Applying the Solow Model

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Topics

We apply the Solow model to study:

1. Cross-country variation in growth rates
2. Implications of automation

Long-run Growth

Long-run Growth

What does the Solow model imply for long-run growth?

Main result

The principle of transition dynamics
Countries grow faster when they are far below their steady state.

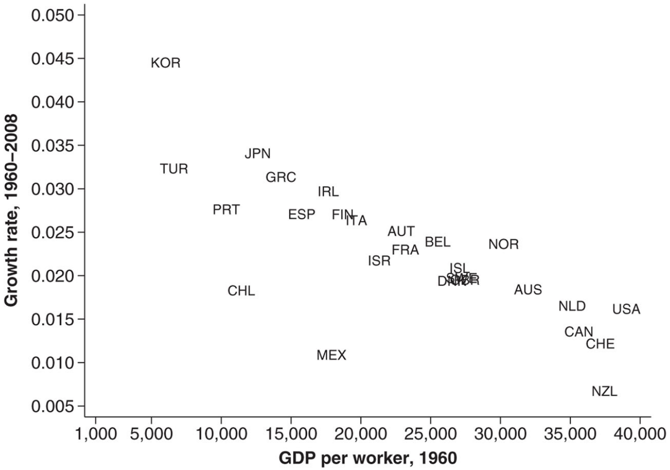
Main reference: Jones (2013), ch. 2, 3

What is the evidence supporting the principle?

- One exercise: if countries have similar steady states, their income levels should converge over time
- initially poor countries should grow faster

Convergence: Evidence

FIGURE 3.5 CONVERGENCE IN THE OECD, 1960–2008



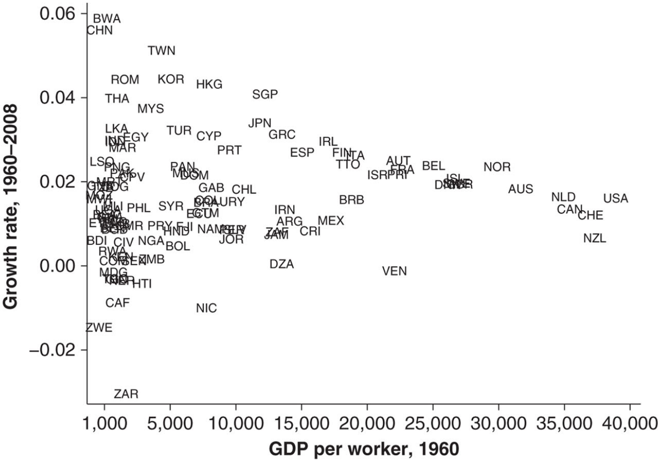
Among OECD countries: those that were initially poor grew faster.

Empirical Evidence

- Should we conclude that transitional growth explains cross-country differences in output growth?
- No!
- Figure 5.8 only shows OECD countries - mostly rich Western European countries + North America.

Empirical Evidence

FIGURE 3.6 THE LACK OF CONVERGENCE FOR THE WORLD, 1960–2008



No luck for a broad set of countries.

Empirical Evidence

- But figure 5.9 is the wrong experiment!
- The Solow model does not say: "poor countries grow faster"
- It says: "countries that are poor **relative to their steady states** grow faster."
- That is true in the data.

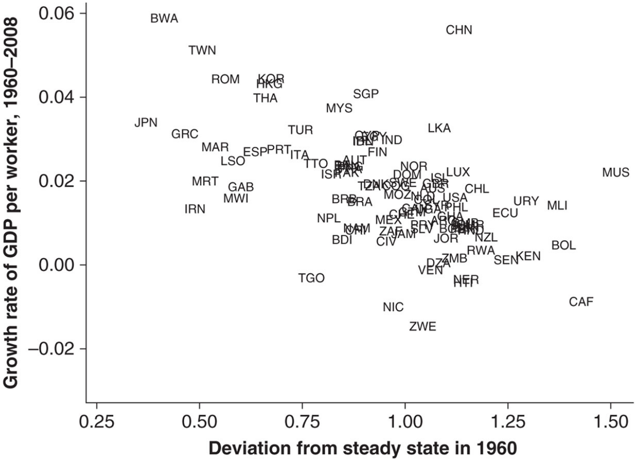
Empirical Evidence

Exercise

For a set of countries gather data on s, n.
Compute steady state output: y^*
Compute output in 1960 relative to steady state: y/y^*
Compute average growth 1960-2000
Plot average growth against y/y^*
What do you expect to find?

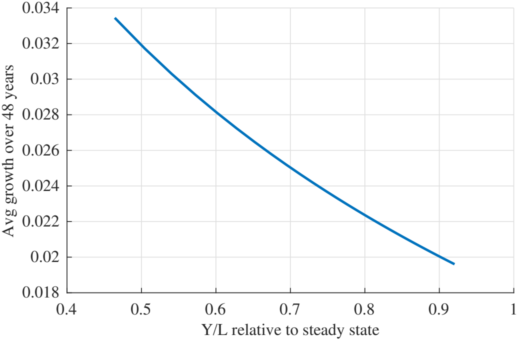
Conditional Convergence

FIGURE 3.8 **"CONDITIONAL" CONVERGENCE FOR THE WORLD, 1960–2008**



Source: Jones (2013)

Convergence: Solow Model

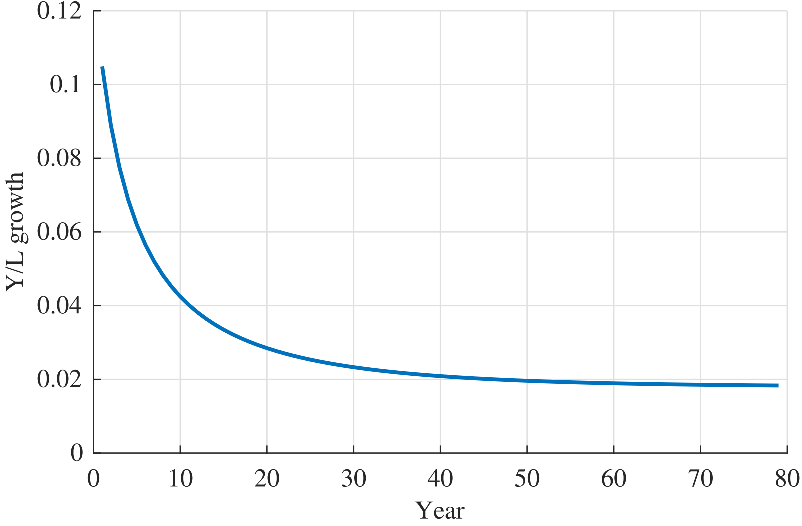


Prediction from a Solow model with capital share 1/3

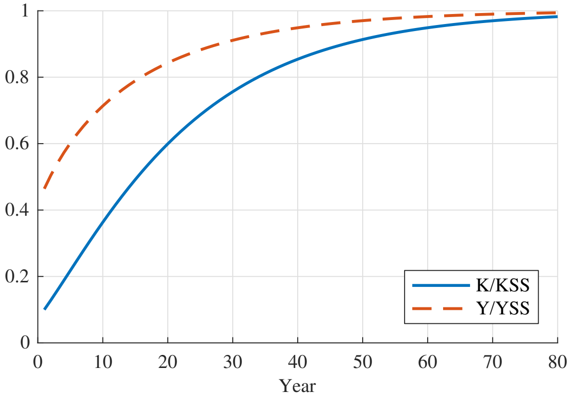
The fit is not bad, though the growth rate varies less than in the data.

Simulating the Solow Model

High growth rates do not last as long as in the data.



Simulating the Solow Model



Convergence is too fast.

In the data, the "**half-life**" is about 30 years – 10 years in the model.

Convergence is even faster when the saving rate is endogenous.

Convergence implications

The Solow model makes a quantitative prediction about growth rates.

Countries **converge fairly quickly** to their steady states (perhaps within 20 years).

Then they all should grow at almost the same rates.

Fact

The Solow model cannot explain why countries grow at different rates for long periods of time.

"Growth accounting" shows that much of variation in long-run growth is due to A, not k.

Did we just invalidate the Principle of Transition Dynamics?

- No, we did not.
- Countries grows faster when their capital stocks are low.
- But this does not account for the observed differences in long-run (40 year) growth rates across countries.
- It does account for growth rates at shorter horizons.

The Tigers

There are a few countries that sustained growth by capital accumulation for a long period of time.

How?

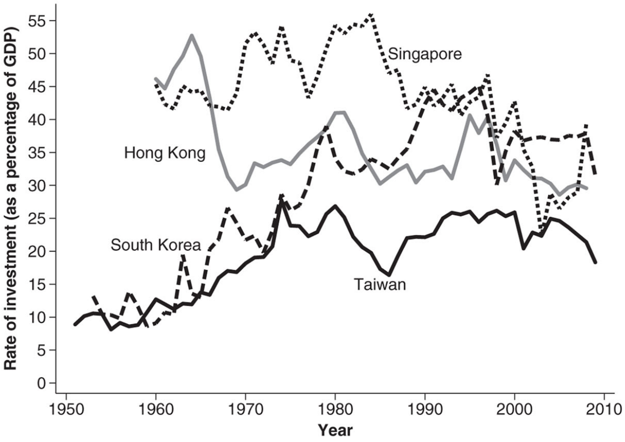
It cannot work with a constant saving rate s - the Solow model shows this.

Such countries must have saving rates that **rise over time**.

Examples are: South Korea, Singapore, Hong-Kong.

The Tigers

FIGURE 2.14 INVESTMENT RATES IN SOME NEWLY INDUSTRIALIZING ECONOMIES

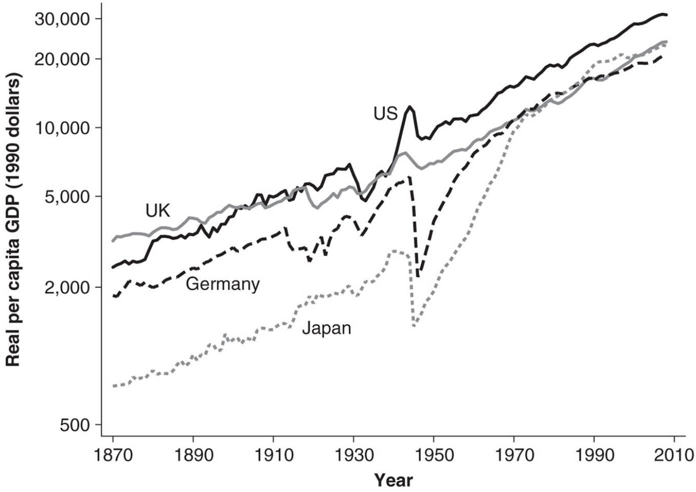


Source: Jones (2013)

Convergence and Post-war Growth

One episode where convergence was very fast: growth after WW2

FIGURE 3.3 **PER CAPITA GDP, 1870–2008**



Convergence and Post-war Growth

Convergence to pre-war trends was very fast after WW2

Many countries were back on their trend paths after 5-7 years

Much of the convergence after the initial years was growth in TFP, not capital accumulation.

Summary

The Solow model's main prediction is the Principle of Transition Dynamics.

In the data:

- no unconditional convergence
- but conditional convergence (consistent with the model).

But the convergence in the data is mostly not due to capital accumulation.

- the model implies very fast convergence
- we see this in the data after capital destruction

Empirical long-run growth rate differences are mostly due to A, not K.

Automation

The Issues

Automation / AI create labor displacing innovation.

A new input (robots / AI) takes the place of labor.

The marginal cost of AI is very small.

What happens to the workers?

A modified Solow model

There is a new input X that represents innovation

$$Y = AX^{\beta}K^{\alpha}L^{\gamma} \tag{1}$$

Constant returns to scale:

$$\beta + \alpha + \gamma = 1 \tag{2}$$

Per capita output:

$$y = Y/L = Ax^{\beta}k^{\alpha} \tag{3}$$

Early innovation: A rises; $\beta = 0$.

"New economy:" x rises; β rises.

- x gets a larger income share.
- capital and labor get smaller shares.

Law of motion

Capital accumulation is unchanged $\dot{k} = sy - (n+\delta)k$

- This fixes steady state $k/y = s/(n+\delta)$.

Production function:

$$y/k = Ax^{\beta}k^{\alpha-1} \tag{4}$$

Steady state capital stock:

$$k^{1-\alpha} = Ax^{\beta}\frac{s}{n+\delta} \tag{5}$$

Factors are paid marginal products

As always with Cobb-Douglas: factor income shares are constant

- capital gets α
- labor gets γ
- x gets the rest: $\beta = 1 - \alpha - \gamma$

Details: factor shares

Labor gets share γ:

$$w = \gamma A x^{\beta} K^{\alpha} L^{\gamma-1} \quad (6)$$
$$= \gamma y \quad (7)$$

Capital gets share α:

$$q = \alpha A x^{\beta} K^{\alpha-1} L^{\gamma} \quad (8)$$
$$= \alpha y / k \quad (9)$$

x gets share β:

$$p = \beta A x^{\beta-1} K^{\alpha} L^{\gamma} \quad (10)$$
$$= \beta y / x \quad (11)$$

"Old fashioned" innovation

A rises by factor $\lambda > 1$

β unchanged.

Implications:

- $k/y = s/(n+\delta)$ unchanged
- k rises by $\lambda^{\beta/(1-\alpha)}$ (from the steady state k solution)
- w and p (price of x) and y do the same
  - from the factor price equations
- q (MPK) unchanged

"New economy:" Higher β

To focus on redistribution effect: adjust A so that y unchanged

- $k/y = s/(n+\delta)$ unchanged
- Then k unchanged
- w, q fall;
- p rises

Pure redistribution of income from factors to x.

Combined Effect

Automation: x rises while income is redistributed from factors to x (β rises).

Distributional effects:

- x owners (innovators) get richer.
- Wages: may stagnate, even though output rises
  - labor share declines (true in the data!)

Investment

- marginal product of capital q falls
- I/Y may fall (but then c/y would have to rise!)

Policy implications

What has changed relative to old-fashioned A growth?

Policy implications

A key idea of economic policy

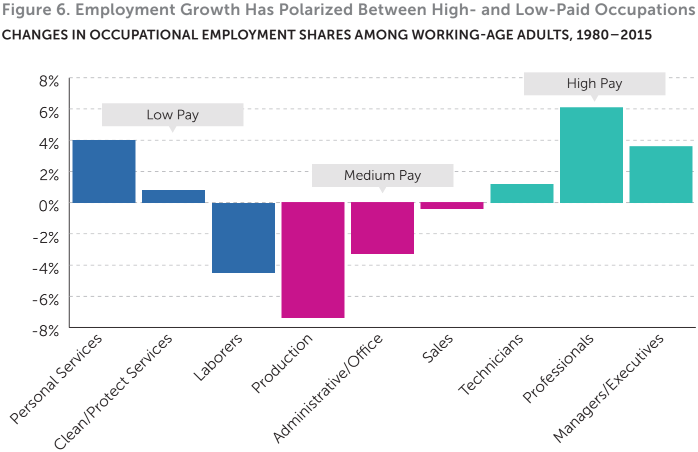
Separate redistribution from efficiency
If you want to redistribute income, use transfers, not subsidies.

One additional concern:

What if the marginal product of some workers falls so much to make them unemployable?

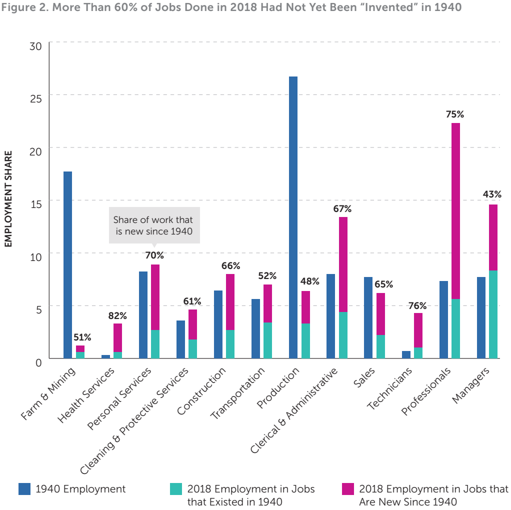
Automation

Automation has replaced “routine” jobs.



Source: Autor (2020)

Automation also creates new jobs



Source: Autor (2020)

What does the future hold?

We don't know.

> *"No economic law dictates that the creation of new work must equal or exceed the elimination of old work. Still, history shows that they tend to evolve together." – Autor (2020), p. 12*

References I

Autor, D. (2020): "The Work of the Future," Tech. rep., MIT Work of the Future Task Force.

Jones, Charles; Vollrath, D. (2013): *Introduction To Economic Growth*, W W Norton, 3rd ed.