

# Overlapping Generations Model Bequests and Altruism

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- If parents leave bequests to their children, does that solve the dynamic efficiency problem?
- The answer is **no** - bequests can only increase the capital stock.

- Why do parents leave bequests to their children?
- Empirically, we don't know (a possible research question).
- Theoretically, there are various ways of modeling bequests:
  - ① **Altruism**: parents value their children's utility.
  - ② **Warm glow**: parents value the bequest itself (a reduced form).
  - ③ **Strategic**: parents promise bequests so kids behave well.

- 1 If parents leave strictly positive bequests, the economy must be dynamically efficient.
- 2 A dynamically inefficient economy is not affected by a bequest motive.
- 3 Households linked by bequests look like they **live forever**.
  - This motivates many models with infinitely lived households.
  - It only works if bequests are altruistic and always positive.

# OLG Model With Altruism

- Each household has  $(1 + n)$  children when old.
- The household values own consumption according to

$$u(c_t^y, c_{t+1}^o)$$

- The household also values the utility of the child.
- Preferences are defined recursively:

$$V(t) = u(c_t^y, c_{t+1}^o) + \beta V(t+1)$$

Expanding this we find that the parent values utility of all future generations:

$$\begin{aligned}V(t) &= u(c_t^y, c_{t+1}^o) + \beta[u(c_{t+1}^y, c_{t+2}^o) + \beta V(t+2)] \\ &= u(c_t^y, c_{t+1}^o) + \beta u(c_{t+1}^y, c_{t+2}^o) \\ &\quad + \beta^2[u(c_{t+2}^y, c_{t+3}^o) + \beta V(t+3)]\end{aligned}$$

and therefore

$$V(t) = \sum_{j=0}^{\infty} \beta^j u(c_{t+j}^y, c_{t+j+1}^o) \quad (1)$$

This looks like

- the **planner's** welfare function,
- the utility function of a household who **lives forever**.

# Budget constraint

- Period budget constraints are

$$c_t^y + s_t = e_1 + b_t \quad (2)$$

$$c_{t+1}^o + (1+n)b_{t+1} = e_2 + R_{t+1}s_t \quad (3)$$

- $b_{t+1}$  is the bequest left to each child by cohort  $t$ .
- Present value budget constraint (set  $n=0$  for simplicity):

$$\begin{aligned} b_t &= c_t^y - e_1 + (c_{t+1}^o - e_2 + b_{t+1})/R_{t+1} \\ &= z_t + b_{t+1}/R_{t+1} \end{aligned}$$

Successively replace the  $b_{t+j}$  with  $z_{t+j} + b_{t+j+1}/R_{t+j+1}$  to obtain

$$b_t = \sum_{j=0}^J \frac{z_t}{D_{t,j}} + \frac{b_{t+J+1}}{D_{t,t+J+1}}$$

where

$$D_{t,j} = \prod_{i=1}^j R_{t+i}$$

is a discount factor.

# Budget constraint

Take  $J \rightarrow \infty$  and assume that

$$\lim_{J \rightarrow \infty} \frac{b_{t+J}}{D_{t,t+J}} = 0$$

Then the present value budget constraint becomes

$$\begin{aligned} & \sum_{j=0}^{\infty} \frac{c_{t+j}^y + c_{t+j+1}^o / R_{t+j+1}}{D_{t,j}} \\ &= b_t + \sum_{j=0}^{\infty} \frac{e_1 + e_2 / R_{t+j+1}}{D_{t,j}} \end{aligned}$$

# Budget constraint

This is a common result:

$$\textit{Present value of spending} = [\textit{Present value of income}] + [\textit{Initial assets}]$$

looks like the budget constraint of an infinitely lived household.

The parent therefore behaves exactly like an infinitely lived individual

- maximizing a single utility function over an infinite horizon
- subject to a single present value budget constraint.

This only works if

- households can borrow and lend at the same interest rate;
- bequests can be negative or are always intended to be positive
- parents are altruistic (not warm glow etc)

Show that the equilibrium allocation is the same as the planner's allocation.  
Why is this important?

- If we think bequests are positive, we can ignore finite lifetimes and write down models with a single, infinitely lived household.

One potential problem:

- We set up the parent's problem as if he could choose the child's actions.
- Why can we do that?

When Are Bequests Positive?

And do they help with dynamic inefficiency?

# When are bequests positive?

Bequests are positive, if a small bequest raises parental utility.

Consider the following perturbation of the optimal plan with  $b = 0$ :

- 1 Reduce old age consumption by  $\varepsilon$ . The utility loss is  $-u_2(t)\varepsilon$ .
- 2 Give  $\varepsilon/(1+n)$  to each child as a bequest.
- 3 Assume the child eats the bequest when young [what if not?] and gains

$$\beta u_1(t+1) \cdot \varepsilon / (1+n) \quad (4)$$

- 4 The household wants to leave a bequest if

$$\beta u_1(t+1) \cdot \varepsilon / (1+n) > u_2(t) \cdot \varepsilon \quad (5)$$

- 5 Apply the parent's FOC to express both gain and loss in terms of  $u_1$ .  
The FOC is

$$u_1(t) = (1+r_{t+1})u_2(t)$$

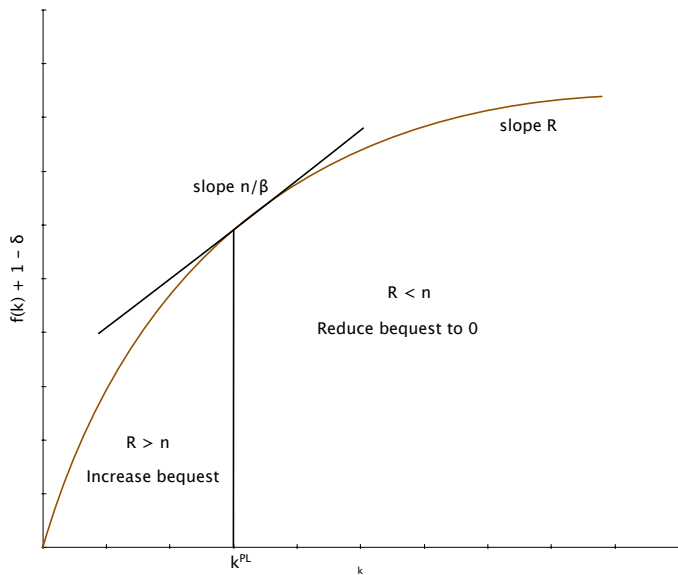
Thus the parent increases his bequest if

$$\beta u_1(t+1) \cdot \varepsilon / (1+n) > u_1(t) / (1+r_{t+1}) \cdot \varepsilon$$

This means:

- A situation where  $R > (1 + n) / \beta$  can never be an equilibrium.
  - Every parent would want to increase his bequest until the MGR holds with equality
  - Then the economy is dynamically *efficient*.
- If without bequests  $R < (1 + n) / \beta$ , households don't want to leave bequests and the bequest motive is irrelevant.
  - Dynamic inefficiency remains.

# Dynamic inefficiency



If the bequest motive is operative ( $b > 0$ ), then:

- The economy attains the modified golden rule.
- Therefore it is dynamically efficient.
- The market equilibrium coincides with the planner's solution (show this!).
- Ricardian equivalence holds even across generations. (We haven't shown that, but it follows directly from the fact that there is a present value budget constraint that holds across generations.)

If the bequest motive is not operative, it does not matter.

- Acemoglu, ch. 5.3, 9.