

## Problem Set 3: Infinite Horizon Model

Econ720. Fall 2009. Lutz Hendricks

### 1 Land Prices with Capital Accumulation

Consider the following economy with land and capital. There is a single good produced from land  $L$  and capital  $K$  according to

$$K_{t+1} = AF(K_t, L_t) + (1 - \delta)K_t - c_t \quad (1)$$

where  $A$  is an exogenous productivity factor,  $\delta$  is the depreciation rate of capital, and  $c$  is consumption. The production function has constant returns to scale. Production takes place in a representative firm which rents capital and land from households. The rental prices are  $r_t$  and  $q_t$ . The purchase price of land is  $p_t$ . The aggregate endowment of land,  $L$ , is fixed.

A single representative household orders consumption sequences according to  $\sum_{t=0}^{\infty} \beta^t u(c_t)$ . He receives income only from holding capital and land.

- (a) Set up the household's Bellman equation. Define a solution to the household problem.
- (b) Define a competitive equilibrium.
- (c) Determine the effects of the following changes on steady state prices and quantities. A qualitative characterization is sufficient (which variables increase/decrease?):  $L$  increases,  $A$  increases.

### 2 Wealth in the utility function

Consider the following modification of the standard growth model where the households derives utility from holding wealth. The representative household solves

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t, k_{t-1})$$

subject to the budget constraint

$$k_{t+1} = R_t k_t + w_t - c_t$$

The utility function is strictly concave and increasing in both arguments. The representative firm rents capital and labor from the household at rental prices  $q_t$  and  $w_t$  to maximize period profits. The production function is given by  $F(K_t, L_t)$  and has standard properties including Inada conditions and constant returns to scale.

- (a) State the household problem.
- (b) Derive and explain the conditions that characterize a solution to the household problem (in sequence language).
- (c) Define a competitive equilibrium.
- (d) Derive a single equation that determines the steady state capital stock.
- (e) Is the steady state unique? Explain the intuition why the steady state is or is not unique.