

# Asset Pricing

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The infinite horizon model can be used to price long-lived assets.

- This is more interesting in stochastic economies.
- It then yields the famous  $\beta$  measure of risk and the CAPM.

# Pricing Assets: An Example

We study an example with these features:

- A representative household with utility  $\sum_{t=0}^{\infty} \beta^t u(c_t)$ .
- The household supplies  $N$  units of labor to the firm.
- Firms produce a perishable good using labor and land.
- Land is in fixed supply  $L$ .

- Firms rent labor and land to produce a single, perishable good.
- The production function is  $F(N_t, L_t; A_t)$ .
  - constant returns in  $(N, L)$
- The productivity sequence  $\{A_t\}$  is given.

# Firm's problem

The firm's problem is standard:

$$\max F(N_t, L_t; A_t) - w_t N_t - r_t L_t \quad (1)$$

FOCs:

$$r = F_L \quad (2)$$

$$w = F_N \quad (3)$$

Solution:  $N_t, L_t$  that satisfy the 2 FOCs.

# Household

The household solves

$$V(l) = \max u(c) + \beta V(l') \quad (4)$$

subject to

$$p(l' - l) = wN + rl - c$$

The first-order conditions are

$$p u'(c) = \beta V'(l')$$

$$V'(l) = u'(c)(r + p)$$

The Euler equation is standard

$$u'(c) = \beta u'(c') \frac{r' + p'}{p}$$

Solution:  $\{c_t, l_t\}$  that solve the Euler equation and budget constraint.

A competitive equilibrium is a set of sequences  $(c_t, l_t, p_t, r_t, w_t)$  that satisfy:

- household: Euler equation and budget constraint;
- firm: 2 FOCs:
- market clearing for land:  $L = l$ ;
- market clearing for goods:  $c = F(N, L)$ .

# The price of land

- We find a difference equation for  $p_t$ .
- Substitute the goods market clearing condition and the first-order condition for  $r$  into the Euler equation to obtain

$$u'(F(N, L)) = \beta u'(F(N', L')) \frac{F_L(N', L') + p'}{p} \quad (5)$$

- Note that this difference equation only contains  $p$  and exogenous variables.

# The price of land

- We solve the difference equation for  $p_t$  by **forward iteration**.

$$\begin{aligned} p_t &= \beta \frac{u'(c_{t+1})}{u'(c_t)} \left\{ F_L(t+1) + \beta \frac{u'(c_{t+2})}{u'(c_{t+1})} [F_L(t+2) + p_{t+2}] \right\} \\ &= \sum_{j=0}^{\infty} F_L(t+j+1) \frac{\beta^{j+1} u'(F(N, L, A_{t+j+1}))}{u'(F(N, L, A_t))} \end{aligned} \quad (6)$$

- In deriving (6) I made use of the fact that  $c_t = F(N, L, A_t)$  and that

$$\frac{u'(c_{t+1})}{u'(c_t)} \frac{u'(c_{t+2})}{u'(c_{t+1})} \cdots \frac{u'(c_{t+1+\mathcal{T}})}{u'(c_{t+\mathcal{T}})} = \frac{u'(c_{t+1+\mathcal{T}})}{u'(c_t)}$$

# The price of land

- The asset price equals the discounted present value of "divdends."

$$p_t = \sum_{j=0}^{\infty} F(t+j+1) MRS(t, t+j+1) \quad (7)$$

- The discount factor is the Marginal Rate of Substitution

$$MRS(t, t+j) = \frac{\beta^j u'(t+j)}{u'(t)} \quad (8)$$

- This is a fairly general result.

# The price of land: Intuition

- Start from the equilibrium price.
- Add  $\varepsilon$  to the date  $t+j$  payoff.
- The household gains  $\beta^j u'(t+j) \varepsilon$ .
- The household's willingness to pay for this:  $u'(t) \varepsilon$ .
- The derivative of the price:

$$\partial p_t / \partial F(t+j) = \frac{\beta^j u'(t+j)}{u'(t)} \quad (9)$$

# The price of land: stationary economy

- We calculate the price of land for a stationary economy, in which  $A_t = A^S$ .

$$p_t^S = F_L(N, L, A^S) \frac{\beta}{1 - \beta}$$

## The price of land: fluctuating economy

- We calculate the price of land for an economy which is subject to deterministic fluctuations.
- In even periods  $A_t = A^H$  and in odd periods  $A_t = A^L$  with  $A^H \geq A^L$ .
- Assume that technology shocks are neutral in the sense that, for given factor inputs, the marginal product of land is independent of  $A_t$ .
- Assume that technology shocks are symmetric in the sense that  $2F^S = F^H + F^L$ , where  $F^j = F(N, L; A^j)$ .

## The price of land: fluctuating economy

- The trick is to break the sum in (6) into two parts, one for even and one for odd periods:

$$\begin{aligned} p_t^R &= \beta F_L \left\{ \sum_{j=0}^{\infty} \beta^{2j} \frac{u'(F(N, L, A_{t+1+2j}))}{u'(F(N, L, A_t))} + \beta \sum_{j=0}^{\infty} \beta^{2j} \frac{u'(F(N, L, A_{t+2+2j}))}{u'(F(N, L, A_t))} \right\} \\ &= \beta F_L \left\{ \frac{u'(F(N, L, A_{t+1}))}{u'(F(N, L, A_t))} \frac{1}{1-\beta^2} + \frac{u'(F(N, L, A_{t+2}))}{u'(F(N, L, A_t))} \frac{\beta}{1-\beta^2} \right\} \end{aligned}$$

- Denote the marginal rate of substitution between odd and even periods by

$$\alpha = u'(F^H)/u'(F^L) < 1$$

# The price of land: fluctuating economy

- If  $t$  is even, then

$$p_t^{even} = (1/\alpha + \beta) F_L \frac{\beta}{1 - \beta^2}$$

- If  $t$  is odd, then

$$p_t^{odd} = (\alpha + \beta) F_L \frac{\beta}{1 - \beta^2}$$

# The price of land: fluctuating economy

- Since  $1 - \beta^2 = (1 + \beta)(1 - \beta)$ ,

$$\frac{p_t^{\text{even}}}{p_t^S} = \frac{1/\alpha + \beta}{1 + \beta} > 1$$

$$\frac{p_t^{\text{odd}}}{p_t^S} = \frac{\alpha + \beta}{1 + \beta} < 1$$

- In even periods, the "risky asset" is always worth more than the "safe asset." In odd periods, the reverse is true.
- The word "risky asset" is misleading; it actually provides insurance against low consumption states.

# The price of land: Intuition

- What is the intuition?
- Consider an even period.
  - Times are good, so that saving is easy.
  - And the return tomorrow is worth a lot because times will be bad.
  - Hence, the demand for land is high and so is the price.
- In odd periods, saving is painful and the return won't be worth much tomorrow. So the price is low.

- The standard growth model is also the standard framework for pricing assets.
- The price of an asset equals the present value of "dividends."
- The discount factors are the Marginal Rates of Substitution.
- This survives in stochastic environments. Just add  $E[.]$ .