

## Problem Set 4: Cash-in-Advance Model

Econ720. Fall 2009. Lutz Hendricks

### 1 Shopping time

Consider the following monetary model.

**Households:** There is a single representative household with preferences over consumption ( $c$ ) and leisure ( $l$ ) given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t); \quad 0 < \beta < 1.$$

The household supplies capital ( $k$ ) and labor ( $n$ ) to a firm, so that income is  $(1 + r_t)k_t + w_t n_t$ . The household also brings  $M_t$  units of money into the period. Let  $P_t$  be the price level in period  $t$  and define  $m_t = M_t/P_t$ . The household's budget constraint is then

$$(1 + r_t)k_t + w_t n_t + m_t = c_t + k_{t+1} + m_{t+1} (1 + \pi_{t+1})$$

where  $1 + \pi_{t+1} = P_{t+1}/P_t$ . Money is held to facilitate consumption transactions. The transactions technology is such that  $s_t$  units of time are required to purchase  $c_t$  given money balances  $m_t$ :

$$s_t = g(c_t, m_t)$$

Obviously,  $g_c > 0$  and  $g_m < 0$ . The household is endowed with one unit of time in each period, which is split between leisure, work, and shopping:

$$1 = l_t + n_t + s_t$$

**Firms:** Firms produce output in competitive markets, using labor and capital, both of which are hired from households on spot markets. Firms have access to a typical neoclassical production function:  $f(k_t, n_t)$ . The depreciation rate is  $0 < \delta < 1$ .

**Questions:** .

1. Define a solution to the household problem.
2. Define a competitive equilibrium. Assume that the money supply is constant.
3. Is money neutral in this economy? Prove your answer using the system of equations that define a competitive equilibrium.
4. Would money still be neutral if the transactions technology used nominal money balances i.e.,  $s_t = g(c_t, M_t)$ ? Explain the intuition. You need not derive your answer.