Overlapping Generations Model
Dynamic Efficiency and Social Security

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Issues

The OLG model can have **inefficient equilibria**.

We solve the problem of a fictitious **social planner**

- This yields a Pareto optimal allocation by construction.

We learn from this:

1. Solving the planning problem may be an easy way of characterizing CE (if it is optimal).
2. Comparing it with the CE points to sources of inefficiency.

2. The Social Planner's Problem

Planner's problem

Imagine an omnipotent **social planner** who

- can assign actions to all agents (consumption, hours worked, ...)
- maximizes some average of individual utilities "**welfare**"
- **only faces resource constraints**.

Solving this problem yields **one Pareto optimal** allocation.

- No economy that faces the same technological constraints can do better for everyone.
  - Obvious?
- A benchmark against which equilibria can be assessed.
- But there may be many Pareto optimal allocations.

2.1. Simple Planner Example

Demographics:

- N agents who live for one period.
- mass N_j for $j = 1, \ldots, n$

Preferences: $\mathscr{U}(c)$

Endowments: y_j

Technology: $\sum_j N_j y_j = \sum_j N_j c_j$

Simple Planner Example

Planner's problem:

$$\max_{c_j} \underbrace{\sum_j \mu_j \mathscr{U}\left(c_j\right)}_{\text{welfare}} + \lambda \underbrace{\left[\sum_j N_j \left(y_j - c_j\right)\right]}_{\text{resource constraint}} \tag{1}$$

μ_j is the weight that the planner places on type j.

FOC:

$$\mu_j \mathscr{U}'\left(c_j\right) = \lambda N_j \tag{2}$$

In words...

Simple Planner Example

A more intuitive FOC:

- Let
  $$\phi_j \equiv \mu_j / N_j \tag{3}$$
  be the weight that the planner puts on each individual of type j.

Then the FOC says:

$$\phi_j \mathscr{U}'(c_j) = \lambda \tag{4}$$

In words...

Simple Planner Example

Each set of weights μ_j produces **one** Pareto optimal allocation.

By varying the weights we can obtain **all** Pareto optimal allocations.

- It makes sense even if comparing utilities across agents does not.

To ensure that the objective function is finite, assume that $\sum_j \mu_j < \infty$.

2.2. OLG Welfare function

The planner maximizes a weighted average of individual utilities.

Welfare is

$$\underbrace{\mu_0 \beta u(c_1^o)}_{\text{initial old}} + \sum_{t=1}^{\infty} \underbrace{\mu_t [u(c_t^y) + \beta u(c_{t+1}^o)]}_{\text{generation } t}$$

Old consumption of the initial old is the earliest quantity that the planner can change.

Planner Constraints

The planner only faces feasibility or **resource constraints**.

In this model:

$$Y = C + I \tag{5}$$

$$\underbrace{F(K_t, N_t)}_{Y} = \underbrace{N_t c_t^y + N_{t-1} c_t^o}_{C} + \underbrace{K_{t+1} - (1-\delta) K_t}_{I} \tag{6}$$

Or, in per capita young terms $(k_t = K_t / N_t)$:

$$f(k_t) = c_t^y + c_t^o/(1+n) + (1+n)k_{t+1} - (1-\delta)k_t$$

because $K_{t+1}/N_t = (K_{t+1}/N_{t+1}) \times (N_{t+1}/N_t)$

Planner's Lagrangian

$$
\begin{aligned}
\Gamma \quad = \quad & \mu_0 \beta u(c_1^o) + \sum_{t=1}^{\infty} \mu_t [u(c_t^y) + \beta u(c_{t+1}^o)] \\
& + \sum_{t=1}^{\infty} \lambda_t \begin{bmatrix} (1-\delta)k_t + f(k_t) \\ -c_t^y - c_t^o/(1+n) - (1+n)k_{t+1} \end{bmatrix}
\end{aligned}
$$

Planner's FOCs:

$$
\begin{aligned}
\mu_t u'(c_t^y) &= \lambda_t \\
\mu_{t-1} \beta u'(c_t^o) &= \lambda_t/(1+n) \\
\lambda_{t+1}[1-\delta+f'(k_{t+1})] &= \lambda_t(1+n)
\end{aligned}
$$

Interpretation

Three ways of using a unit of goods at date t:

$$\lambda_t = \mu_t u'\left(c_t^y\right) \tag{7}$$

$$\lambda_t = (1+n)\,\mu_{t-1} u'\left(c_{t+1}^o\right) \tag{8}$$

$$\lambda_t = \frac{f'(k_{t+1}) + 1 - \delta}{1+n}\lambda_{t+1} \tag{9}$$

All uses must give the same marginal utility (λ_t).

Planner's problem

Static optimality:

$$\lambda_t = \mu_t u'(c_t^y) = \mu_{t-1}(1+n)\beta u'(c_t^o)$$

Intuition...

Euler equation

$$\mu_t u'(c_t^y)[1-\delta+f'(k_t)] = \mu_{t-1} u'(c_{t-1}^y)(1+n)$$

Using the static condition, the Euler equation becomes

$$u'(c_t^y) = \beta u'(c_{t+1}^o)[1-\delta+f'(k_{t+1})] \tag{10}$$

which looks like the Euler equation of the household.

This is not surprising: the planner should respect the individual FOCs unless there are externalities.

Interpretation of the Euler equation

- A feasible perturbation does not change welfare.
- In $t-1$:
  - $c_{t-1}^{y} \downarrow$ by $(1+n)$
  - $k_t \uparrow$ by 1 (per capita of the date t young)
- In t:
  - output $\uparrow$ by $f'(k_t)$ (per capita t young)
  - raise c_t^y by $1-\delta+f'(k_t)$ or
  - raise c_t^o by $(1+n)(1-\delta+f'(k_t))$
- From $t+1$ onwards: nothing changes
  - especially not k_{t+1}

Planner's Solution

Sequences $\left\{c_t^y, c_t^o, k_{t+1}\right\}_{t=1}^{\infty}$ that satisfy:

- Static and Euler equation.
- Feasibility.
- A transversality condition or $k_{t+1} \geq 0$.
  - We talk about those later.

2.3. Comparison with Equilibrium

The same:

- Euler equation
- Resource constraint = goods market clearing.

Different:

- CE has 2 budget constraints (one redundant by Walras' law)
- Planner has static condition

Missing in the C.E.: a mechanism for transferring goods from young to old (planner's static condition).

Planner's Steady State

Euler in steady state:

$$\frac{\mu_t}{\mu_{t-1}} u'(c^y)[1-\delta+f'(k)] = u'(c^y)(1+n)$$

For a steady state to exist, weights must be of the form

$$\mu_t = \omega^t, \quad \omega < 1$$

Otherwise the ratios μ_{t+1}/μ_t in the FOCs are not constant.

Then the Euler equation becomes

$$\omega \ (1-\delta+f'(k_{MGR})) = (1+n)$$

This is the **Modified Golden Rule**. ($\omega = 1$ is the Golden Rule).

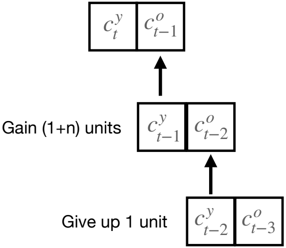
Because $\omega < 1$: $k_{MGR} < k_{GR}$ and the MGR is **dynamically efficient**.

How does the planner avoid dynamic inefficiency?

If the planner desires lots of old age consumption, he can implement a "transfer scheme" of the following kind:

Take a unit of consumption from each young and give $(1+n)$ units to each old at the same date.

There is no need to save more than the GR.



Of course, there aren't really any transfers in the planner's world.

3. Social Security

Social Security

A transfer scheme akin to Social Security can replicate the Planner's allocation and avoid dynamic inefficiency.

Social Security consists of

- a **payroll tax** on workers;
- a **transfer** payment to the retired.

Note how directly the planner's solution points to a policy implementation.

Two flavors of Social Security

Fully funded:

- For each worker, the government invests the tax payments.
- This is equivalent to a forced saving plan.
- A system that is gaining popularity around the world.

Pay-as-you-go:

- Current transfers are paid from current tax revenues.
- The U.S. system.

3.1. Household with Social Security

The household maximizes

$$u\left(c_t^y\right) + \beta u\left(c_{t+1}^o\right)$$

subject to the present value budget constraint

$$w_t - \tau_t^y - \frac{\tau_{t+1}^o}{1+r_{t+1}} = c_t^y + \frac{c_{t+1}^o}{1+r_{t+1}} \tag{11}$$

Lump-sum taxes do not change the Euler equation (prove this):

$$\beta(1+r_{t+1})u'([1+r_{t+1}]s_{t+1} - \tau_{t+1}^o) = u'(w_t - s_{t+1} - \tau_t^y)$$

Household with Social Security

The saving function remains the same

$$s_{t+1} = s\left(w_t - \tau_t^y, -\tau_{t+1}^o, r_{t+1}\right) \tag{12}$$

For given prices, Social Security reduces saving for two reasons:

- Higher income when old.
- Lower income when young.

Household with Social Security

If a tax change does not alter the present value of taxes,

$$d\tau^y + \frac{d\tau^o}{1 + r_{t+1}} = 0$$

then the optimal consumption path does not change.

- Reason: present value budget constraint and first-order condition unchanged.
- This is the Permanent Income Hypothesis.

3.2. Fully funded Social Security

Young pay τ_t^y.

Old pay $\tau_{t+1}^o = -(1+r_{t+1})\ \tau_t^y < 0$.

Government supplies revenues as capital to firms.

For the household:

- Forced saving at rate of return r.
- No change to the present value budget constraint.

Result: fully funded SS is neutral

- Private saving (of the young) drops by the Social Security tax amount.

Exercise

Write out the equilibrium definition for the model with Fully Funded Social Security.

3.3. Pay-as-you-go Social Security

Assume population growth at rate n: $N_t = (1+n)N_{t-1}$.

Tax collection from the current young: $N_t\ \tau_t^y$.

Transfer payments to the current old: $-N_{t-1}\ \tau_t^o$.

The budget balances in each period:

$$\tau_t^o = -\tau_t^y\ (1+n) \tag{13}$$

From the household's perspective:

- Forced saving with return n.
- Saving drops by an amount different from τ_t^y.

Pay-as-you-go SS is not neutral.

Dynamic efficiency

- If SS reduces the steady state capital stock, it can alleviate dynamic inefficiency.
- Note that the argument is not reversible:
  - in a dynamically efficient economy, "reverse social security" is not a Pareto improvement.
  - why not?

Reading

- Acemoglu (2009), ch. 9.
- Krusell (2014), ch. 7

References I

Acemoglu, D. (2009): *Introduction to modern economic growth*, MIT Press.

Krusell, P. (2014): "Real Macroeconomic Theory," Unpublished.