

Dynamic Contracts

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- Many markets work through **intertemporal contracts**:
- Labor markets, credit markets, intermediate input supplies, ...
- Contracts solve (or create) a number of problems:
 - ① Insurance: firms insure workers against low productivity shocks.
 - ② Incentives: work hard to keep your job.
 - ③ Information revelation: you can lie once, but not over and over again.

If there are no frictions, agents can write complete contracts.

Frictions prevent this:

- ❶ Lack of commitment: borrowers can walk away with the loan.
- ❷ Private information: firms don't observe how hard employees work.

We study optimal contracts for these frictions.

An analytical trick

- Dynamic contracts generally depend on the entire history of play.
 - "Three strikes and you are out"
- The set of possible histories grows exponentially with t .
- A trick, due to Abreu, Pearce & Stachetti, makes this tractable.
- Use the **promised expected future utility** as a state variable.
- Then the current payoff can (often) be written as a function of today's play and promised value.

- Thomas & Worrall (1990); Kocherlakota (1996).
- The problem:
 - A set of agents suffer income shocks.
 - They borrow / lend from a "money lender".
 - They cannot commit to repaying loans.
 - How can a contract be written that provides some insurance?

- The world lasts forever.
- There is one non-storable good.
- A money lender can borrow / lend from "abroad" at interest rate β^{-1} .
- A set of agents receive random endowments y_t .
- They can only trade with the money lender.

$$E \sum_{t=0}^{\infty} \beta^t u(c_t)$$

Note: β determines time preference and interest rate.

- Each household receives iid draws y_t .
- y takes on S discrete values, \bar{y}_s .
- Probabilities are Π_s .

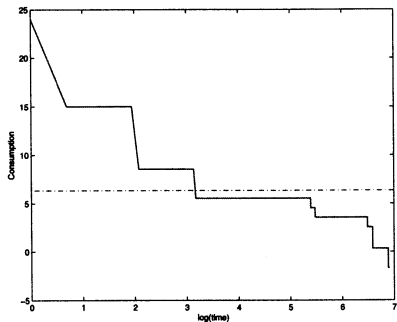
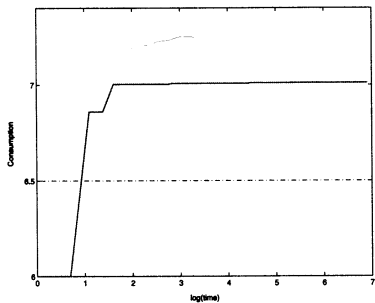
- Households could achieve full insurance by trading Arrow securities.
- Consumption would be constant at the (constant) mean endowment.

We consider 3 frictions:

- 1 Households cannot commit not to walk away with a loan.
- 2 Households have private information about y_t .
- 3 Households have private information and a storage technology.

The optimal contracts in the 3 cases are dramatically different.

Sample consumption paths



Ljungqvist & Sargent (2007)

Sample consumption paths

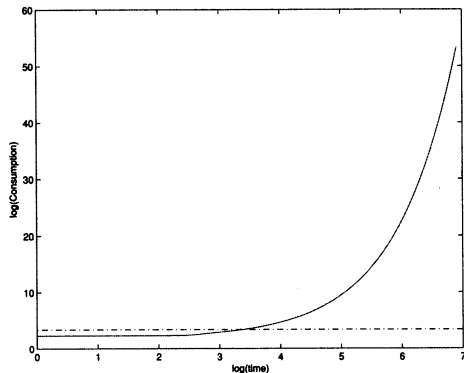


Figure 19.2.2: Typical consumption path in environment c.

Ljunqvist & Sargent (2007)

How to set up the problem

Assumptions:

- 1 the money lender offers the contract to the household
- 2 the household can accept or reject
- 3 the household accepts any contract that is better than autarky

How to set up the problem

- The optimal contract can be written as an **optimization problem**:
 - max profits
 - subject to: participation constraints.
- The state is the promised future value of the contract.
- To characterize, take first-order conditions.

Assumption:

- Households can walk away from their debt.
- As punishment, they live in autarky afterwards.

The contract must be self-enforcing.

Applications:

- Loan contracts.
- Labor contracts.
- International agreements.

- We can study an economy with one person - there is no interaction.
- A contract specifies an allocation for each history: $h_t = \{y_0, \dots, y_t\}$
- An allocation is simply household consumption:

$$c_t = f_t(h_t) \tag{1}$$

- The money lender collects y_t and pays c_t .

- Money lender's profit:

$$P = E \sum_{t=0}^{\infty} \beta^t (y_t - f_t(h_t)) \quad (2)$$

- Agent's value:

$$v = E \sum_{t=0}^{\infty} \beta^t u(f_t(h_t)) \quad (3)$$

- These are complicated!

Participation constraint

- With commitment, the lender would max P subject to the resource constraint.
 - What would the allocation look like?
- Lack of commitment adds a participation constraint:

$$E_{\tau} \underbrace{\sum_{t=\tau}^{\infty} \beta^{t-\tau} u(f_t(h_t))}_{\text{stay in contract}} \geq \underbrace{u(y_t) + \beta v_{AUT}}_{\text{walk away}} \quad (4)$$

- This must hold for every history h_t .

- If the agent walks, he receives

$$v_{AUT} = E \sum_{t=0}^{\infty} \beta^t u(y_t) = \frac{E u(y_t)}{1 - \beta} \quad (5)$$

- The contract is not recursive in the natural state variable y_t .
- History dependence seems to destroy a recursive formulation.
- We are looking for a state variable x_t so that we can write:

$$\begin{aligned}c_t &= g(x_t, y_t) \\ x_{t+1} &= l(x_t, y_t)\end{aligned}$$

- The correct state variable is the promised value of continuation in the contract:

$$v_t = E_{t-1} \sum_{j=0}^{\infty} \beta^j u(c_{t+j}) \quad (6)$$

- The household enters the period with promised utility v_t , then learns y_t .
- The contract adjusts c_t and v_{t+1} to fulfill the promise v_t .
- Proof: Abreu, Pearce, Stachetti.

Recursive formulation

- The state variable for the lender is v .
- The objective is to design payoffs, c_s and w_s , for this period to max discounted profits

$$P(v) = \max_{c_s, w_s} \sum_{s=1}^S \Pi_s [(\bar{y}_s - c_s) + \beta P(w_s)] \quad (7)$$

- w_s is the value of v' promised if state s is realized today.

Constraints

- 1 Promise keeping:

$$\sum_{s=1}^S \Pi_s [u(c_s) + \beta w_s] \geq v \quad (8)$$

- 2 Participation:

$$u(c_s) + \beta w_s \geq u(\bar{y}_s) + \beta v_{AUT}; \quad \forall s \quad (9)$$

- 3 Bounds:

$$c_s \in [c_{\min}, c_{\max}] \quad (10)$$

$$w_s \in [v_{AUT}, \bar{v}] \quad (11)$$

Cannot promise less than autarky or more than the max endowment each period.

$$P(v) = \max_{c_s, w_s} \sum_{s=1}^S \Pi_s [(\bar{y}_s - c_s) + \beta P(w_s)] \quad (12)$$

$$+ \mu \left[\sum_{s=1}^S \Pi_s [u(c_s) + \beta w_s] - v \right] \quad (13)$$

$$+ \sum_s \lambda_s [u(c_s) + \beta w_s - u(\bar{y}_s) - \beta v_{AUT}] \quad (14)$$

Note: Participation constraints may not always bind. Then $\lambda_s = 0$.

$$c_s : \Pi_s = u'(c_s)[\lambda_s + \mu\Pi_s] \quad (15)$$

$$w_s : -\Pi_s P'(w_s) = \lambda_s + \mu\Pi_s \quad (16)$$

Assumption: P is differentiable. (Verify later)

Envelope:

$$P'(v) = -\mu \quad (17)$$

What do these say in words?

- Simplify:

$$u'(c_s) = -P'(w_s)^{-1} \quad (18)$$

- This implicitly defines the consumption part of the contract:

$$c_s = g(w_s).$$

- Properties:

- Later we see that $P(v)$ is concave ($P' < 0, P'' < 0$).
- Therefore: $u''(c_s) dc_s = \frac{P''(w_s)}{[P'(w_s)]^2} dw_s$ and $dc/dw > 0$.
- A form of consumption smoothing / insurance.

$$P'(w_s) = P'(v) - \lambda_s / \Pi_s \quad (19)$$

Two cases:

- 1 Participation constraint does not bind:

$$\lambda_s = 0$$

$$w_s = v$$

- 2 Participation constraint binds:

$$\lambda_s > 0$$

$$P'(w_s) < P'(v) \Rightarrow w_s > v$$

Participation constraint does not bind

- $w_s = v$ regardless of the realization y_s .
- Consumption follows from

$$\begin{aligned}u'(c_s) &= -P'(v)^{-1} \\ c_s &= g_2(v)\end{aligned}$$

- The household is fully insured against income shocks in the range where $\lambda_s = 0$.
- Intuition: this happens for low y .
- The lender may lose in such states: he pays out the promise.

Participation constraint binds

- The constraint:

$$u(c_s) + \beta w_s = u(\bar{y}_s) + \beta v_{AUT} \quad (20)$$

implies

$$c_s < \bar{y}_s \quad (21)$$

because $w_s \geq v \geq v_{AUT}$ (any contract must be better than autarky - otherwise the agent walks).

- The household gives up consumption in good times in exchange for future payoffs.
- To make this incentive compatible, the lender has to raise future payoffs: $w_s > v$.

- When the participation constraint binds, c and w are solved by

$$\begin{aligned}u(c_s) + \beta w_s &= u(\bar{y}_s) + \beta v_{AUT} \\ u'(c_s) &= -P'(w_s)^{-1}\end{aligned}$$

- This solves for

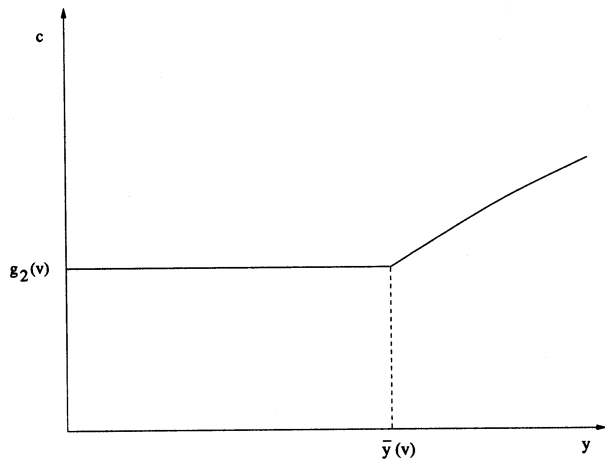
$$\begin{aligned}c_s &= g_1(\bar{y}_s) \\ w_s &= l_1(\bar{y}_s)\end{aligned}$$

- v does not matter!
- Intuition: The current draw y_s is so good that walking into autarky pays more than v .
- The continuation contract must offer at least $u(\bar{y}_s) + \beta v_{AUT}$, regardless of what was promised in the past.

The optimal contract

- Intuition: For low y the participation constraint does not bind, for high y it does.
- The threshold value $\bar{y}(v)$ satisfies:
 - 1 Consumption obeys the no-participation equation $u'(c_s) = -P'(v)^{-1}$.
 - 2 The participation constraint binds with $w_s = v$:
$$u(c_s) + \beta v = u(\bar{y}[v]) + \beta v_{AUT}$$
- $\bar{y}'(v) > 0$: Higher promised utility makes staying in the contract more attractive.

Consumption function



Ljungqvist & Sargent (2007)

Properties of the contract

- 1 For $y \leq \bar{y}(v)$: Pay constant $c = g_2(v)$ and keep c, v constant until the participation constraint binds.
- 2 For $y > \bar{y}(v)$: Incomplete insurance. $v' > v$.
- 3 v never decreases.
- 4 c never decreases.
- 5 As time goes by, the range of y 's for which the household is fully insured increases.
- 6 Once a household hits the top $y = \bar{y}_S$: c and v remain constant forever.

- With two-sided commitment, the firm would offer a constant c .
 - It would collect profits from lucky agents and pay to the unlucky ones.
 - Because of risk aversion, the average c would be below the average y .
 - The firm earns profits.
- With lack of commitment:
 - Unlucky households are promised enough utility in the contract, so they stay. Full insurance.
 - Lucky households have to give up some consumption to pay for future payouts in bad states.
 - To compensate, the firm offers higher future payments every time a "profit" is collected.

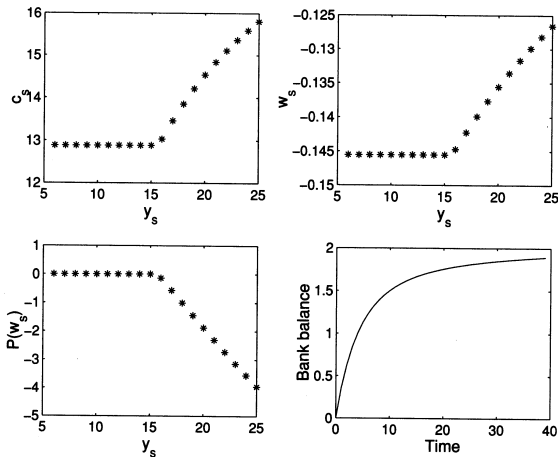
Think about this in the context of a labor market.

- "Young" households are poor (low v and c).
- Earnings rise with age.
- Earnings volatility declines with age (because the range of full insurance expands).
- Old workers are costly to employ. Firms would like to fire them.

This broadly lines up with labor market data.

- Inequality is first rising, then falling.
- Young households are all close to v_0 initially.
- Old households are perfectly insured in the limit.
- Middle aged households differ in their histories and thus payoffs.

Numerical example



Outcomes as function of highest \bar{y} experienced.

Ljungqvist & Sargent (2007)

- Ljungqvist & Sargent, "Recursive Macroeconomic Theory," 2nd ed. ch. 19.
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- Kocherlakota, N. R. (1996). Implications of efficient risk sharing without commitment. *The Review of Economic Studies*, 63(4), 595.
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