Problem Set 2: OLG Models With Money

Econ720. Fall 2009. Lutz Hendricks

1 Money and Storage

Consider a two-period OLG model with fiat money and a storage technology. The timing is a follows:

- The old enter period t holding aggregate capital $K_t = N_{t-1}k_t$ and nominal money balances of $M_t = m_t\ N_{t-1}$. Each old person produces $f(k_t)$. All capital is used up in production (complete depreciation). Assume that $f(.)$ obeys Inada conditions.
- The government pays a lump-sum transfer of $x_t p_t$ units of money to each old person: $M_{t+1} = M_t + N_{t-1}x_t p_t$.
- $N_t = (1+n)^t$ young households are born. Each is endowed with e units of a single good.
- The young buy money (m_{t+1}/p_t) from the old, consume c_t^y and save k_{t+1}.
- The old consume their income. The utility function is $u(c_t^y) + \beta\, u(c_{t+1}^o)$.

(a) State the household's budget constraints when young and old.
(b) Derive the household's optimality conditions. Define a solution to the household problem.
(c) The government's only role is to grow the aggregate money supply at the constant rate μ: $M_{t+1} = (1+\mu)\, M_t$. Define a competitive equilibrium.
(d) Define a steady state as a system of 5 equations in 5 unknowns.
(e) Find the money growth rate (μ) that maximizes steady state consumption per young person, $(N_t\, c_t^y + N_{t-1}\, c_t^o)/N_t$.

2 Money in the Utility Function in an OLG Model

Consider a two-period overlapping generations model with money. Each generation is of constant size N. Young households supply one unit of labor when young and earn the wage w_t. Each consumes c_t^y and saves the remainder either as capital (s_{t+1}) or as real money (m_t^d/p_t). Old households consume their savings, including interest income (c_{t+1}^o). The utility function is $u(c_t^y) + \beta\, u(c_{t+1}^o) + v(m_t^d/p_t)$. Assume $v' > 0$.

At time 0 each member of the old generation was endowed with m_0 pieces of paper. No new paper is ever issued. Old persons sell their money at price $1/p_t$ to young people, in order to fund her consumption. The young then hold the paper until old at the end of period $t+1$, and sell it to fund c_{t+1}^o. This continues throughout time.

(a) Derive a set of 4 equations that characterize optimal household behavior. Show that the household's first-order conditions imply rate of return dominance, i.e., the real return on money is less than the real return on capital (assuming both capital and money are held in equilibrium).

(b) Wages and interest rates are determined by competitive factor markets. Firms use capital K_t and labor of young workers L_t to produce output. Assume that output is $F(K_t, L_t)$, satisfying Inada conditions. F has constant returns to scale. Solve the firm's problem.

(c) Define a competitive equilibrium. Capital depreciates at rate δ.

(d) Assume that the utility functions u and v are logarithmic. Solve *in closed form* for the household's money demand function, $m_t^d/p_t = \varphi(w_t, r_{t+1}, \pi_{t+1})$, and for its saving function, $s_{t+1} = \phi(w_t, r_{t+1}, \pi_{t+1})$. $\pi_{t+1} \equiv p_{t+1}/p_t$.