

# Problem Set 5: Growth Model in Continuous Time

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## 1 Capital income tax

[Romer 2.9] Consider a Ramsey economy on its balanced growth path. At time 0 the government starts to tax capital income, so that the interest rate facing the household is

$$r(t) = (1 - \tau) f'(k_t)$$

where I have assumed that  $\delta = 0$ . Tax revenues are rebated to the household in a lump-sum fashion. The change in the policy is unanticipated.

1. How does the tax affect the  $\dot{k} = 0$  and the  $\dot{c} = 0$  loci?
2. How do the balanced growth values of  $c$  and  $k$  change?
3. Describe the changes at time 0 and the transition path thereafter.
4. Show that the saving rate on the balanced growth path ( $[y - c]/y$ ) is decreasing in  $\tau$ .
5. Imagine there are two countries that differ only in  $\tau$ . Do the residents of the high  $\tau$  countries have an incentive to invest in the low  $\tau$  country or vice versa?
6. How do your answers change if the tax revenues are used to pay for government purchases instead of being rebated?

## 2 Continuous Time CIA Model. Cash and Credit Goods.

Crusoe solves the following problem:

$$\max \int_0^{\infty} e^{-\rho t} u(c_t, g_t) dt$$

subject to the budget constraint

$$\dot{k}_t + c_t + g_t + \dot{M}_t/p_t = f(k_t) + x_t$$

and the CIA constraint

$$c_t \leq M_t/p_t$$

The notation is standard. There are two consumption goods, which are perfect substitutes in production but not in consumption. The cash good  $c$  is subject to the CIA constraint, while the credit good  $g$  is not. Denote real balances by  $m_t = M_t/p_t$ .  $x_t$  is a money transfer from the government.

(a) Write down the household's Hamiltonian. Which are his states and controls? Derive first-order conditions for two cases: either the CIA constraint always binds or it never binds.

(b) Define a competitive equilibrium. Assume that the government lets the money stock grow at the constant rate  $g(M)$ .

(c) Derive a set of equations that characterize the steady state. Show that the nominal interest rate equals zero, if the CIA constraint does not bind.

(d) Determine the effects of a higher money growth rate on the steady state allocation. Assume that the utility function takes the form  $u(c, g) = U(c) + V(g)$ , where  $U$  and  $V$  are strictly concave functions.