rightarrow$  higher reservation wage  $\rightarrow$  longer unemployment.

## More dispersed wage offers

- Result: A mean preserving spread in the wage offer distribution raises the reservation wage and ex ante utility.
- Intuition:
  - Making bad wage offers worse is costless - they are rejected anyway.
  - Making good wage offers better is valuable.
- Proof: Ljunqvist & Sargent.

- Each period the worker is fired with probability  $\alpha$ .
- A fired worker must wait 1 period before drawing a new wage.
- Value when unemployed:

$$v_U = c + \beta \int v(w') dF(w')$$

- Value when employed at wage  $w$ :

$$v_E(w) = w + \beta(1 - \alpha)v(w) + \beta\alpha v_U$$

- Bellman equation

$$v(w) = \max\{v_E(w), v_U\}$$

- Reservation wage solves

$$v_E(\bar{w}) = v_U$$

$$\bar{w} + \beta(1 - \alpha)v(\bar{w}) + \beta\alpha v_U = v_U$$

- With  $v(\bar{w}) = v_U$ :

$$\bar{w} = \frac{v_U}{1 - \beta} = c + \beta \int v(w') dF(w')$$

# Firing: Implications

- How does the firing probability affect unemployment?
- The reservation wage equations are the “same” with and without firing:

$$\bar{w} = c + \beta \int v(w') dF(w')$$

- The value function is lower with firing
  - because quitting is never optimal
- Therefore  $\bar{w}$  is lower with firing.
- If jobs do not last as long, there is no point holding out for the perfect offer.

# Equilibrium: Bath Tub Model

- An easy way of embedding the decision problem into a partial equilibrium.
- Exit: Assume the worker "dies" with probability  $\alpha$ .
- Layoffs: Workers are laid off with probability  $\xi$ .
- Entry:  $\alpha$  identical unemployed workers enter each period.

# Laws of motion

- Total population: constant at 1.
- Keep track of the unemployment rate  $U_t$ .
- Inflows:
  - $\alpha$  new entrants.
  - $\xi(1 - \alpha)(1 - U)$  layoffs.
- Outflows:
  - $\alpha U$  deaths.
  - $[1 - F(\bar{w})](1 - \alpha)U$  job matches.
- Law of motion

$$U_{t+1} - U_t = \alpha + \xi(1 - \alpha)(1 - U_t) - \alpha U_t - (1 - \alpha)[1 - F(\bar{w})]U_t$$

# What is missing?

- Not satisfactory: The job finding rate / wage offer distribution should be endogenous.
  - Think about analyzing policies...
- Matching and search models address this.
  - by introducing endogenous supply of jobs
  - and wage bargaining.

- Ljungqvist & Sargent, Recursive Macroeconomic Theory, 2nd ed., ch. 6.3.
- Williamson (2006), "Notes on macroeconomic theory," ch. 7, works out a similar model with exogenous job separations.