

Money in OLG Models

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- We study the value of money in OLG models.
- We develop an important model of money.
- The model can be used to price other assets.

The central question of monetary economics:

Why and when is money valued in equilibrium?

By money we mean bits of green paper with pictures of dead presidents.

Rate of return dominance is a problem:

- Why would anyone hold money in the presence of other assets that offer a higher rate of return?

The answer of the OLG model:

- Money is a bubble.
- Its value derives solely from the expectation that money will be valued tomorrow.
- It is only valued if no other asset offers a higher rate of return.

Can money alleviate dynamic inefficiency?

- Previous models lacked a long-lived asset that would facilitate intergenerational trade.
- Money could solve this problem.

An OLG Model of Money

- Start with the standard two period OLG model without production or bonds.
- The population grows at rate n .
- Money is introduced as follows.
 - In period 1, the initial old are given M bits of green paper
 - In every subsequent period, the government prints additional paper and hands it to the current old *in proportion* to current paper holdings.
 - Effectively, money pays (nominal) interest if held from young to old age.
 - The money growth rate is constant:

$$M_{t+1}/M_t = 1 + \theta$$

Beginning of t :

- N_t young are born and receive e_1 goods.
- N_{t-1} old
 - receive e_2 goods
 - carry over M_{t-1}/N_{t-1} units of money
 - receive money transfer $\theta M_{t-1}/N_{t-1}$.

During t

- the young sell goods to the old
- the old sell money to the young

At the end of t :

- the young hold M_t

- This is a standard two period household problem with endowments.
- Preferences are

$$u(c_t^y, c_{t+1}^o)$$

- The household receives endowments e_1, e_2 of (perishable) goods.
- The price of goods in period t is P_t .
- The budget constraints are therefore

$$\begin{aligned}P_t(e_1 - c_t^y) &= P_t x_t \\P_{t+1}(c_{t+1}^o - e_2) &= x_t(1 + \theta)P_t\end{aligned}$$

- The lifetime budget constraint is:

$$e_1 - c_t^y = \frac{c_{t+1}^o - e_2}{(1 + \theta)P_t/P_{t+1}}$$

- Note that money acts exactly like a bond that pays gross interest

$$R_{t+1} = (1 + \theta)P_t/P_{t+1}$$

- The Lagrangian is the same as in previous models:

$$\Gamma = u(c_t^y, c_{t+1}^o) + \lambda_t \left[e_1 - c_t^y + \frac{e_2 - c_{t+1}^o}{R_{t+1}} \right]$$

- FOCs:

$$u_1(t) = \lambda_t$$

$$u_2(t) = \lambda_t / R_{t+1}$$

- Euler equation:

$$u_1(t) = R_{t+1} u_2(t)$$

A solution to the household problem is a triple (c_t^y, c_{t+1}^o, x_t) which satisfies

- the Euler equation and
- the two budget constraints.

Optimal behavior can be characterized by a savings function (which is now a *money demand function*)

$$x_t = s(R_{t+1}, e_1, e_2) \quad (1)$$

Equilibrium

The government is simply described by a money growth rule:

$$M_{t+1}/M_t = 1 + \theta$$

Market clearing:

- Money market:

$$M_t = N_t P_t x_t$$

or

$$m_t = M_t / (N_t P_t) = s(R_{t+1})$$

- Goods market:

$$e_1 + e_2 / (1 + n) = c_t^y + c_t^o / (1 + n)$$

Equilibrium Definition

A sequence of prices and quantities

$$(c_t^y, c_t^o, x_t, P_t, M_t)$$

such that

- 1 M_t obeys the money growth equation. [1 eqn]
- 2 Markets clear [1 independent eqn]
- 3 Households behave optimally [3 eqn]

Characterizing Equilibrium

- We look for a difference equation in terms of the economy's state variables.
- State variables are M and P .
- But in this model (and typically) only the ratio $m = M/PN$ matters.

Characterizing Equilibrium

Start from the money market clearing condition

$$m_t = s(R_{t+1}) \quad (2)$$

Substitute out R using

$$R_{t+1} = (1 + \theta)P_t/P_{t+1} \quad (3)$$

We need an expression for inflation. From

$$\frac{M_{t+1}}{M_t} = \frac{m_{t+1}}{m_t} \frac{P_{t+1}}{P_t} \frac{N_{t+1}}{N_t}$$

we have

$$R_{t+1} = (1 + \theta)P_t/P_{t+1} = (1 + n)m_{t+1}/m_t$$

The law of motion is

$$m_t = s((1 + n)m_{t+1}/m_t) \quad (4)$$

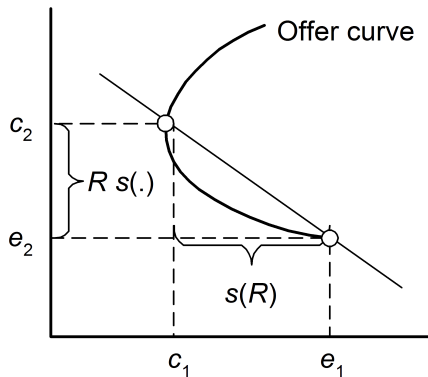
- We want to determine the shape of the law of motion.
- The key idea is to use the household's intertemporal consumption allocation to figure out how money evolves over time.

Household consumption choice

The lifetime budget constraint has slope $-R_{t+1}$.

Plot the tangencies between budget constraint and indifference curves for all interest rates \rightarrow

offer curve.



What do we know about the offer curve?

- 1 It goes through the endowment point.
- 2 At low levels of R the household would like to borrow (but cannot).
- 3 For interest rates where the household saves very little, income effects are small \Rightarrow savings rise with R_{t+1} .
- 4 The offer curve intersects each budget line only once.

Money demand

- Money demand equals saving of the young:

$$m_t = s(R_{t+1}) = e_1 - c_t^y \quad (5)$$

- Hence: the horizontal axis shows m_t .
- Money demand also equals capital income of the old:

$$(1+n)m_{t+1} = R_{t+1}s(R_{t+1}) = c_{t+1}^o - e_2 \quad (6)$$

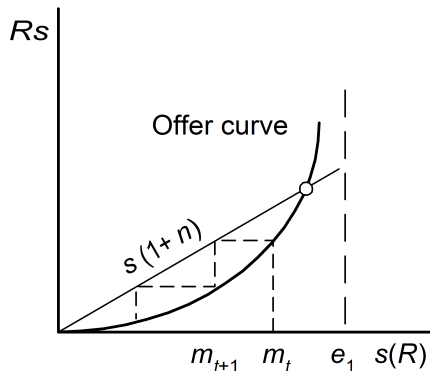
- Hence: the vertical axis shows $(1+n) m_{t+1}$.
- The offer curve therefore describes the law of motion for m :

$$(1+n)m_{t+1} = F(m_t)$$

where F is the offer curve.

Law of motion

Using a line of slope $(1+n)$ we can find the path of m_t for any start value m_0 .



Steady state

There is a unique monetary **steady state** (intersection of offer curve and ray through origin).

It is *unstable*.

Properties of the steady state:

- Per capita real money balances, m , are constant over time.
- The gross rate of return on money is

$$\begin{aligned}R_{t+1} &= (1 + \theta)P_t/P_{t+1} \\ &= (1 + n)m_{t+1}/m_t\end{aligned}$$

Therefore in steady state

$$R = 1 + n$$

- Steady state inflation is

$$P_{t+1}/P_t = \frac{1 + \theta}{1 + n}$$

Assumption: the offer curve is not backward bending.

Take m_0 as given for now.

What if $m_0 > m_{ss}$?

- This cannot happen because m_t would blow up towards ∞ .
- But then consumption will exceed total output at some point.

If $m_0 < m_{ss}$: m_t

collapses towards 0.

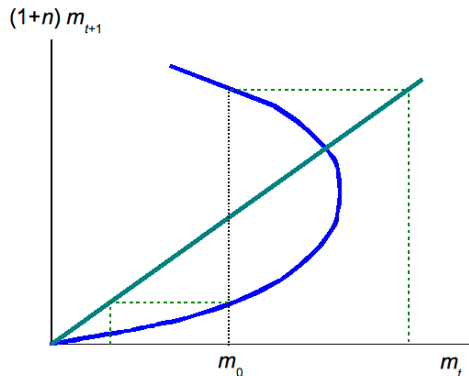
- Because M grows at a constant rate, this must happen through inflation.
- Along this path R falls over time \Rightarrow inflation accelerates.

Intuition:

- If $m_0 = m_{ss}$ people save just enough to keep m constant.
- If m_0 is a bit lower, then R is a bit lower. People save less.
- That requires a lower m_1 , hence more inflation.
- That leads people to save less again, etc.

Dynamics: Backward bending Offer Curve

We have **multiple equilibria** and **complex dynamics**.



Initial money stock

- Nothing in the model pins down m_0 . Any value below m_{SS} is acceptable.
- There is a continuum of equilibrium paths.
- The reason: money is a bubble.
- As long as expectations are such that people are willing to hold m_0 , we have an equilibrium.
- $m_0 = 0$ is also an equilibrium.

Does money solve the dynamic inefficiency problem?

- It might because it permits intergenerational trade.

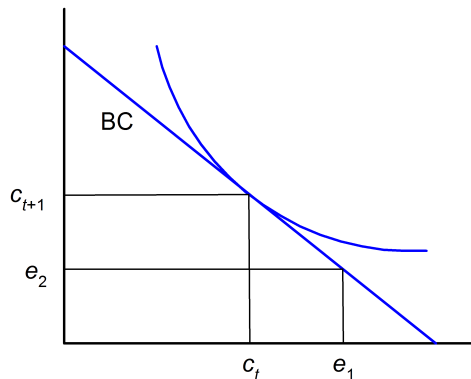
Two cases:

- ① Samuelson case: the offer curve at the origin is flatter than $1 + n$;
- ② Classical case: it is steeper than $1 + n$.

Samuelson case

The economy without money is **dynamically inefficient**.

The reason: the slope of the offer curve at the origin equals the non-monetary interest rate.



- There is no trade. The interest rate equals the slope of the IC through the endowment point.
- The economy is dynamically inefficient, if the interest rate is lower than n .
- Then the offer curve is flatter than the $1 + n$ line.

- The non-monetary economy is dynamically **efficient**.
- The offer curve is too steep to intersect the $1 + n$ line.
- A monetary equilibrium does not exist.
- The main result is therefore:

Money is valued in equilibrium only in an economy that would be dynamically inefficient without money.

Is this a good theory of money?

Good features of the OLG model of money are:

- 1 The outcome that money is valued in equilibrium is not assumed (e.g. because money yields utility or is simply required for transactions).
- 2 The value of money depends on expectations and is fragile.

The problem:

- 1 The model does not generate rate of return dominance.
- 2 A key feature of money seems to be missing: liquidity.

How to construct a theory of money that resolves the problems without introducing new ones is an open question.

- Blanchard & Fischer (1989), ch. 4.1 [A clear exposition.]
- Krueger, "Macroeconomic Theory," ch. 8 discusses offer curves.
- Sargent & Ljungqvist, ch. 9 [Detailed.]
- McCandless & Wallace