Models of Creative Destruction

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Motivation

- We study models of process innovation ("quality ladders").
- New issues:

1. Innovations replace existing monopolies - creative destruction.
2. Multiple firms can produce the same good - price competition.

A Baseline Model

- Demographics: There is a single, infinitely lived household.
- Preferences:

$$\int_0^\infty e^{-\rho t} u(C_t) dt \tag{1}$$

Commodities

At date t we have:

- 1 final good Y. Used for consumption, R&D, and production of intermediates.
- A unit measure of intermediate inputs, indexed by v.

Each intermediate good can be produced with many different "qualities" $q(v, t)$.

Innovation takes the form of introducing better qualities.

Final Goods Technology

- There is one final good that can be used for consumption, investment in R&D, and production of intermediate inputs:

$$Y_t = C_t + X_t + Z_t \tag{2}$$

- Final goods are produced from labor and intermediates:

$$Y_t = (1-\beta)^{-1} L_t^{\beta} \int_0^1 q(v,t) x(v,t)^{1-\beta} dv \tag{3}$$

- There is a unit measure of intermediates.
- $q(v,t)$ is the best available quality of intermediate v at t.
- Assumption: Only the best quality is used in equilibrium.

Final Goods Technology

- Why is only the best quality used?
- For each good v, a large number of qualities are offered (by monopolists): $q(s, v, t)$.
- They are perfect substitutes in the production of final goods.
- Think of the production function as

$$Y_t = (1-\beta)^{-1} L_t^{\beta} \int_0^1 X(v,t)^{1-\beta} dv \tag{4}$$

- $X(v, t)$ is input of all vintages of good v:

$$X(v,t) = \left[\int_{-\infty}^{t} q(s,v,t)^{1/(1-\beta)} x(s,v,t) ds \right] \tag{5}$$

Final Goods Technology

- When patent owners for all vintages s compete (see Ch. 12), pricing ensures that only the vintage with the highest q is used in equilibrium.

$$X(v,t) = q(v,t)^{1/(1-\beta)} x(v,t) \tag{6}$$

where $q(v,t) = \max_s q(s,v,t)$.

- Exercise: Derive conditions such that this is true. (See end of slides for an answer sketch.)

Technology: Innovation

- Each innovation takes the quality from $q(v,t)$ to $\lambda q(v,t)$.
- The quality step is $\lambda > 1$.
- Innovation takes place separately for each v.
- Investing $Z(v,t)$ goods creates a flow of quality improvements:

$$n(v,t) = \eta Z(v,t)/q(v,t) \tag{7}$$

- This really means: the Poisson arrival rate of innovations is $n(v,t)$.
- Over a short interval:

$$q(v,t+\Delta t) = \begin{cases} q(v,t) & \text{with probability } 1 - n(v,t)\Delta t \\ \lambda q(v,t) & \text{with probability } n(v,t)\Delta t \end{cases} \tag{8}$$

Technology: Intermediate Goods

- Intermediates perish in production.
- Their marginal cost is $\psi q(v,t)$.
- Note: $q(v,t)$ shows up in various places in such a way to ensure balanced growth.

Market Arrangements

- Final goods: perfect competition.
- Innovators received permanent patents for the qualities they create.
  - Other firms can improve on their qualities.
- Intermediate goods firms are the same as innovators (or innovators sell qualities at competitive prices).
  - They are monopolists
  - but there is a competitive fringe of firms offering lower qualities
- Assumption: Current monopolists cannot innovate.
  - not binding: they would not want to innovate b/c their gain in profits is lower than the gain for new entrants.
- Free entry into innovation.
- Households own the innovating firms and receive their profits.

Equilibrium

- Allocation: C_t, X_t, Z_t, Y_t and $q(v,t)$.
- Prices: $p^x(v,t), V(v,t)$, r_t, w_t.
- Such that:

1. Agents "maximize" (below).
2. Markets clear.
3. Zero profits for innovators.

- A wrinkle: $q(v,t)$ is stochastic. So the equilibrium def is slightly wrong.
- Assumption: Invoke a law of large numbers to ensure that aggregates are deterministic.

Equilibrium Characterization

Household

- Again: avoid writing out the budget constraint.
- Just note that the household owns a portfolio of assets (shares of intermediate goods firms) with deterministic rate of return $r(t)$.
- Euler equation:

$$g(C(t)) = \frac{r(t) - \rho}{\theta} \tag{9}$$

- Value of assets held:

$$a(t) = \int_0^1 V(v,t)dv \tag{10}$$

- $V(v,t)$ is the value of the intermediate input firm v.
- TVC: $\lim_{t\to\infty} e^{-rt}a(t) = 0$ [with constant interest rate].
- We need to find r to find the growth rate.

Free Entry

- As usual: we find r from free entry:
    - Value of a patent = present value profits, discounted at r.
    - Free entry: $V(v, t|q)$ = cost of a one-step quality improvement.

Free Entry

- What is the cost of a one-step quality improvement?
- Suppose current quality is $q(v,t)/\lambda$. (simplifies notation)
- Success rate of innovation from the production function:

$$n(v,t) = \eta Z(v,t)/[q(v,t)/\lambda] \tag{11}$$

- New quality is $q(v,t)$ with value $V(v,t|q)$.
- Investing $Z(v,t)$ for period Δt yields an innovation with probability $n(v,t)\Delta t$.
- Marginal cost: $Z(v,t)\Delta t$.
- Marginal benefit: a patent valued at $V(v,t|q)$ with probability $\eta Z(v,t)/[q(v,t)/\lambda]\Delta t$.

Free Entry

- If marginal benefit < marginal cost: no innovation (not interesting).
- Otherwise: innovation continues until

$$\underbrace{Z(v,t)\Delta t}_{\text{marginal cost}} = \underbrace{V(v,t|q)\frac{\lambda\eta}{q(v,t)}Z(v,t)\Delta t}_{\text{marginal benefit}} \tag{12}$$

- Or:

$$\frac{q(v,t)}{\lambda\eta} = V(v,t|q) \tag{13}$$

Value of innovation

- Next we need to find the present value of profits.
- **General asset pricing equation** (which we will derive later...):

$$rp = \dot{p} + d \quad (14)$$

- In words:
  - the current payoff of the asset consists of capital gain $\dot{p}$ and divident d.
  - rate of return = [current payoff] / [current price]

Value of innovation

- Applying the asset pricing equation to the value of the firm.
- Current price: $p = V(v, t, |q)$.
- Dividend: Flow profit: $\pi(v, t) = d$.
- Lose profit flow at rate $z(v, t|q)$ - endogenous, chosen by competitors.
- Capital gain: $\dot{V}(v, t|q) - z(v, t|q)V(v, t|q)$.
- Pricing equation:

$$r(t)V(v, t|q) = \dot{V}(v, t|q) + \pi(v, t|q) - z(v, t|q)V(v, t|q) \tag{15}$$

- We need to find profits to find r...

Digression: Capital Gain

- One might expect the capital gain to be

$$(1-z)\dot{V} - zV \tag{16}$$

- Write out payoffs over interval Δt

$$Vr\Delta t = \pi\Delta t + (1 - z\Delta t)\dot{V}\Delta t - z\Delta t V \tag{17}$$

- Take $\Delta t \to 0$ and the term $(1 - z\Delta t) \to 1$.

Final goods demand

- To find profits we need prices and demand for intemediates.
- Technology for final goods:

$$Y_t = (1-\beta)^{-1} L_t^{\beta} \int_0^1 q(v,t) x(v,t)^{1-\beta} dv \tag{18}$$

- Demand for intermediates is iso-elastic:

$$x(v,t) = \left(\frac{q(v,t)}{p^x(v,t)} \right)^{1/\beta} L \tag{19}$$

Intermediate goods

- Assume drastic innovation.
- Owner of current best quality can set monopoly price:

$$p^x(v,t) = \frac{\psi q(v,t)}{1-\beta} \tag{20}$$

- Normalize $\psi = 1 - \beta$.
- Then demand is

$$x(v,t) = L \tag{21}$$

- Profits:

$$\pi(v,t) = [p^x(v,t) - \psi q(v,t)]x(v,t) \tag{22}$$
$$= \beta q(v,t) L \tag{23}$$

Balanced Growth

- r is constant
- Assume there is innovation in one sector.
- In any sector with innovation, free entry implies:

$$\frac{q(v,t)}{\lambda\eta} = V(v,t|q) \tag{24}$$

- For a given quality: $\dot{V}(v,t|q) = 0$.
- Intuition: Replacement probability and profits are constant over time.

Balanced Growth

- Pricing equation:

$$rV(v,t|q) = \dot{V}(v,t|q) + \pi(v,t|q) - z(v,t|q)V(v,t|q) \quad (25)$$
$$= 0 + \beta q(v,t)L - z(v,t|q)V(v,t|q) \quad (26)$$

or

$$V(v,t|q) = \frac{\beta q(v,t)L}{r + z(v,t|q)} = \frac{q(v,t)}{\lambda \eta} \quad (27)$$

- This means: $z(v,t|q) = z^*$ in all sectors with innovation.

- Could there be sectors without innovation?
- No - V is present value of expected profits.
- Without innovation in sector v: $z(v,t|q)=0$.
- That raises the value of the firm to

$$V(v,t|q)=\frac{\beta q(v,t)L}{r}>\frac{q(v,t)}{\lambda\eta} \tag{28}$$

- There would be strictly positive profits for entrants.

Balanced Growth

- We have almost found r, except that we still need to know z^*:

$$r = \lambda \eta \beta L - z^* \tag{29}$$

- We get z^* from the balanced growth condition $g(C) = g(Y)$.

Output Growth

- Define average quality: $Q(t) = \int_0^\infty q(v,t)dv$.
- Final output with $x(v,t) = L$:

$$Y_t = (1-\beta)^{-1} L_t^\beta \int_0^1 q(v,t) L^{1-\beta} dv \quad (30)$$

$$= (1-\beta)^{-1} L Q(t) \quad (31)$$

- Output growth: $g(Y) = g(Q)$.

Quality Growth

- Consider an interval Δt - small.
- Fraction $z^*\Delta t$ varieties experience 1 innovation.
- The rest experiences no innovation.
- For small Δt the probability of multiple innovation is negligible.
- Therefore:

$$Q(t+\Delta t) = \int_0^1 [(z^*\Delta t)\lambda q(v,t) + (1-z^*\Delta t)q(v,t)]dv \tag{32}$$

$$= (z^*\Delta t)\lambda Q(t) + (1-z^*\Delta t)Q(t) \tag{33}$$

- Growth rate:

$$g(Q(t)) = (\lambda - 1)z^* \tag{34}$$

Balanced Growth

$$g(Q) = (\lambda - 1)z^* \quad (35)$$

$$= g(C) \quad (36)$$

$$= \frac{\lambda\eta\beta L - z^* - \rho}{\theta} \quad (37)$$

Solve for z^* and

$$g(C) = \frac{\lambda\eta\beta L - \rho}{\theta + (\lambda - 1)^{-1}} \quad (38)$$

Properties of Balanced Growth

- No transitional dynamics.
- Symmetry: all varieties share the same rate of innovation z^* - this is what makes the model tractable.
- The static allocation is not optimal
  - monopoly pricing distorts $x(v, t)$
- The growth rate may be above or below the Pareto optimal one (see Ch. 14.1).

Reading

- Acemoglu, Introduction to Modern Economic Growth, ch. 14.
- Aghion and Howitt: The Economics of Growth.

Only best quality is used in equilibrium

- Let's focus on one good and suppress the (v, t) arguments for notational clarity.
- In the production function (4) all qualities s of the same good are perfect substitutes.
- The Firm minimizes the cost of $X(v,t) = \int q(s)^{(1/1-\beta)} x(s) ds$.
- The cost is $\int p(s) x(s) ds$.
- The firm uses the goods with the highest ratio of "quality" to price: $q(s)^{1/(1-\beta)}/p(s)$.
- The monopolist charges markup ψ: $p(s_{Mon}) = \psi q_{Mon}$.
- Competitors charge at least marginal cost $p(s) = q(s)$.
- The innovation is drastic if the monopolist has the highest quality/price ratio:

$$\lambda^{1/(1-\beta)}/(\lambda\psi) > 1 \tag{39}$$