

Problem Set 1: OLG Models
Econ720. Fall 2009. Prof. Lutz Hendricks

1 An Economy with Land

Consider a two-period OLG model in which production requires land and labor. Agents hold land as their savings and rent it to firms for production. The total amount of land is fixed at M units per young household. The cohort size is constant and normalized to one. Each household supplies one unit of labor when young. [This is essentially an economy with a capital good which is in fixed supply.]

- (a) Write down the household's budget constraints. Note that the household sells his land holdings at price q_{t+1} when old. Denote the purchase price of land by q_t and its rental price by r_{t+1} .
- (b) Derive the household's FOCs and Euler equation. Utility is

$$u(c_t^y) + \beta u(c_{t+1}^o)$$

- (c) Derive the equilibrium wage and rental rate from the firm's problem. The production function has constant returns in land and labor: $F(M, L)$
- (d) Define a competitive equilibrium.
- (e) Derive an implicit solution for q in steady state.

2 OLG Model with Assets

Consider a two-period OLG model. There are two types of households, indexed by h . In each period, a mass of 0.5 households is born of each type. Households receive endowments (e^y, e^o) when young and old, respectively. Households issue or purchase one period bonds when young. Preferences are given by $\ln(c_{h,t}^y) + \beta_h \ln(c_{h,t+1}^o)$.

1. Define a solution to the household problem. Solve for the household's bond supply function.
2. Solve for the equilibrium bond interest rate.
3. Your solution for R should reveal the following features: (i) If old endowments are larger, R is higher. (ii) If β_h increases, R decreases. (iii) R is time invariant. Provide intuition for these features.
4. Now add a durable good to the economy. It is in fixed supply, K . It pays a dividend d per period. Households trade "shares" of this good in an asset market at price p_t , measured in units of consumption goods. Define a competitive equilibrium for this economy.
5. Why do you find that the number of equations equals the number of objects to be determined? Usually, we find that we have one additional equation, which is redundant by Walras' law.
6. Derive an equation that determines the equilibrium price sequence p_t .