

# Problem Set 5: Innovation and Growth

Econ720. Fall 2011. Prof. Lutz Hendricks

## 1 Stochastic patent duration

[Due to Matt Doyle] Consider a version of the “Expanding Variety of Goods” model in which innovators’ monopoly power diminishes over time. Otherwise the model is standard.

Demographics: There is a single representative household.

Endowments: The household is endowed with  $L$  units of labor, which can only be used for work.

Preferences:

$$U = \int_0^\infty \frac{c^{1-\theta} - 1}{1-\theta} \cdot e^{-\rho t} dt. \quad (1)$$

Technology:

- Final goods are produced from labor and intermediate inputs according to

$$Y = AL^{1-\alpha} \cdot \sum_{j=1}^N (X_j)^\alpha, \quad (2)$$

where  $0 < \alpha < 1$ ,  $Y$  is output,  $L$  is labor input,  $X_j$  is the input of the  $j$ 'th type of the intermediate good, and  $N$  is the number of varieties.

- It takes one unit of final goods to produce one unit of intermediates.
- It costs  $\eta$  units of the final good to create a new type of intermediate good.

Market arrangements:

- The final goods sector is perfectly competitive.
- Intermediate goods producers hold monopolies.
- There is free entry for innovators.
- Households own all firms in the economy.

Patents: Upon innovation, the innovator receives a patent. If intermediate good  $j$  is currently monopolized, it becomes competitive in the next instant  $dT$  with probability  $p \cdot dT$ , where  $p \geq 0$ . Thus, if good  $j$  is invented at time  $t$ , the probability of it still being monopolized at some future date  $v \geq t$  is  $e^{-p \cdot (v-t)}$ .

Notation: Denote by  $N^c$ , the number of intermediate goods produced competitively and by  $N$  the total number of intermediate goods.

**Answer the following questions:**

1. Define an equilibrium.
2. Set up and solve the household's problem. Assume that the interest rate is constant. Derive an expression for the growth rate of consumption. Interpret your results.
3. Derive the quantity of  $X_j$  produced when the  $j$ 'th producer is a monopolist. Also derive the quantity of  $X_j$  produced when the  $j$ 'th intermediate good is produced competitively.
4. Write down the zero profit condition for entrepreneurs. This condition implies:

$$r(t) = \pi^m / \eta - p \quad (3)$$

(you are not required to derive Equation (3)). Using Equation (3), show that:

$$r = (L/\eta) \cdot A^{1/(1-\alpha)} \cdot \frac{1-\alpha}{\alpha} \cdot \alpha^{2/(1-\alpha)} - p \quad (4)$$

Compare this to the analogous condition where monopoly power is permanent.

5. Solve for a balanced growth values of  $\dot{c}/c$ ,  $N^c/N$ , and  $Y/N$ . Hint: Use the following approximation:  $\dot{N}^c = p \cdot (N - N^c)$ .
6. Suppose  $p \rightarrow \infty$ , so that intermediate goods become competitive instantly. What happens to growth? Explain (you do not need to derive the result—just give the intuition).