

Cash-in-Advance Model

Prof. Lutz Hendricks

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- We study a second model of money.
- OLG models have 2 shortcomings:
 - ① They fail to explain rate of return dominance.
 - ② Money has no transaction value.
- CIA models focus on **transactions demand** for money.

- The overall model structure is that of the standard growth model.
- The firm hires K and L from the household.
- A single representative household:
 - works for the firm
 - saves in the form of money and capital
- The transaction technology requires that **some goods are purchased with money.**

Timing within periods

- 1 The household enters the period with capital k_t and a stock of money m_{t-1}^d .
- 2 He then receives a transfer of money τ_t from the government. His period t money holdings are

$$m_t = m_{t-1}^d + \tau_t$$

- 3 The household produces and sells his output for money to be received at the “end of the period.”
- 4 He uses m_t to buy goods from other households (c_t and k_{t+1}).
- 5 The household is paid for the goods he sold in step 3 so that his end of period money stock is m_t^d .

Note that money earned in period t cannot be used until $t + 1$.

Household problem

We simply add one constraint to the household problem: the CIA constraint.

The household solves

$$\max \sum_{t=1}^{\infty} \beta^t u(c_t)$$

subject to the budget constraint

$$k_{t+1} + c_t + m_t^d/p_t = f(k_t) + (1 - \delta)k_t + m_t/p_t$$

and the CIA constraint

$$m_t/p_t \geq c_t + k_{t+1} - (1 - \delta)k_t$$

and the law of motion

$$m_{t+1} = m_t^d + \tau_{t+1}$$

Household problem

Remarks

- Exactly what kinds of goods have to be bought with cash is arbitrary.
- The CIA constraint holds with equality if the rate of return on money is less than that on capital (the nominal interest rate is positive).

Houshold: Dynamic Program

Individual state variables: m, k .

Bellman equation:

$$V(m, k) = \max u(c) + \beta V(m', k') \\ + \lambda(BC) + \gamma(CIA)$$

We need to impose

$$m_t = m_{t-1}^d + \tau_t$$

Then we can use m_{t+1} as a control (this would not work under uncertainty).

Bellman Equation

$$\begin{aligned} V(m, k) = & \max u(c) + \beta V(m', k') \\ & + \lambda [f(k) + (1 - \delta)k + m/p - c - k' - (m' - \tau')/p] \\ & + \gamma [m/p - c - k' + (1 - \delta)k] \end{aligned}$$

$\lambda > 0$: multiplier on budget constraint

γ : multiplier on CIA constraint - could be 0.

First-order conditions

$$\begin{aligned}u'(c) &= \lambda + \gamma \\ \beta V_m(\bullet') &= \lambda/p \\ \beta V_k(\bullet') &= \lambda + \gamma\end{aligned}$$

Kuhn Tucker:

$$\begin{aligned}\gamma[m/p - c - k' + (1 - \delta)k] &= 0 \\ \gamma &\geq 0\end{aligned}$$

Thus:

$$u'(c) = \beta V_k(\bullet')$$

Envelope conditions:

$$V_m = (\lambda + \gamma)/p$$

$$V_k = \lambda[f'(k) + 1 - \delta] + \gamma[1 - \delta]$$

Eliminate derivatives of V :

$$\begin{aligned}\beta[\lambda' + \gamma'] / p' &= \lambda / p \\ (\lambda + \gamma) / \beta &= \lambda' f'(k') + [1 - \delta][\lambda' + \gamma'] \\ \beta u'(c') p / p' &= \lambda \\ u'(c) &= \lambda + \gamma\end{aligned}$$

Obtain the Euler equation

$$u'(c) = \beta^2 u'(c'')(p' / p'') f'(k') + (1 - \delta) \beta u'(c') \quad (1)$$

Solution: Binding CIA constraint

A solution to the household problem is:

A value function (V) and policy functions ($m'(m, k), k'(m, k)$) that "solve" the Bellman equation.

Or: Sequences $\{c_t, m_{t+1}, k_{t+1}\}$ that satisfy:

- 1 the Euler equation;
- 2 the budget constraint;
- 3 the CIA constraint (with the law of motion).
- 4 transversality conditions:

$$\lim_{t \rightarrow \infty} \beta^t u'(c_t) k_t = 0$$

$$\lim_{t \rightarrow \infty} \beta^t u'(c_t) m_t / p_t = 0$$

Interpretation of the Euler equation

Today:

- Give up $dc = -\varepsilon$.

Tomorrow:

- $dk' = \varepsilon$.
- Eat the undepreciated capital: $dc' = (1 - \delta)\varepsilon$.
- Produce additional output $f'(k')\varepsilon$.
- Save it as money: $dm'' = f'(k')\varepsilon p'$.

The day after:

- Eat an additional dm''/p'' .

- Why isn't there a simple Euler equation for the perturbation:
 - ① $dc = -\varepsilon$. $dm' = p\varepsilon$.
 - ② $dc' = \varepsilon p/p'$.
- Answer: This would leave cash on the table today.
- Therefore, the Euler equation for this perturbation is:

$$\begin{aligned}u'(c) &= \lambda + \gamma \\ &= \beta u'(c') p/p' + \gamma\end{aligned}$$

With $\gamma = 0$:

$$\beta \lambda' / p' = \lambda / p$$

$$\lambda / \beta = \lambda' [f'(k') + 1 - \delta]$$

$$\beta u'(c') p / p' = \lambda$$

$$u'(c) = \lambda$$

Simplify: We get the standard Euler equation

$$u'(c) = \beta u'(c') [f'(k') + 1 - \delta] \quad (2)$$

and the "no arbitrage" condition:

$$f'(k') + 1 - \delta = p/p' \quad (3)$$

Solution: $\{c_t, k_{t+1}, m_{t+1}\}$ that satisfy:

- Euler
- Budget constraint
- No arbitrage.
- Boundary conditions.

When does the CIA constraint bind?

- The CIA constraint binds unless the return on money equals that on capital, i.e. the nominal interest rate is zero.
- No arbitrage:

$$1 + i = (1 + r)(1 + \pi) = [f'(k) + 1 - \delta] p' / p = 1$$

- Holding money has no opportunity cost.
- The presence of money does not distort the intertemporal allocation.
- We have the standard Euler equation.

Equilibrium

- The government's only role is to hand out lump-sum transfers of money.
- The money growth rule is

$$\tau_t = gm_{t-1}^d$$

- Money holdings in period t are

$$\begin{aligned} m_t &= m_{t-1}^d + \tau_t \\ &= (1+g)m_{t-1}^d \end{aligned}$$

- Goods: $c + k' = f(k) + (1 - \delta)k$.
- Money market: implicit in the notation.

An **equilibrium** is a sequence $(k_t, m_t, c_t, \tau_t, p_t)$ that satisfies

- 1 the money growth rule and definition of τ (sort of a government budget constraint);
- 2 the household optimality conditions (see above) (3 equations)
- 3 the goods market clearing condition.

Steady State

Binding CIA constraint

- In steady state all real, per capita variables are constant $(c, k, m/p)$.
- This requires $\pi = g$ to hold real money balances constant.
- The Euler equation implies

$$1 = \beta^2(1 + \pi)^{-1}f'(k') + (1 - \delta)\beta$$

- Using $1 + \pi = 1 + g$ this can be solved for the capital stock:

$$f'(k_{ss}) = (1 + g)[1 - \beta(1 - \delta)]/\beta^2 \quad (4)$$

- Higher inflation reduces k_{ss} .

Assuming that the CIA constraint binds:

$$f(k) = m/p \quad (5)$$

Goods market clearing with constant k implies

$$c = f(k) - \delta k \quad (6)$$

A steady state is a vector $(c, k, m/p)$ that satisfies (4) through (6).

Properties of the Steady State

CIA binding

Definition

Money is called **neutral** if changing the level of M does not affect the real allocation.

It is called **super neutral** if changing the growth rate of M does not affect the real allocation.

Money is not super neutral

- Higher inflation (g) implies a lower k .
- Inflation increases the cost of holding money, which is required for investment (inflation tax).

Properties of the Steady State

CIA binding

Exercise:

- Show that super-neutrality would be restored, if the CIA constraint applied only to consumption ($m/p \geq c$).
- What is the intuition for this finding?

The velocity of money is one

- - Higher inflation reduces money demand only by reducing output.
 - This is a direct consequence of the rigid CIA constraint and probably an undesirable result.
 - Obviously, this would not be a good model of hyperinflation.
 - This limitation can be avoided by changing the transactions technology (see RQ).

Steady State

CIA constraint does not bind

$$f'(k) + 1 - \delta = (1 + g)^{-1} \quad (7)$$

$$= 1/\beta \quad (8)$$

$$f(k) - \delta k = c \quad (9)$$

Result:

A steady state only exists if $\beta = 1 + g$.

Then: The steady state coincides with the (Pareto optimal) non-monetary economy.

Steady State

CIA constraint does not bind

- Why is there no steady state with $1 + g < \beta$?
- $\beta R = \beta / (1 + g) > 1$.
- The household would choose unbounded consumption. Cf.

$$u'(c) = \beta R u'(c') \quad (10)$$

- **The Friedman rule maximizes steady state welfare.**
- Friedman Rule: Set nominal interest rate to 0.
- Proof: Under the Friedman rule, the steady state conditions of the CE coincides with the non-monetary economy's.
- Intuition:
 - It is optimal to make holding money costless b/c money can be costlessly produced.
 - This requires that the rate of return on money $\frac{1}{1+\pi}$ equal that on capital.

Is this a good theory of money?

Recall the central questions of monetary theory:

- ① Why do people hold money, an asset that does not pay interest (rate of return dominance)?
- ② Why is money valued in equilibrium?
- ③ What are the effects of monetary policy: one time increases in the money supply or changes in the money growth rate?

Is this a good theory of money?

Positive features:

- 1 Rate of return dominance.
- 2 Money plays a liquidity role.

Drawbacks:

- 1 The reason why money is needed for transactions is not modeled.
- 2 The form of the CIA constraint is arbitrary (and important for the results).
- 3 The velocity of money is fixed.

- Blanchard & Fischer (1989), 4.2.