

## Problem Set 1: OLG Models

Econ720. Prof. Lutz Hendricks. August 30, 2011

---

### 1 An Economy with Land

Consider a two-period OLG model in which production requires land and labor.

Demographics: Each period a cohort of size  $N = 1$  is born. Each person lives for 2 periods.

Preferences: Households value consumption when young and old according to

$$u(c_t^y) + \beta u(c_{t+1}^o)$$

Endowments: At time 0, the old each hold  $M$  units of land. Each young person supplies one unit of labor.

Technology: Output is produced from land and labor according to the production function  $F(M, L)$ .  $F$  has constant returns to scale.

Markets: Households rent labor to firms at wage  $w_t$ . Households rent land to firms at rental price  $r_{t+1}$ . Land is traded at price  $q_t$ .

#### Questions:

1. Write down the household's budget constraints. Note that the household sells his land holdings at price  $q_{t+1}$  when old.
2. Derive the household's FOCs and Euler equation.
3. State and solve the firm's problem.
4. Define a competitive equilibrium.
5. Derive an implicit solution for  $q$  in steady state.

### 2 OLG Model with Assets

Demographics: There are two types of households, indexed by  $h$ . In each period, a mass of 0.5 households is born of each type. Each person lives for 2 periods.

Endowments: Households receive endowments  $(e^y, e^o)$  when young and old, respectively.

Preferences:  $\ln(c_{h,t}^y) + \beta_h \ln(c_{h,t+1}^o)$ .

Technologies: None.

Markets: Households trade goods and one period bonds that are issued and purchased by households.

**Questions:**

1. Define a solution to the household problem. Solve for the household's bond supply function.
2. Solve for the equilibrium bond interest rate.
3. Your solution for  $R$  should reveal the following features: (i) If old endowments are larger,  $R$  is higher. (ii) If  $\beta_h$  increases,  $R$  decreases. (iii)  $R$  is time invariant. Provide intuition for these features.
4. Now add a durable good to the economy. It is in fixed supply,  $K$ . It pays a dividend  $d$  per period. Households trade "shares" of this good in an asset market at price  $p_t$ , measured in units of consumption goods. Define a competitive equilibrium for this economy.
5. Why do you find that the number of equations equals the number of objects to be determined? Usually, we find that we have one additional equation, which is redundant by Walras' law.
6. Derive an equation that determines the equilibrium price sequence  $p_t$ .