

Partial Equilibrium R&D Models

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- We study models where **intentional innovation** drives productivity growth.
- We start with partial equilibrium to see how consumers and firms behave.
- Then we embed this into a GE model.

- Historians often view innovation as the result of research that is not profit driven.
- Economists treat innovation as producing goods that are sold in markets ("blueprints").
- There are historical examples of both types of innovation.

How to model innovation

- Current models are somewhat reduced form.
- The issue how existing knowledge feeds into future innovation is treated as a knowledge spillover.
- Knowledge is treated as a scalar - like capital.
- In fact, the only difference between blueprints and machines is **non-rivalry**:
 - blueprints can be used simultaneously in the production of several goods.

Partial Equilibrium Innovation

Partial Equilibrium: Firm

- We study the problem of an innovating firm.
- Innovation **reduces marginal cost**.
 - No innovation: marginal cost ψ .
 - Innovation costs μ . Reduces marginal cost to ψ/λ .
- Alternative: innovation creates **new goods**.

Partial Equilibrium: Firm

- Assume a large number of firms with marginal cost ψ .
- The industry faces a **demand curve** $Q = D(p)$.
 - Price elasticity: ε_D .
- Without innovation: perfect competition. All firms are identical.
- With innovation: one firm can underprice all others.
 - It becomes a monopolist, subject to a fringe of competitors with MC ψ .

Partial Equilibrium

Drastic / non drastic innovation

- A **drastic** innovation gives the innovator a monopoly.
 - The monopoly price is below competitor's marginal cost ψ .
- A **non-drastic** innovation leaves the monopoly price above ψ .
 - Charging monopoly prices results in 0 sales.
 - The demand curve facing the firm becomes infinitely elastic at price ψ .

- Assume a **drastic innovation**:
 - The firm has a monopoly afterwards.
- Monopoly price:

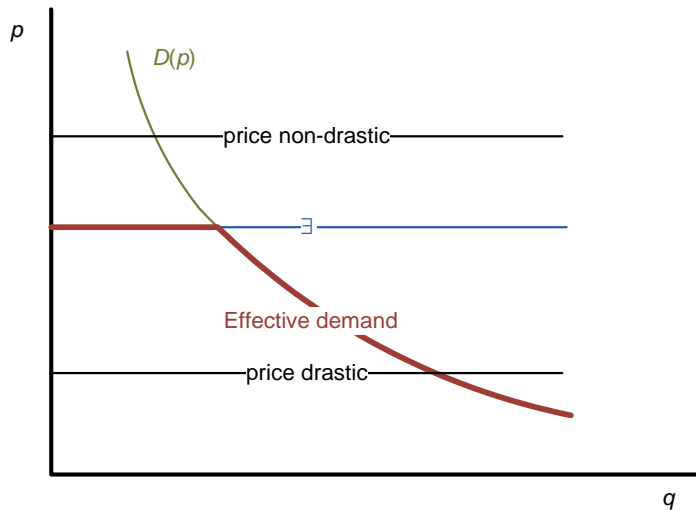
$$p^M = \frac{\psi/\lambda}{1 - 1/\varepsilon_D} \quad (1)$$

- Monopoly profit:

$$\pi_1^I = D(p^M) [p^M - \psi/\lambda] - \mu \quad (2)$$

- This is also the (private) value of the innovation.

Effective demand curve



Partial Equilibrium

Limit pricing

- What is the optimal price for a **non-drastic** innovation?
- Capture the entire market.
- Set price to competitors' marginal cost.

$$p_1 = \psi$$

- Reason: Price elasticity becomes infinite at that point.

- There may be **underinvestment** in innovation:
- The innovator does not capture the entire consumer surplus.

- There may be **overinvestment** in innovation.
- Competitors do not take into account the externality of "stealing" the incumbent's profits.
- Private value of innovation for competitor: monopoly profit.
- Social value: change in monopoly profit + change in consumer surplus.

Replacement effect

- The private value of an innovation is smaller for an incumbent than for a competitor.
- Reason: The incumbent only *gains* the *change* in profits.
- Part of the opportunity cost of innovation is the loss of existing monopoly profits.

The Policy Tradeoff

- No innovation with perfect competition (but see Boldrin & Levine...).
- Innovators must charge price $>$ average cost to recoup the costs of innovation.
- Patent protection is the device that gives innovators monopoly power.
- The trade-off:
 - Stronger patents - higher profits - more innovation.
 - Stronger patents distort prices.

Dixit Stiglitz Model

Dixit Stiglitz Model

- Virtually all models of innovation have a Dixit Stiglitz block.
- An analytically tractable demand structure with
 - **love for variety.**
 - constant elasticity demand functions.
- Constant demand elasticity means: monopoly price = $MC / (1 - 1 / \text{elasticity})$
 - basically exogenous

Dixit Stiglitz Model

- The world is static.
- There are N consumption goods c_i with prices p_i .
- There is one "other" consumption good y with price 1.
 - Its purpose is to absorb income effects.
- Household income is m .

- Households like: $u(C, y)$.
- C is a CES composite consumption good:

$$C = \left(\sum_{i=1}^N c_i^\theta \right)^{1/\theta} \quad (3)$$

- $\theta = (\varepsilon - 1) / \varepsilon > 0$.
- Elasticity of substitution $\varepsilon > 1$.
- The trick: constant substitution elasticity implies constant price elasticity.

- Consider the symmetric case: $c_i = \bar{C}/N$.
- Then

$$C = \left(\sum_{i=1}^N [\bar{C}/N]^\theta \right)^{1/\theta}$$
$$= \left(N [\bar{C}/N]^\theta \right)^{1/\theta} \tag{4}$$

$$= N^{(1-\theta)/\theta} \bar{C} \tag{5}$$

- Spreading a given number of units (\bar{C}) over more varieties (N) increases utility.
- One model of innovation: Invent new goods ($N \uparrow$).

- The household's demand functions are iso-elastic.
- The household solves:

$$\max u(C, y)$$

subject to

$$\sum_{i=1}^N p_i c_i + y = m \quad (6)$$

$$\max u \left(\left[\sum_{i=1}^N c_i^\theta \right]^{1/\theta}, m - \sum p_i c_i \right)$$

FOC

$$\begin{aligned} \frac{\partial u}{\partial y} &= \frac{\partial u}{\partial c_i} \frac{1}{p_i} \\ &= \frac{\partial u}{\partial C} \frac{1}{\theta} \left[\sum_{i=1}^N c_i^\theta \right]^{1/\theta-1} \theta \frac{c_i^{\theta-1}}{p_i} \end{aligned}$$

Demand functions

- A useful feature:

$$[c_i/c_j]^{-1/\varepsilon} = p_i/p_j \quad (7)$$

- Equal for all goods:

$$c_i^{-1/\varepsilon} / p_i \quad (8)$$

- Demand function:

$$c_i = X p_i^{-\varepsilon} \quad (9)$$

for some endogenous constant X (which we need to find).

- Price elasticity is constant at ε .

Demand functions

Claim:

The demand functions take the form

$$c_i/C = (p_i/P)^{-\varepsilon} \quad (10)$$

where C is the composite consumption good

$$C = \left[\sum_{i=1}^N c_i^\theta \right]^{1/\theta} \quad (11)$$

and P is the "ideal price index" for the household (the cost minimizing cost of C):

$$P = \left(\sum p_i^{1-\varepsilon} \right)^{1/(1-\varepsilon)} \quad (12)$$

- Now we have a simple two good problem:

$$\max u(C, y) \quad (13)$$

subject to

$$PC + y = m \quad (14)$$

- FOC:

$$u_y/u_C = 1/P \quad (15)$$

- Example: $u(C, y) = \alpha \ln(C) + (1 - \alpha) \ln(y)$.

- $P = \alpha y / (1 - \alpha) C$
- with budget constraint: $y = (1 - \alpha)m$ and $PC = \alpha m$.

- Another way of thinking about the household problem:

① For given C , find the cost minimizing c_i . Define the price index as

$$PC = \sum p_i c_i \quad (16)$$

② $\max u(C, y)$ subject to $PC + y = m$.

- The cost minimizing price index is

$$P = \left(\sum p_i^{1-\varepsilon} \right)^{1/(1-\varepsilon)} \quad (17)$$

Ideal price index I

By definition:

$$PC = \sum p_i c_i \quad (18)$$

We need to express C and $\sum p_i c_i$ as functions of prices to solve for P .
First-order conditions determine relative demands:

$$c_i / c_1 = p_i^{-\varepsilon} / p_1^{-\varepsilon} \quad (19)$$

Sub into expression for

$$\begin{aligned} \sum p_i c_i &= c_1 \sum p_i (c_i / c_1) \\ &= c_1 p_1^\varepsilon \sum p_i^{1-\varepsilon} \end{aligned}$$

Ideal price index II

Sub the same into expression for

$$\begin{aligned}C &= c_1 \left(\sum (c_i/c_1)^{(\varepsilon-1)/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)} \\&= c_1 \left(\sum (p_i/p_1)^{1-\varepsilon} \right)^{\varepsilon/(\varepsilon-1)} \\&= c_1 p_1^\varepsilon \left(\sum p_i^{1-\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}\end{aligned}$$

Take the ratio:

$$P = \frac{c_1 p_1^\varepsilon}{c_1 p_1^\varepsilon} \frac{\sum p_i^{1-\varepsilon}}{\left(\sum p_i^{1-\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}}$$

Simplify to get the solution for P .

Demand functions I

The demand functions take the form

$$c_i/C = (p_i/P)^{-\varepsilon} \quad (20)$$

Proof:

$$p_i c_i = p_i c_1 (p_i/p_1)^{-\varepsilon}$$

$$\begin{aligned} \sum p_i c_i &= PC = c_1 p_1^\varepsilon \sum p_i^{1-\varepsilon} \\ &= c_1 p_1^\varepsilon P^{1-\varepsilon} \end{aligned}$$

$$PC P^{\varepsilon-1} = c_1 p_1^{\varepsilon}$$

Rearrange. QED.

- Assume a Dixit-Stiglitz composite consumption good in preferences.
- Then demand is isoelastic.
 - the elasticity is determined by the elasticity of substitution across varieties in C .
- The cost of the optimal bundle C is given by P .
- The household reduces to a 2 good problem with standard solution.

- Each firm has a monopoly over a variety i .
- The demand elasticity is ε .
- Optimal monopoly pricing implies a constant markup over marginal cost:

$$p_i = \frac{\psi}{1 - 1/\varepsilon} \quad (21)$$

- Assumption: The firm is small enough to neglect its effect on C and P .

- Assume symmetry.
- Price index:

$$\begin{aligned} P &= \left(\sum p_i^{1-\varepsilon} \right)^{1/(1-\varepsilon)} \\ &= N^{\frac{1}{1-\varepsilon}} \frac{\psi}{1 - 1/\varepsilon} \end{aligned}$$

- More goods of the same price \rightarrow it costs less to achieve the same utility.

$$\begin{aligned}\pi_i &= c_i (p_i - \psi) \\ &= C P^\varepsilon p_i^{-\varepsilon} (p_i - \psi) \\ &= C N^{\varepsilon/(1-\varepsilon)} \frac{\varepsilon}{\varepsilon - 1} \psi\end{aligned}\tag{22}$$

More varieties can increase profits:

- Direct effect: P falls - more competitors erode profits.
- "Aggregate demand externality": C may rise (depends on preferences)
 - Higher N raises marginal utility for a given variety.
 - Innovators impose pecuniary externality on competitors.

Continuum of varieties

- Nothing changes when i is continuous.
- Replace all Σ with \int .

- Acemoglu, "Introduction to Modern Economic Growth," ch. 12.
- Romer, "Advanced macro," ch. 3.1-3.4.
- Jones, Charles (2004). "Growth and Ideas."
<http://elsa.berkeley.edu/~chad/cv.html#Papers>.