Final Exam. Econ720. Spring 2009

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- Answer all questions.
- Write legibly! Write legibly! Write legibly!
- Write on only one side of each sheet.
- The total time is 2:00 hours.
- A good answer should explain what you are doing. For example: "To find the consumption function, I take first order conditions, then use the budget constraint to solve for c." Then comes the math...

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1 Yield Curve in the Lucas Fruit Tree Model

Consider a standard Lucas Fruit Tree model.

- Demographics: There is a representative consumer who lives forever.
- Preferences:
$$E \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$
u has standard properties (strictly concave etc.).
- Endowments: The household is endowed with one tree that yields y_t units of the consumption good in each period. y_t is an i.i.d. random variable.
- Market arrangements: Households trade in competitive markets: (a) goods, (b) trees, (c) bonds of different maturities.
- Bond markets: A bond of maturity$i$ pays one unit of consumption i periods from now. Its price is $p_{t,i}$. Its per period yield to maturity is $r_{t,i} = (1/p_{t,i})^{1/i} - 1$. There are bonds for maturities $i = 1, ..., n$.

Questions:

1. State the household's dynamic program. Hint: Think of the household as bringing bonds of maturity $0, ..., n-1$ into the period and as choosing bonds of the same maturity for next period. But note the important point: the bond b_i brought into the period cost p_i while the bond b_i' costs p_{i+1} to purchase (explain why this is true).
2. Derive first-order conditions and envelope equations. Derive Euler equations for each asset.

3. Determine the equilibrium price of bonds of maturity i, $p_{t,i}$. Hint: Use backward induction, starting from the bond that matures tomorrow.

4. Show that the standard Lucas asset pricing equation holds for bonds.

5. For which value of y_t is the yield curve flat in the sense that $r_{t,i} = r_t$ for all i? In which states is the yield curve upward sloping / downward sloping? Definition: The yield curve plots $r_{t,i}$ against i.

6. Explain the intuition for #5.

2 One period unemployment benefits

Consider the decision problem of a worker who lives forever and has preferences

$$E \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t c_t \tag{2}$$

The worker can be in three states:

1. employed, receiving wage w;

2. unemployed after being employed last period, receiving exogenous benefits b;

3. unemployed after being unemployed last period, receiving no benefits.

While unemployed, the worker received wage offers drawn from the distribution $F(w)$. Jobs end with probability δ.

Questions:

1. State the Bellman equations that determine the values of being employed, unemployed after being employed, and unemployed after being unemployed last period. (Recall the general form of these Bellman equations: rV=[current payoff] + [prob of event] $\times$[“capital gain”]).

2. Show that the reservation wage is independent of how long the worker has been unemployed. You need not show (yet) that the optimal plan has a reservation wage property.

3. Explain why the reservation wage is independent of how long the worker has been unemployed.

4. Show that the worker follows a reservation wage strategy. Hint: the first step is to solve for the value of being employed as a function of the wage and the value of being unemployed.

3 Answers

3.1 Answer: Yield Curve in the Lucas Fruit Tree Model

[Based on a question due to Steve Williamson]

1. Household problem: The household enters the period holding shares s and bonds that mature $i = 0, ..., n-1$ periods from today. He chooses holdings of the same assets for tomorrow. The trick is that a bond that has maturity i tomorrow has maturity $i+1$ today and costs $p_{t,i+1}$, not $p_{t,i}$.

$$V(s, b_0, ..., b_{n-1}; y) = \max u(c) + E\beta V(s', b_0', ..., b_{n-1}'; y') \tag{3}$$

subject to the budget constraint

$$c + \sum_{i=0}^{n-1} p_{i+1} b_i' + ps' = (p+y)\, s + \sum_{i=0}^{n-1} p_i b_i \tag{4}$$

2. First-order conditions: Standard for the stocks, which yields the usual asset pricing equation. For the bond:

$$b_i' : \; u'(c) p_{i+1} = \beta E V_{b_i}(.') \tag{5}$$

Envelope:

$$V_{b_i} = u'(c) p_i \tag{6}$$

Euler:

$$u'(c) p_{i+1} = \beta E u'(c') p_i'$$

3. Solve this by backward induction:

$$p_0 = 1 \tag{7}$$

Sub that into the Euler equation and iterate to find

$$p_{t,i} = \beta^i E \frac{u'(c_{t+i})}{u'(c_t)} \tag{8}$$

with $c_t = y_t$.

4. Note that each asset has a standard pricing equation of the form

$$u'(c_t) = \beta^i E u'(c_{t+i}) (1 + r_{t,i})^i \tag{9}$$

where $r_{t,i}$ is not stochastic and $Eu'(c_{t+i}) = Eu'(y_{t+i})$ does not depend on the current state y.

5. The yield curve plots $\left[u'\left(c_t\right)/Eu'\left(c_{t+i}\right)\right]^{1/i}/\beta$ against i. It is flat the at $1/\beta$ for the value of c_t that satisfies $u'\left(c_t\right) = Eu'\left(c_{t+i}\right)$. For lower c_t the ratio in brackets is above 1 and the yield curve slopes down.

6. Intuition: As usual, low consumption relative to the future is associated with high yields. [More...]

3.2 Answer: One period unemployment benefits

[Based on a question due to Steve Williamson]

1.

$$\begin{aligned} rV_e\left(w\right) &= w + \delta\left(V_u^1 - V_e(w)\right) && (10) \\ rV_u^1 &= b + \int_0^{\bar{w}} max\left[V_e(w), V_u^0\right] - V_u^1 dF(w) && (11) \\ rV_u^0 &= 0 + \int_0^{\bar{w}} max\left[V_e(w), V_u^0\right] - V_u^0 dF(w) && (12) \end{aligned}$$

2. In (11) and (12) the "max" terms are the same. This means that the reservation wage, assuming there is one, must be the same.

3. Intuition: the two states only differ in the current payoff, which can no longer be affected by any worker choices. Looking forward, the two problems are the same: the worker will be in state 0 if unemployed.

4. From (10), we can solve for

$$V_e(w) = \frac{w + \delta V_u^1}{r + \delta} \qquad (13)$$

V_e is strictly increasing in the wage. Applying that to (11) and (12) implies the reservation wage property.