

Search and Unemployment

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Search and Unemployment

- We study models of "involuntary" unemployment.
- Unemployment is a productive activity: search for a new job.
- Types of models:
 - 1 Decision theoretic (McCall model).
 - 2 Matching: A matching function creates new jobs.
 - 3 Search: Random encounters and bargaining.

- A partial equilibrium model of a worker searching for a job.
- The worker lives forever, in discrete time.
- Preferences:

$$\sum_{t=0}^{\infty} \beta^t y_t$$

- y_t is income.
- When employed: $y = w$. When unemployed: $y = c$.

- Each period: Draw a wage offer from distribution $F(W) = \Pr(w \leq W)$.
- Support: $[0, B]$.
- Once employed, the job lasts forever and yields lifetime income $\frac{w}{1-\beta}$.

Bellman equation

- State: current wage offer w .
- Control: accept / reject.
- Bellman:

$$v(w) = \max \left\{ \frac{w}{1 - \beta}, c + \beta \int_0^B v(w') dF(w') \right\}$$

Reservation wage property

- Plot the values of accepting and rejecting.
- Accept if the present value of earnings, $\frac{w}{1-\beta}$, exceeds the reservation value

$$Q = c + \beta \int_0^B v(w') dF(w') \quad (1)$$

- If reject:

$$v(w) = v(\bar{w}) = \frac{\bar{w}}{1-\beta} \quad (2)$$

for all $w \leq \bar{w}$.

Reservation wage property

Q and the reservation wage \bar{w} are determined from the indifference condition

$$\begin{aligned}\frac{\bar{w}}{1-\beta} &= Q \\ &= c + \beta \int_0^{\bar{w}} \frac{\bar{w}}{1-\beta} dF(w') + \beta \int_{\bar{w}}^B \frac{w'}{1-\beta} dF(w')\end{aligned}$$

Reservation wage property

Simplify:

$$\begin{aligned}\int_0^{\bar{w}} \frac{\bar{w}}{1-\beta} dF(w') + \int_{\bar{w}}^B \frac{w'}{1-\beta} dF(w') &= \int_0^B \frac{\bar{w}}{1-\beta} dF(w') + \int_{\bar{w}}^B \frac{w' - \bar{w}}{1-\beta} dF(w') \\ &= \frac{\bar{w}}{1-\beta} + \int_{\bar{w}}^B \frac{w' - \bar{w}}{1-\beta} dF(w')\end{aligned}$$

Rearrange:

$$\bar{w} - c = \beta \int_{\bar{w}}^B \frac{w' - \bar{w}}{1-\beta} dF(w') \quad (3)$$

$$= \beta E \left\{ \frac{w' - \bar{w}}{1-\beta} \mid w' \geq \bar{w} \right\} \Pr(w' \geq \bar{w}) \quad (4)$$

Reservation wage property

$$\bar{w} - c = \beta E \left\{ \frac{w' - \bar{w}}{1 - \beta} \mid w' \geq \bar{w} \right\} \Pr (w' \geq \bar{w}) \quad (5)$$

- Cost of searching = expected discounted wage gain.
- Higher $c \rightarrow$ higher reservation wage \rightarrow longer unemployment.

More dispersed wage offers

- Result: A mean preserving spread in the wage offer distribution raises the reservation wage and ex ante utility.
- Intuition:
 - Making bad wage offers worse is costless - they are rejected anyway.
 - Making good wage offers better is valuable.
- Proof: Ljunqvist & Sargent.

Equilibrium

Bath tub model

- An easy way of embedding the decision problem into a partial equilibrium.
- Exit: Assume the worker "dies" with probability α .
- Layoffs: Workers are laid off with probability ζ .
- Entry: α identical unemployed workers enter each period.

Equilibrium

Laws of motion

- Total population: constant at 1.
- Keep track of the unemployment rate U_t .
- Inflows:
 - α new entrants.
 - $\zeta (1 - \alpha) (1 - U)$ layoffs.
- Outflows:
 - αU deaths.
 - $[1 - F(\bar{w})] (1 - \alpha) U$ job matches.
- Law of motion

$$U_{t+1} - U_t = \alpha + \zeta (1 - \alpha) (1 - U_t) - \alpha U_t - (1 - \alpha) [1 - F(\bar{w})] U_t \quad (6)$$

- Not satisfactory: The job finding rate / wage offer distribution should be endogenous.
- Matching and search models address this.

- Ljungqvist & Sargent, ch. 6.3.
- Williamson (2006), "Notes on macroeconomic theory," ch. 7, works out a similar model with exogenous job separations.