Problem Set 4: Cash-in-Advance Model

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1 Shopping time

Demographics: There is a single representative household who lives forever.

Preferences: The household values consumption (c) and leisure (l) according to

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t); \qquad 0 < \beta < 1.$$

Endowments: In each period, the agent is endowed with 1 unit of time that can be used for leisure (l), work (n), and shopping (s):

$$1 = l_t + n_t + s_t$$

The household is endowed with k_0 units of capital and M_0 units of money in period 0.

Technology: The transactions technology is such that s_t units of time are required to purchase c_t given money balances $m_t = M_t/P_t$:

$$s_t = g(c_t, m_t)$$

where P_t is the price of the good. Obviously, $g_c > 0$ and $g_m < 0$.

Goods are produced from capital and labor with the production function $f(k_t, n_t)$, which has nice properties. The resource constraint is $f(k, n) + (1 - \delta) k = c + k'$.

Markets: The usual markets for goods, money, capital and labor rental operate. There is no government and the money supply is constant.

Questions:

1. Write out the household problem in sequence language. Hint: the budget constraint should contain the term $m_{t+1}(1 + \pi_{t+1})$ where π is inflation.
2. Write out the household's Dynamic Program. Hint: It is best to substitute all constraints into the objective function, except for the budget constraint.
3. Derive and interpret the first-order conditions (not yet substituting out the Lagrange multiplier).
4. Define the "total marginal utility from consumption" as

   $$v(c, l, m) = u_c(c, l) - g_c(c, m)\ u_l(c, l)$$

   Derive the static optimality condition and the Euler equation. Show that the return on money is lower than the return on capital.
5. Define a solution to the household problem in sequence language.
6. Define a competitive equilibrium.
7. Is money neutral in this economy? Prove your answer using the system of equations that define a competitive equilibrium.
8. Would money still be neutral if the transactions technology used nominal money balances i.e., $s_t = g(c_t, M_t)$? Explain the intuition. You need not derive your answer.