

# Money in the utility function

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August 7, 2009

# Money in the utility function

- A shortcut for getting money valued in equilibrium: assume that households gain utility from holding money.
- "Sidrauski" model.
- Benefits: Tractability.
- Drawbacks: Arbitrary specification of utility affects results.

# The Economic Environment

- Much of the model is a standard growth model.
- The government prints paper (costlessly).
- Households gain utility from holding paper.

Households solve:

$$\max \int_{t=0}^{\infty} e^{-\rho t} u(c_t, m_t) dt \quad (1)$$

subject to  $k_0, m_0$  given and

$$p(c + \dot{k}) + \dot{M} = p(w + rk + x) \quad (2)$$

$M$  is nominal money.

$m = M/p$ .

$x$  are lump-sum transfers (of money).

# Household

## Budget constraint in real terms

$$\dot{k} + \dot{M}/p = w + rk + x - c \quad (3)$$

Note that

$$\begin{aligned} \dot{m} &= \dot{M}/p - (M/p^2)\dot{p} \\ &= \dot{M}/p - m\pi \end{aligned}$$

where  $\pi$  is the inflation rate ( $\pi = \dot{p}/p$ ).

Therefore

$$\dot{k} + \dot{m} = w + rk + x - c - \pi m \quad (4)$$

# Household

## Budget constraint

- We seem to have 2 state variables  $(k, m)$  but only one law of motion.
- The reason: the correct state variable is wealth:  $A = k + m$ .
- To transform the budget constraint into a law of motion for  $A$ , write it as

$$\dot{A} = w + rA + x - c - (r + \pi)m \quad (5)$$

- Every unit of wealth held in money reduces income by the nominal interest rate  $(r + \pi)$ .

$$H = u(c, m) + \lambda[w + rA + x - c - (r + \pi)m] \quad (6)$$

FOC:

$$u_c = \lambda$$

$$u_m = \lambda(r + \pi)$$

$$\dot{\lambda} = (\rho - r)\lambda$$

TVC:

$$\lim_{t \rightarrow \infty} e^{-\rho t} \lambda_t A_t = 0$$

Static condition:

$$u_c = u_m / (r + \pi) \quad (7)$$

Intertemporal condition:

$$\dot{\lambda} / \lambda = g(u_c) = -(r - \rho) \quad (8)$$

where  $g(z) \equiv \dot{z}/z$  denotes a growth rate.

# Household: Separable utility

If the utility function is **separable**,

$$u(c, m) = v(c) + \bar{v}(m) \quad (9)$$

then

$$u_c = v'(c) \quad (10)$$

and

$$g(u_c) = v''(c)c/v'(c) = -\sigma g_c \quad (11)$$

Then a very common expression emerges:

$$g(c) = (r - \rho) / \sigma \quad (12)$$

**Firms** solve the standard static profit maximization problem:

$$r = f'(k) \quad (13)$$

$$w = f(k) - f'(k)k \quad (14)$$

- The government grows the money supply at the constant rate  $\mu = g(M)$ .
- Implied lump-sum transfers are

$$x = \dot{M}/p = \mu m \quad (15)$$

- Money and factor market clearing are implicit in the notation.
- Goods market clearing is feasibility:

$$\dot{k} + c = f(k) - \delta k \quad (16)$$

# Equilibrium

An equilibrium is a set of functions

$$w_t, r_t, p_t, c_t, k_t, m_t, A_t, x_t, M_t$$

that satisfy

- the transfer equation (2) and the money growth equation
- the definition  $m = M/p$
- 2 firm FOCs
- $(c, m, A, k)$  obey the household's FOCs (for  $c$  and  $m$ ), the budget constraint, and the definition of  $A$ .
- The goods market clears.

These are 9 variables and 10 equations, one of which is redundant by Walras' law.

The boundary conditions are initial values for  $M$  and  $k$  and the TVC.

# Characterization

- We reduce the CE to 4 equations in  $(c, p, k, m)$ .
- Household first-order conditions:

$$\begin{aligned}g(u_c[c, m]) &= -(f'(k) - \delta - \rho) \\ u_c(c, m) &= u_m(c, m) / (f'(k) - \delta + \pi)\end{aligned}$$

- Goods market clearing:

$$\dot{k} + c = f(k) - \delta k$$

- Money growth rule:

$$\dot{m} = (\mu - \pi)m$$

- Assume: the utility function is additively separable

$$u(c, m) = \bar{u}(c) + v(m) \quad (17)$$

- Then money has absolutely no effect on the real sector.
- The evolution of  $c$  and  $k$  is determined by the Euler equation and the goods market clearing condition alone.

# Steady state

In steady state  $c, k, m$  are constant.

The Euler equation then determines the steady state capital stock:

$$r = f'(k) - \delta = \rho \quad (18)$$

Goods market clearing then yields consumption:

$$c = f(k) - \delta k$$

Constant real balances require  $\pi = \mu$ .

The static optimality condition yields an implicit equation for  $m$ :

$$u_m(c_{SS}, m_{SS}) = (\rho + \mu)u_c(c_{SS}, m_{SS}) \quad (19)$$

$\Rightarrow$

$$m_{SS} = m^d(c_{SS}, \rho + \mu) \quad (20)$$

# Super-neutral money

- Changes in money growth ( $\mu$ ) only affect the inflation rate, but not real variables ( $k_{SS}, c_{SS}$ ).
- Intuition: inflation does not alter the intertemporal tradeoff between consumption today and tomorrow.
- Inflation only affects the relative levels of goods and money consumed

- What is the effect of inflation on real money balances? Differentiate (3) to obtain

$$u_{mm}dm = (\rho + \mu)u_{cm}dm + u_c d\mu \quad (21)$$

⇒

$$dm/d\mu = u_c / [u_{mm} - (\rho + \mu)u_{cm}] \quad (22)$$

- Unless money and consumption are too strong complements ( $u_{cm}$  large and positive), higher inflation is associated with lower real money balances and thus lower steady state utility.

# The Friedman Rule

- Which money growth rate maximizes steady state utility?
- Since  $\mu$  does not affect  $c_{ss}$ , we only need to know how to maximize  $m_{ss}$ .
- If we set  $(\rho + \mu) = 0$ , then  $u_m = 0$ , which is the best we can do: satiate the household with money.
- If  $u_m > 0$  even asymptotically, the problem does not have a solution.
- The intuition is quite general:
  - If money provides some kind of benefit, the best we can do is to make it costless to hold money.
  - That will be the case when money pays the same rate of return as capital (the Friedman rule).

- Blanchard & Fischer 4.5