

# Partial Equilibrium R&D Models

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- We study models where **intentional innovation** drives productivity growth.
- We start with partial equilibrium to see how consumers and firms behave.
- Then we embed this into a GE model.

- Historians often view innovation as the result of research that is not profit driven.
- Economists treat innovation as producing goods that are sold in markets ("blueprints").
- There are historical examples of both types of innovation.

# How to model innovation

- Current models are somewhat reduced form.
- The issue how existing knowledge feeds into future innovation is treated as a knowledge spillover.
- Knowledge is treated as a scalar - like capital.
- In fact, the only difference between blueprints and machines is **non-rivalry**:
  - blueprints can be used simultaneously in the production of several goods.

# Partial Equilibrium Innovation

# Partial Equilibrium: Firm

- We study the problem of an innovating firm.
- Innovation reduces marginal cost.
  - No innovation: marginal cost  $\psi$ .
  - Innovation costs  $\mu$ . Reduces marginal cost to  $\psi/\lambda$ .
- The firm faces a demand curve  $Q = D(p)$ .
  - Price elasticity:  $\varepsilon_D$ .

# Partial Equilibrium

## Drastic / non drastic innovation

- Assume a large number of firms with marginal cost  $\psi$ .
- A **drastic** innovation gives the innovator a monopoly.
  - The monopoly price is below competitor's marginal cost  $\psi$ .
- A **non-drastic** innovation means:
  - Charging monopoly prices results in 0 sales.
  - The demand curve facing the firm becomes infinitely elastic at price  $\psi$ .

- Assume a **drastic innovation**:
  - The firm has a monopoly afterwards.
- Monopoly price:

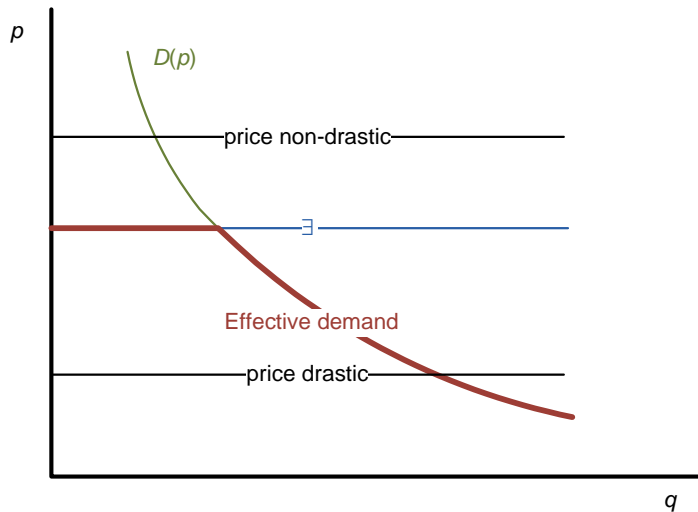
$$p^M = \frac{\psi/\lambda}{1 - 1/\varepsilon_D} \quad (1)$$

- Monopoly profit:

$$\pi_1^I = D(p^M) [p^M - \psi/\lambda] - \mu \quad (2)$$

- This is also the (private) value of the innovation.

# Effective demand curve



# Partial Equilibrium

## Limit pricing

- What is the optimal price for a **non-drastic** innovation?
- Capture the entire market.
- Set price to competitors' marginal cost.

$$p_1 = \psi$$

- Reason: Price elasticity becomes infinite at that point.

# Partial Equilibrium

## Optimality

- The private value of innovation is less than the social value.
- The social value is the sum of consumer and producer surplus.
- The firm does not capture the entire consumer surplus.
- The monopoly price is above the socially optimal price.

# Partial Equilibrium

## Replacement effect

- The private value of an innovation is smaller for an incumbent than for a competitor.
- Reason: The incumbent only gains the *change* in profits.
- Part of the opportunity cost of innovation is the loss of existing monopoly profits.

# Partial Equilibrium

## Business stealing effect

- Innovation effort can be inefficiently large.
- Competitors do not take into account the externality of "stealing" the incumbent's profits.
- Private value of innovation for competitor: monopoly profit.
- Social value: change in monopoly profit + change in consumer surplus.

# Dixit Stiglitz Model

# Dixit Stiglitz Model

- Virtually all models of innovation have a Dixit Stiglitz block.
- It contains:
  - Demand: CES preferences with **love for variety**.
  - Supply: Monopolistic competition.

# Dixit Stiglitz Model

## Demand

- The world is static.
- There are  $N$  consumption goods  $c_i$ .
- There is one "other" consumption good  $y$ .
- Household income is  $m$ .
- Preferences:  $u(C, y)$  with

$$C = \left( \sum_{i=1}^N c_i^\theta \right)^{1/\theta} \quad (3)$$

- $\theta = (\varepsilon - 1) / \varepsilon > 0$ .
- Elasticity of substitution  $\varepsilon > 1$ .

- Consider the symmetric case:  $c_i = \bar{C}/N$ .
- Then

$$\begin{aligned} C &= \left( N [\bar{C}/N]^\theta \right)^{1/\theta} \\ &= N^{(1-\theta)/\theta} \bar{C} \end{aligned} \tag{4}$$

- Spreading a given number of units ( $\bar{C}$ ) over more varieties ( $N$ ) increases utility.
- One model of innovation: Invent new goods ( $N \uparrow$ ).

# Household

## Demand functions

- The household's demand functions are iso-elastic.
- This makes the firm's behavior very simple: markups are constant.
- The household solves:

$$\max u(C, y)$$

subject to

$$\sum_{i=1}^N p_i c_i + y = m \quad (5)$$

$$\max u \left( \left[ \sum_{i=1}^N c_i^\theta \right]^{1/\theta}, m - \sum p_i c_i \right)$$

FOC

$$\begin{aligned} \frac{\partial u}{\partial y} &= \frac{\partial u}{\partial c_i} \frac{1}{p_i} \\ &= \frac{\partial u}{\partial C} \frac{1}{\theta} \left[ \sum_{i=1}^N c_i^\theta \right]^{1/\theta-1} \theta \frac{c_i^{\theta-1}}{p_i} \end{aligned}$$

- Equal for all goods:

$$c_i^{-1/\varepsilon} / p_i \quad (6)$$

- Demand function:

$$c_i = X p_i^{-\varepsilon} \quad (7)$$

for some constant  $X$ . **Iso-elastic.**

- Another way of thinking about the household problem:

- 1 For given  $C$ , find the cost minimizing  $c_i$ . Define the price index as

$$PC = \sum p_i c_i \quad (8)$$

- 2  $\max u(C, y)$  subject to  $PC + y = m$ .

- The cost minimizing price index is

$$P = \left( \sum p_i^{1-\varepsilon} \right)^{1/(1-\varepsilon)} \quad (9)$$

Proof:

$$P = \frac{\sum p_i c_i}{C}$$

$$\begin{aligned}\sum p_i c_i &= c_1 \sum p_i (c_i / c_1) \\ &= c_1 p_1^\epsilon \sum p_i^{1-\epsilon}\end{aligned}$$

(because  $c_i / c_1 = p_i^{-\epsilon} / p_1^{-\epsilon}$ )

# Household II

## Ideal price index

$$\begin{aligned}C &= c_1 \left( \sum (c_i/c_1)^{(\varepsilon-1)/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)} \\&= c_1 \left( \sum (p_i/p_1)^{1-\varepsilon} \right)^{\varepsilon/(\varepsilon-1)} \\&= c_1 p_1^\varepsilon \left( \sum p_i^{1-\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}\end{aligned}$$

Therefore,

$$P = \frac{c_1 p_1^\varepsilon}{c_1 p_1^\varepsilon} \frac{\sum p_i^{1-\varepsilon}}{\left( \sum p_i^{1-\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}}$$

The demand functions take the form

$$c_i/C = (p_i/P)^{-\varepsilon} \quad (10)$$

Proof:

$$p_i c_i = p_i c_1 (p_i/p_1)^{-\varepsilon}$$

$$\begin{aligned} \sum p_i c_i &= PC = c_1 p_1^\varepsilon \sum p_i^{1-\varepsilon} \\ &= c_1 p_1^\varepsilon P^{1-\varepsilon} \end{aligned}$$

$$PC P^{\varepsilon-1} = c_1 p_1^{\varepsilon}$$

Rearrange. QED.

- Each firm has a monopoly over a variety  $i$ .
- The demand elasticity is  $\varepsilon$ .
- Optimal monopoly pricing implies a constant markup over marginal cost:

$$p_i = \frac{\psi}{1 - 1/\varepsilon} \quad (11)$$

- Assumption: The firm is small enough to neglect its effect on  $C$  and  $P$ .

- Assume symmetry.
- Price index:

$$\begin{aligned} P &= \left( \sum p_i^{1-\varepsilon} \right)^{1/(1-\varepsilon)} \\ &= N^{\frac{1}{1-\varepsilon}} \frac{\psi}{1 - 1/\varepsilon} \end{aligned}$$

- More goods of the same price  $\rightarrow$  it costs less to achieve the same utility.

$$\begin{aligned}\pi_i &= c_i(p_i - \psi) \\ &= C P^\varepsilon p_i^{-\varepsilon} (p_i - \psi) \\ &= C N^{\varepsilon/(1-\varepsilon)} \frac{\varepsilon}{\varepsilon - 1} \psi\end{aligned}\tag{12}$$

### More varieties:

- $C$  changes - depends on preferences.
- $P$  falls - profits fall (recall  $\varepsilon > 1$ ).

Aggregate profits ( $N\pi$ ) may rise in  $N$ .

- "Aggregate demand externality"
- Higher  $N$  raises marginal utility for a given variety.

# Continuum of varieties

- Nothing changes when  $i$  is continuous.
- Replace all  $\Sigma$  with  $\int$ .

- Acemoglu, "Introduction to Modern Economic Growth," ch. 12.
- Romer, "Advanced macro," ch. 3.1-3.4.
- Jones, Charles (2004). "Growth and Ideas."  
<http://elsa.berkeley.edu/~chad/cv.html#Papers>.