

# Overlapping Generations Model: Dynamic Efficiency and Social Security

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- The OLG model can have **inefficient equilibria**.
- We solve the problem of a fictitious **social planner**
  - This yields a Pareto optimal allocation by construction.
- We learn from this:
  - 1 Solving the planning problem may be an easy way of characterizing CE (if it is optimal).
  - 2 Comparing it with the CE points to sources of inefficiency.

# The Social Planner's Problem

# Planner's problem

- Imagine an omnipotent social planner.
- She can assign actions to all agents (consumption, hours worked, ...).
- She maximizes some average of individual utilities.
- She only faces resource constraints.

- The planner's objective function is assumed to be a weighted average of individual utilities:

$$\omega_0 \beta u(c_1^o) + \sum_{t=1}^{\infty} \omega_t [u(c_t^y) + \beta u(c_{t+1}^o)]$$

- Utility comparisons across persons don't make sense, but it is often used.
- By varying the weights ( $\omega_t$ ) we can obtain all Pareto optimal allocations.
- To ensure that the objective function is finite, suitable conditions need to be imposed on the weights.

# Planner's problem

The planner only faces feasibility constraints:

$$(1 - \delta)k_t + f(k_t) = c_t^y + c_t^o/(1 + n) + (1 + n)k_{t+1}$$

Lagrangian:

$$\begin{aligned} \Gamma = & \omega_0 \beta u(c_1^o) + \sum_{t=1}^{\infty} \omega_t [u(c_t^y) + \beta u(c_{t+1}^o)] \\ & + \sum_{t=1}^{\infty} \lambda_t \left[ \begin{array}{c} (1 - \delta)k_t + f(k_t) \\ -c_t^y - c_t^o/(1 + n) - (1 + n)k_{t+1} \end{array} \right] \end{aligned}$$

FOCs:

$$\begin{aligned}\omega_t u'(c_t^y) &= \lambda_t \\ \omega_{t-1} \beta u'(c_t^o) &= \lambda_t / (1+n) \\ \lambda_t [1 - \delta + f'(k_t)] &= \lambda_{t-1} (1+n)\end{aligned}$$

Static optimality:

$$\omega_t u'(c_t^y) = \omega_{t-1} (1+n) \beta u'(c_t^o)$$

Euler equation:

$$\omega_t u'(c_t^y) [1 - \delta + f'(k_t)] = \omega_{t-1} u'(c_{t-1}^y) (1+n)$$

Sequences  $\{c_t^y, c_t^o, k_{t+1}\}_{t=1}^{\infty}$  that satisfy:

- Static and Euler equation.
- Feasibility.
- A transversality condition or  $k_{t+1} \geq 0$ .

# Interpretation of the Euler equation

- A feasible perturbation does not change welfare.
- Consider taking  $(1+n)$  units of consumption from the young at date  $t-1$  and investing them until date  $t$ . This yields

$$[1 - \delta + f'(k_t)]$$

units of additional consumption for the young at date  $t$ .

- Using the static condition,

$$\omega_t u'(c_t^y)/(1+n) = \omega_{t-1} \beta u'(c_t^o)$$

the Euler equation becomes

$$u'(c_t^y) = \beta u'(c_{t+1}^o)[1 - \delta + f'(k_{t+1})] \quad (1)$$

- which looks conspicuously like the Euler equation of the household.
- This is not surprising: the planner should respect the individual FOCs unless there are externalities.

# Planner's Steady State

For a steady state to exist, weights must be of the form

$$\omega_t = \omega^t, \quad \omega < 1$$

Then the Euler equation condition becomes

$$\omega (1 - \delta + f'(k_{MGR})) = (1 + n)$$

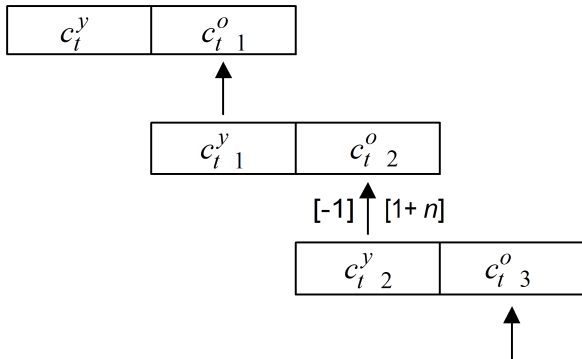
This is the **Modified Golden Rule**. ( $\omega = 1$  is the Golden Rule).  
Because  $\omega < 1$ :  $k_{MGR} < k_{GR}$  and the MGR is dynamically efficient.

## How does the planner avoid dynamic inefficiency?

If the planner desires lots of old age consumption, he can implement a "transfer scheme" of the following kind:

Take a unit of consumption from each young and give  $(1+n)$  units to each old at the same date.

There is no need to save more than the GR.



Of course, there aren't really any transfers in the planner's world.

Social Security

A transfer scheme akin to Social Security can replicate the Planner's allocation and avoid dynamic inefficiency.

Social Security consists of

- a **payroll tax** on workers;
- a **transfer** payment to the retired.

# Two flavors of Social Security

- **Fully funded:**
  - For each worker, the government invests the tax payments.
  - This is equivalent to a forced saving plan.
- **Pay-as-you-go:**
  - Current transfers are paid from current tax revenues.

# Fully funded Social Security

- Young: pay tax  $\tau_t^y$ .
- Old: receive transfers  $\tau_{t+1}^o = -(1 + r_{t+1}) \tau_t^y$ .
- Government supplies revenues as capital to firms.
- For the household:
  - Forced saving at rate of return  $r$ .
  - No change to the present value budget constraint.
- Therefore, if prices remain fixed:
  - No change to optimal consumption plan.
  - Private saving (of the young) drops by the Social Security tax amount.

- Assume population growth at rate  $n$ :  $N_t = (1 + n) N_{t-1}$ .
- Tax collection from the current young:  $N_t \tau_t^y$ .
- Transfer payments to the current old:  $-N_{t-1} \tau_t^o$ .
- The budget balances in each period:

$$\tau_t^o = -\tau_t^y (1 + n) \quad (2)$$

- From the household's perspective:
  - Forced saving with return  $n$ .
  - Saving drops by an amount different from  $\tau_t^y$ .

# Household with Social Security

The household maximizes

$$u(c_t^y) + \beta u(c_{t+1}^o)$$

subject to the present value budget constraint

$$w_t - \tau_t^y - c_t^y = \frac{c_{t+1}^o - \tau_{t+1}^o}{1 + r_{t+1}} \quad (3)$$

Lump-sum taxes do not change the Euler equation (prove this):

$$\beta(1 + r_{t+1})u'([1 + r_{t+1}]s_{t+1} + \tau_{t+1}^o) = u'(w_t - s_{t+1} - \tau_t^y)$$

- The saving function remains the same

$$s_{t+1} = s(w_t - \tau_t^y, -\tau_{t+1}^o, r_{t+1}) \quad (4)$$

- For given prices, Social Security reduces saving for two reasons:
  - Higher income when old.
  - Lower income when young.

- If a tax change does not alter the present value of taxes,

$$d\tau^y + \frac{d\tau^o}{1+r_{t+1}} = 0$$

then the optimal consumption path does not change.

- Reason: present value budget constraint and first-order condition unchanged.
- This is the Permanent Income Hypothesis.

# Fully Funded Social Security

- We prove that unchanged  $(w_t, r_t)$  clear the markets with Social Security.
- Household:
  - By PIH: no change in consumption plan.
  - Household fully dissaves the tax:  $\Delta s_{t+1} = -\tau_t^y$ .
- Government saves:  $s_{t+1}^G = N_t \tau_t^y$ .
- Capital market clearing:

$$\Delta K_{t+1} = N_t \Delta s_{t+1} + s_{t+1}^G = 0 \quad (5)$$

- Fully funded SS is neutral.
  - Essentially, the government just relabels some private savings as public.

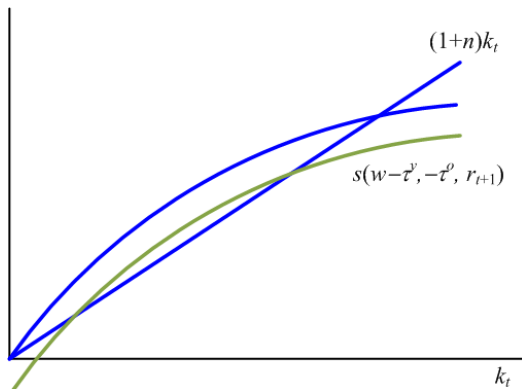
- We prove that unchanged  $(W_t, R_t)$  imply excess demand for  $K$ .
- Household:  $\Delta s_{t+1} < 0$ .
- Government: Balanced budget.
- Capital market:  $\Delta K_{t+1} = N_t \Delta s_{t+1} < 0$ .

# Illustration

- Capital market clearing:

$$k_{t+1}(1+n) = s(w(k_t) - \tau_t^y, w(k_{t+1}) - \tau_{t+1}^o, r(k_{t+1})) \quad (6)$$

- Assume that the saving function is well-behaved (e.g. log utility and Cobb-Douglas).



# Complications

- Since prices change, we cannot guarantee that Pay-as-you-go SS reduces steady state  $k$ .
- Totally differentiate the saving function:

$$[1 + n - s_3 f''(k_{t+1})] dk_{t+1} = -s_1 d\tau^y - s_2 d\tau^o < 0$$

- A sufficient condition for  $dk_{t+1} < 0$  is that  $s_3 > 0$ . Then the law of motion unambiguously shifts down.

- SS reduces the steady state capital stock. It can alleviate dynamic inefficiency.
- Note that the argument is not reversible:
  - in a dynamically efficient economy, “reverse social security” is not a Pareto improvement.
  - why not?

- Acemoglu, ch. 9.