

# Aggregation

Prof. Lutz Hendricks

August 7, 2009

# Notes on Aggregation

- We have assumed a representative household.
- How restrictive is this assumption?
- If households are not identical, do they "aggregate" into a representative household?

# Example with Heterogeneity

# Example with Heterogeneity

- Consider a Cass-Koopmans model with two types of households,  $i = 1, 2$ .
- Demographics:
  - The population of each type is constant ( $N^i$ ).
- Preferences are identical:  $\int_0^\infty e^{-\rho t} \frac{c^{1-\sigma} - 1}{1-\sigma} dt$ .
- Endowments:
  - Each household starts with capital  $k_0^i$ .
  - Each has one unit of type  $i$  time at any moment.

# Example with Heterogeneity

- Technology:

$$\begin{aligned} Y_t &= K_t^\theta [(L_t^1)^{1-\theta} + (L_t^2)^{1-\theta}] \\ &= \dot{K}_t + \delta K_t + C_t. \end{aligned}$$

- Note: Each household supplies a different type of labor.

- The household problem is entirely standard.
- Solution is  $k_t^i$  and  $c_t^i$  which satisfy Euler equation

$$g(c_t^i) = (r - \rho) / \sigma \quad (1)$$

and budget constraint:

$$\dot{k}^i = rk^i + w^i - c^i \quad (2)$$

- Boundary conditions:  $k_0^i$  given and TVC.

- Factor prices equal marginal products.
- $q = F_k$  and  $w^i = F_{L^i}$ .

A CE consists of functions of time  $c^i, k^i, w^i, r, q, K, L^i$  that satisfy

- 2x2 household conditions
- 3 firm first order conditions
- Factor market clearing:  $K = \sum k^i L^i$  and  $L^i = N^i$
- Goods market clearing:  $F(K, L^1, L^2) - \delta K = \dot{K} + \sum L^i c^i$
- Identity:  $r = q - \delta$

The same objects (but as scalars):  $c^i, k^i, w^i, r, q, K, L^i$ .

These satisfy, in **sequential** order:

- Labor inputs are exogenous.
- $F_K = \rho + \delta$  determines  $K$ .
- $r = \rho$ .
- $w^i = (1 - \theta) (K/L^i)^\theta$  determines  $w^i$ .

We then have an additional 3 equations:

- ① capital market clearing:

$$K = \sum k^i L^i \quad (3)$$

- ② household budget constraints with  $\dot{k}^i = 0$ :

$$c^i = \rho k^i + w^i \quad (4)$$

The 3 equations are supposed to determine 4 variables:  $c^i, k^i$ .

- The steady state is not unique.
- Any  $k^i$  that sum to  $K$  are a steady state.
- For any  $k^i$  pair we pick, the budget constraints tell us the corresponding steady state consumption levels.

# Why is the steady state not unique?

- Both households have the same marginal propensity to consume:  $\rho$ .
- Redistribute a bit of  $k^1$  to  $k^2$ . Aggregate  $C$  is unchanged. All markets clear.
- Effectively, the households behave as if they were one - a **representative household**.
- This is **good**: when it works, we don't have to explicitly model heterogeneous households.

# The Representative Household

# The representative household

How hard is it to get a representative household?

One perspective:

*Any aggregate demand curve is consistent with optimal behavior by a set of households.*

## Theorem

*(Debreu-Mantel-Sonnenschein) Let  $\varepsilon > 0$  be a scalar and  $N < \infty$  be a positive integer. Consider a set of prices  $P_\varepsilon = \{p \in \mathbb{R}_+^N : p_j/p_{j'} \geq \varepsilon \forall j, j'\}$  and any continuous function  $x : P_\varepsilon \rightarrow \mathbb{R}_+^N$  that satisfies Walras' Law and is homogeneous of degree 0. Then there exists an exchange economy with  $N$  commodities and  $H < \infty$  households, where the aggregate demand is given by  $x(p)$  over the set  $P_\varepsilon$ .*

# Why is aggregation so hard?

- The problem is income effects.
- Changing prices effectively redistributes income across households.
- If the income elasticities of various goods are very different, demand curves could be upward sloping over some intervals.
- But there is hope if income effects are not too strong.

## Theorem

*(Gorman aggregation) Consider an economy with a finite number  $N$  of commodities and a set  $H$  of households. Suppose that the preferences of household  $i \in H$  can be represented by an indirect utility function of the form*

$$v^i(p, y^i) = a^i(p) + b(p) y^i$$

*then these preferences can be represented by those of a representative household with indirect utility*

$$v(p, y) = \int a^i(p) di + b(p) y$$

*where  $y$  is aggregate income.*

- Key feature of Gorman preferences:
  - All households have the same constant propensity to consume out of income.
- This is why redistributing income does not change consumption.
- Then aggregate income is sufficient to figure out demand.

One example of preferences in the Gorman class: **Dixit-Stiglitz** type CES utility:

$$U(x_1, \dots, x_N) = \left[ \sum_{j=1}^N (x_j - \xi_j)^\theta \right]^{1/\theta}$$

where the  $\xi_j$  are individual specific.

When we study R&D driven innovation, we will use these preferences.

- The growth model has CES preferences:

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

- This is not a coincidence.
- CES preferences are consistent with balanced growth.
- This is because the marginal propensity to consume is constant on the balanced growth path.
- This is why redistribution does not change aggregate consumption.

- Acemoglu, "Introduction to modern economic growth," ch. 5.