

Aggregation

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- We have assumed a representative household.
- How restrictive is this assumption?
- If households are not identical, do they "aggregate" into a representative household?

Example with Heterogeneity

Example with Heterogeneity

- Consider a Cass-Koopmans model with two types of households, $i = 1, 2$.
- Demographics:
 - The population of each type is constant (N^i).
- Preferences are identical: $\int_0^{\infty} e^{-\rho t} \frac{c^{1-\sigma} - 1}{1-\sigma} dt$.
- Endowments:
 - Each household starts with capital k_0^i .
 - Each has one unit of type i time at any moment.

- Technology:

$$\begin{aligned} Y_t &= K_t^\theta [(L_t^1)^{1-\theta} + (L_t^2)^{1-\theta}] \\ &= \dot{K}_t + \delta K_t + C_t. \end{aligned}$$

- Note: Each household supplies a different type of labor.

- The household problem is entirely standard.
- Solution is k_t^i and c_t^i which satisfy Euler equation

$$g(c_t^i) = (r - \rho) / \sigma \quad (1)$$

and budget constraint:

$$\dot{k}^i = rk^i + w^i - c^i \quad (2)$$

- Boundary conditions: k_0^i given and TVC.

- Factor prices equal marginal products.
- $q = F_k$ and $w^i = F_{L^i}$.

A CE consists of functions of time $c^i, k^i, w^i, r, q, K, L^i$ that satisfy

- 2x2 household conditions
- 3 firm first order conditions
- Factor market clearing: $K = \sum k^i L^i$ and $L^i = N^i$
- Goods market clearing: $F(K, L^1, L^2) - \delta K = \dot{K} + \sum L^i c^i$
- Identity: $r = q - \delta$

The same objects (but as scalars): $c^i, k^i, w^i, r, q, K, L^i$.

These satisfy, in **sequential** order:

- Labor inputs are exogenous.
- $F_K = \rho + \delta$ determines K .
- $r = \rho$.
- $w^i = (1 - \theta)(K/L^i)^\theta$ determines w^i .

We then have an additional 3 equations:

- ① capital market clearing:

$$K = \sum k^i L^i \quad (3)$$

- ② household budget constraints with $\dot{k}^i = 0$:

$$c^i = \rho k^i + w^i \quad (4)$$

The 3 equations are supposed to determine 4 variables: c^i, k^i .

- The steady state is not unique.
- Any k^i that sum to K are a steady state.
- For any k^i pair we pick, the budget constraints tell us the corresponding steady state consumption levels.

Why is the steady state not unique?

- Both households have the same marginal propensity to consume: ρ .
- Redistribute a bit of k^1 to k^2 . Aggregate C is unchanged. All markets clear.
- Effectively, the households behave as if they were one - a **representative household**.
- This is **good**: when it works, we don't have to explicitly model heterogeneous households.

The Representative Household

The representative household

How hard is it to get a representative household?

One perspective:

Any aggregate demand curve is consistent with optimal behavior by a set of households.

Theorem

(Debreu-Mantel-Sonnenschein) Let $\varepsilon > 0$ be a scalar and $N < \infty$ be a positive integer. Consider a set of prices $P_\varepsilon = \{p \in \mathbb{R}_+^N : p_j/p_{j'} \geq \varepsilon \forall j, j'\}$ and any continuous function $x : P_\varepsilon \rightarrow \mathbb{R}_+^N$ that satisfies Walras' Law and is homogeneous of degree 0. Then there exists an exchange economy with N commodities and $H < \infty$ households, where the aggregate demand is given by $x(p)$ over the set P_ε .

Why is aggregation so hard?

- The problem is income effects.
- Changing prices effectively redistributes income across households.
- If the income elasticities of various goods are very different, demand curves could be upward sloping over some intervals.
- But there is hope if income effects are not too strong.

Gorman aggregation

Theorem

(Gorman aggregation) Consider an economy with a finite number N of commodities and a set H of households. Suppose that the preferences of household $i \in H$ can be represented by an indirect utility function of the form

$$v^i(p, y^i) = a^i(p) + b(p)y^i$$

then these preferences can be represented by those of a representative household with indirect utility

$$v(p, y) = \int a^i(p) di + b(p)y$$

where y is aggregate income.

- Key feature of Gorman preferences:
 - All households have the same constant propensity to consume out of income.
- This is why redistributing income does not change consumption.
- Then aggregate income is sufficient to figure out demand.

One example of preferences in the Gorman class: **Dixit-Stiglitz** type CES utility:

$$U(x_1, \dots, x_N) = \left[\sum_{j=1}^N (x_j - \xi_j)^\theta \right]^{1/\theta}$$

where the ξ_j are individual specific.

When we study R&D driven innovation, we will use these preferences.

- The growth model has CES preferences:

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

- This is not a coincidence.
- CES preferences are consistent with balanced growth.
- This is because the marginal propensity to consume is constant on the balanced growth path.
- This is why redistribution does not change aggregate consumption.

- Acemoglu, "Introduction to modern economic growth," ch. 5.