

Contracts: Private Information

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Asymmetric Information

- We study a 2nd contracting friction: private information.
- Payoffs must be based on agents' **reports** of their information.
- We are looking for incentive compatible contracts in which agents report the truth.
- Applications:
 - Labor contracts: Employer cannot observe effort vs. luck. (Additional moral hazard.)
 - Investment contracts: Investor can hide income.

- The same as in the money lender model.
- Both sides commit to a contract.
- Promised utility is v^0 (exogenous).
- Lender cannot observe y or c .

- As before, consumers' preferences are

$$E \sum_{t=1}^{\infty} \beta^t u(c_t)$$

- Consumption must be $\geq a$.
- $u'(c) \rightarrow \infty$ as $c \rightarrow a$.
- $u'(c) \rightarrow 0$ as $c \rightarrow \infty$.
- u is bounded above.

$$P(v) = \max_{b_s, w_s} \sum_{s=1}^S \Pi_s [-b_s + \beta P(w_s)]$$

Notation:

- v : Promised utility by contract. As before.
- b_s : Payment to agent who reports \bar{y}_s .

Constraints

- 1 Promise keeping:

$$v = \sum_{s=1}^S \Pi_s V_{s,s} \quad (1)$$

- $V_{s,k} = u(\bar{y}_s + b_k) + \beta w_k$: Value of agent with \bar{y}_s who reports \bar{y}_k .

- 2 Incentive compatibility:

$$C_{s,k} = V_{s,s} - V_{s,k} \geq 0 \quad \forall s, k \quad (2)$$

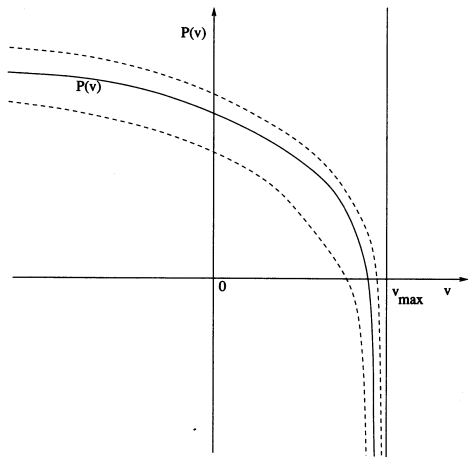
- 3 Bounds:

$$b_s \geq a - \bar{y}_s$$
$$w_s \leq v_{\max} = \sup \frac{u(c)}{1 - \beta}$$

Properties of $P(v)$

- Low v :
 - Household has low current utility, high $u'(c)$.
 - It is cheap to raise v : $P'(v)$ should be small.
- High v :
 - Household has high $u'(c)$.
 - $P'(v)$ should be large.
- Suggests that $P''(v) < 0$ and $P \rightarrow -\infty$ as $v \rightarrow v_{\max}$ (and $c \rightarrow \infty$).

Properties of $P(v)$



Ljungqvist & Sargent (2007)

Incentive compatibility I

- Result: Reporting lower y results in higher transfer b_s and lower future payoff w_s .
- This follows directly from incentive compatibility.
- Downward constraint:

$$V_{s,s} - V_{s,s-1} = u(\bar{y}_s + b_s) + \beta w_s - u(\bar{y}_s + b_{s-1}) - \beta w_{s-1} \geq 0$$

- Upward constraint:

$$V_{s-1,s-1} - V_{s-1,s} = u(\bar{y}_{s-1} + b_{s-1}) + \beta w_{s-1} - u(\bar{y}_{s-1} + b_s) - \beta w_s \geq 0$$

Incentive compatibility II

- Add the two:

$$u(\bar{y}_s + b_s) - u(\bar{y}_{s-1} + b_s) \geq u(\bar{y}_s + b_{s-1}) - u(\bar{y}_{s-1} + b_{s-1}) \quad (3)$$

- A given increment in \bar{y}_{s-1} to \bar{y}_s implies a larger utility increment for b_s than for b_{s-1} .
- Therefore:

$$b_{s-1} \geq b_s \quad (4)$$

- If reporting a higher state reduces transfers, $C_{s,s-1}$ requires that it has a higher future payoff:

$$w_{s-1} \leq w_s \quad (5)$$

Local constraints are enough

- A bit of algebra shows: If $C_{s,s-1}$ and $C_{s,s+1}$ hold, then all $C_{s,k}$ hold.

- It can be shown that $P(v)$ is concave.
- Intuition above.
- Proof: Ljungqvist and Sargent (2007), 19.5.3.
- Assumption: P is strictly concave.

Downward constraints always bind

- Result: For the optimal contract, the downward constraints bind ($C_{s,s-1}$), the upward constraints don't ($C_{s,s+1}$).
- Agents would like to report lower than the true income.
- Proof idea:
 - Can raise profits by shrinking the w_s gaps until all downward constraints bind.
 - If expected w_s remains unchanged, the household is happier (risk aversion).
 - So the firm can raise profits by offering a less attractive contract.

Downward constraints always bind

Proof (by contradiction)

- Suppose that some downward constraint does not bind: $C_{s,s-1} > 0$:

$$u(\bar{y}_s + b_s) + \beta w_s - u(\bar{y}_s + b_{s-1}) - \beta w_{s-1} > 0$$

- We construct an alternative contract that yields higher profits.
- Since $b_s \leq b_{s-1}$: $u(\bar{y}_s + b_s) - u(\bar{y}_s + b_{s-1}) \leq 0$.
- Therefore $w_s > w_{s-1}$ (strictly).
- Reduce w_2 until $C_{2,1} = 0$.
- Then reduce w_3 until $C_{3,2} = 0$. Etc.
- Add a constant to all w_s to keep promised value unchanged.
- The new contract satisfies all constraints (check that upward constraints don't bind).

Proof (by contradiction)

- $EP(v) = \sum \Pi_s P(w_s)$.
- EW_s is unchanged.
- $w_s - w_{s-1}$ has been reduced.
- The new contract is a mean-reducing spread of the old one.
- Since $P(v)$ is strictly concave, $EP(v)$ has increased.

Characterization

Coinsurance

- When a higher y_s is drawn, $u(\cdot) + \beta w_s$ and firm profits both rise.
- Contrast with the frictionless case where the risk neutral firm fully insures the risk averse household.
- **Household utility** rises because the downward constraint binds:

$$C_{s,s-1} = 0:$$

$$u(\bar{y}_s + b_s) + \beta w_s - u(\bar{y}_s + b_{s-1}) - \beta w_{s-1} = 0$$

\implies

$$u(\bar{y}_s + b_s) + \beta w_s - u(\bar{y}_{s-1} + b_{s-1}) - \beta w_{s-1} > 0$$

Firm profits:

$$-b_s + \beta P(w_s) \geq -b_{s-1} + \beta P(w_{s-1}) \quad (6)$$

Proof:

- Suppose (6) does not hold.
- Then change the contract to: $(b_s, w_s) \rightarrow (b_{s-1}, w_{s-1})$.
- Profits rise.
- Household utility is unchanged b/c the downward constraint $C_{s,s-1}$ binds.

The optimal contract design problems is:

$$\begin{aligned} P(v) = & \max_{b_s, w_s} \sum_{s=1}^S \Pi_s [-b_s + \beta P(w_s)] \\ & + \lambda \left[\sum_{s=1}^S \Pi_s [u(\bar{y}_s + b_s) + \beta w_s] - v \right] \\ & + \sum_{s=2}^S \mu_s [u(\bar{y}_s + b_s) + \beta w_s - u(\bar{y}_s + b_{s-1}) - \beta w_{s-1}] \end{aligned}$$

First order conditions

$$b_s : -\Pi_s + \lambda \Pi_s u'(\bar{y}_s + b_s) + \mu_s u'(\bar{y}_s + b_s) - \mu_{s+1} u'(\bar{y}_{s+1} + b_s) = 0$$

$$w_s : \Pi_s \beta P'(w_s) + \lambda \Pi_s \beta + \mu_s \beta - \mu_{s+1} \beta = 0$$

where $\mu_1 = \mu_{S+1} = 0$ (there are no such terms in the FOCs).

Simplify:

$$\Pi_s [1 - \lambda u'(\bar{y}_s + b_s)] = \mu_s u'(\bar{y}_s + b_s) - \mu_{s+1} u'(\bar{y}_{s+1} + b_s)$$

$$\Pi_s [P'(w_s) + \lambda] = \mu_{s+1} - \mu_s$$

Envelope:

$$P'(v) = -\lambda$$

First order conditions

Sum the FOCs for w_s :

$$\begin{aligned}\sum \Pi_s P'(w_s) + \lambda &= \sum \mu_{s+1} - \mu_s \\ &= \mu_{S+1} - \mu_1 \\ &= 0\end{aligned}$$

Therefore:

$$P'(v) = -\lambda = \sum \Pi_s P'(w_s) \quad (7)$$

Marginal profits are a **martingale**: $P'(v) = EP'(v')$.

$$P'(w_s) = P'(v) + \frac{\mu_{s+1} - \mu_s}{\Pi_s} \quad (8)$$

Intuition:

- If truth-telling constraints were non-binding: $\mu_s = \mu_{s+1} = 0$.
- Then the full info optimality condition returns:

$$P'(w_s) = P'(v) = -\lambda = 1/u'(\bar{y}_s + b_s) \quad (9)$$

- On average, this still holds: $\sum \Pi_s P'(w_s) = P'(v)$.
- But now there is an additional cost to raising w_s : it increases the incentive to lie in state $s+1$.
- μ_{s+1} is that cost.
- But higher w_s also reduces the incentive to lie in state s .
- This saves the planner μ_s .

Spreading continuation values

One can show:

- $w_S > v$: When the household draws the best income, he is rewarded.
- $w_1 < v$: When the household draws the worst income, he is punished.

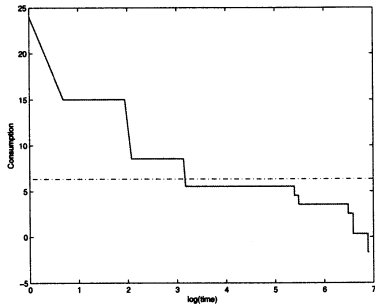
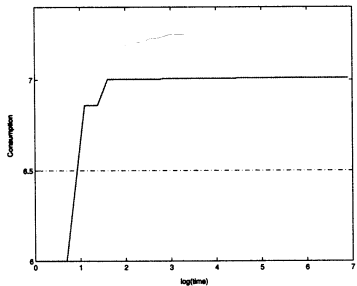
Sketch of proof:

- If $w_S < v$: it violates the martingale property.
- Then the household would be punished in all states (since w is increasing in s).
- If $w_S = v$: the martingale property would require $w_s = v$ for all s .
- This would violate incentive compatibility (no punishment for reporting bad incomes).

- **Result:** $v \rightarrow -\infty$ almost surely.
- $P'(v)$ is a non-positive Martingale.
- Theorem: A non-positive Martingale converges almost surely.
- Therefore, v converges.
- But v cannot converge to a strictly positive value.
- If it did, incentive compatibility would require strictly positive fluctuations in w_s .
- Then $P'(v)$ would not converge.

- The type of contract depends on the friction.
- When the friction is commitment:
 - Raise the rewards over time to prevent agents from walking off.
- When the friction is asymmetric information:
 - Make the payoff an increasing function of the reported income.
 - For low reports: punish the worker (to induce truth-telling)
 - Payoffs drift down over time.

Summary: Typical consumption profiles



Ljungqvist & Sargent (2007)

Private Storage

- Modify the model so that agents can store goods.
- But agents cannot borrow (the planner can).
- The gross return is the same for planner and agent (R).
- Main result:
 - 1 The optimal contract provides no risk sharing across households.
 - 2 The optimal allocation is the same as in an economy where each household can borrow / lend at rate R .

- The world lasts for T periods.
- Agents observe histories of incomes: $h_t = \{y_1, \dots, y_t\}$.
- Agents report $\hat{y}_t(h_t)$ (that may not be truthful) and make storage decisions $\hat{k}_t(h_t)$ (without report).
- Agents receive transfers $b_t(\hat{h}_t)$.
- Budget constraint:

$$c(h_t) + \hat{k}(h_t) = y(h_t) + R\hat{k}_{t-1}(h_{t-1}) + b_t(\hat{h}_t[h_t]) \quad (10)$$

$$\hat{k}(h_t) \geq 0 \quad (11)$$

- \hat{h} is the reported history ending in $\hat{y}_t(h_t)$.

- Preferences:

$$\Gamma(\hat{y}, \hat{k}; b) = \max \sum_{t=1}^T \beta^{t-1} \sum_{h_t} \pi(h_t) u(c(h_t)) \quad (12)$$

- Strategies: $\hat{k}(h_t), \hat{y}(h_t)$.
- Take as given transfer rule b .

- Budget constraint:

$$K_t + \sum_{h_t} \pi(h_t) b_t(\hat{h}_t[h_t]) = RK_{t-1} \quad (13)$$

- $K_T \geq 0$.
- Incentive compatibility: For any history, lifetime utility must be higher under truth-telling than under any lying strategy:

$$\Gamma(\hat{k}, \hat{y}; b) \geq \Gamma(\tilde{k}, \tilde{y}; b)$$

for any alternative strategy (\tilde{k}, \tilde{y}) .

Planner's problem:

Choose b to max $\Gamma(\hat{k}, \hat{y}; b)$

subject to:

- 1 Incentive compatibility: $\Gamma(\hat{k}, \hat{y}; b) \geq \Gamma(\tilde{k}, \tilde{y}; b)$.
- 2 Budget constraint.

Private storage restricts allocations

- Result: Any allocation that can be implemented with private storage can also be implemented when $k = 0$.
- Intuition:
 - Private storage makes it harder to manipulate continuation values through a contract (self-insurance).
 - This makes incentive problems more severe.

Characterizing the optimal contract

- The constraints are complicated.
 - Need to consider lifetime utility for any feasible reporting strategy.
- The only method: guess and verify.
- Find a problem with a smaller set of constraints.
- Show that the optimal allocation is incentive compatible and feasible with the larger set of constraints.

Characterizing the optimal contract

In this model, the optimal contract solves

$$\max \sum_{t=1}^T \beta^{t-1} \sum_{h_t} \pi(h_t) u(c_t(h_t)) \quad (14)$$

subject to

$$\sum_{t=1}^T R^{1-t} [y_t(h_T) - c_t(h_t(h_T))] \geq 0 \quad \forall h_T \quad (15)$$

In words: The allocation the household could achieve through self-insurance with the borrowing constraint $k_T \geq 0$.

Proof: Cole & Kocherlakota (2001).

The trick: Only consider lying strategies where the household reports y_{s-1} instead of y_s .

Characterizing the optimal contract

- The planner only relaxes the individual's borrowing constraints:
 $k_T \geq 0$ instead of $k_t \geq 0$.
- The planner cannot achieve insurance across agents.

- Ljungqvist & Sargent, "Recursive methods," 2nd ed. ch. 19.