

Applying the Solow Model

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Why Are Some Countries Rich and Others Poor?

What does the Solow model tell us about why some countries are much richer than others?

How important are:

- physical capital?
- human capital?
- productivity?

To analyze this: assume countries are close to steady state.

Solving for the steady state

- Recall the law of motion

$$\dot{k} = sf(k) - (n + \delta)k \quad (1)$$

where $f(k) = A^{1-\alpha}k^\alpha$.

- For simplicity: we abstract from productivity growth.
- Exercise: repeat the analysis with productivity growth.

Solving for the steady state

- Impose steady state: $\dot{k} = 0$:

$$sy = (n + \delta)k \quad (2)$$

- The capital-output ratio is

$$k/y = \frac{s}{n + \delta} \quad (3)$$

- Production function

$$y/k = A^{1-\alpha} k^{\alpha-1} \quad (4)$$

- Solve for k

$$k = A(k/y)^{1/(1-\alpha)} \quad (5)$$

$$= A \left(\frac{s}{n + \delta} \right)^{1/(1-\alpha)} \quad (6)$$

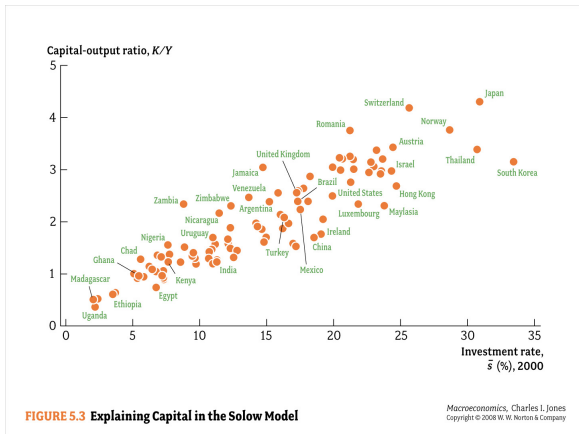
$$y = (k/y) \times k \quad (7)$$

$$= \frac{s}{n + \delta} \times A \left(\frac{s}{n + \delta} \right)^{1/(1-\alpha)} \quad (8)$$

$$= A \left[\frac{s}{n + \delta} \right]^{\alpha/(1-\alpha)} \quad (9)$$

Reality check

- A key prediction of the model: $k/y = s/(n + \delta)$.
- Countries with higher saving rates have higher capital output ratios.



Why does Y/L differ across countries?

- Our static production model answered: K/L and A differ:

$$y = A^{1-\alpha} k^\alpha \quad (10)$$

- The Solow model gives a similar answer: s and A differ:

$$y^* = A \left[\frac{s}{n + \delta} \right]^{\alpha/(1-\alpha)} \quad (11)$$

- This is the same answer in disguise: higher s means higher K/L .

Why does Y/L differ across countries?

- But the answer is **quantitatively** different:
 - Production model: Double $A \rightarrow$ raise y by $2^{1-\alpha} = 2^{2/3} = 1.59$.
 - Solow model: Double $A \rightarrow$ double y^* .
- In the Solow model, the contribution of A to output gaps is larger - why?
- Draw a picture...

How important is K/L for cross-country Y/L gaps?

- Our previous answer was: K/L accounts for a factor near 4.
- In the Solow model:

$$\frac{y_{US}^*}{y_{poor}^*} = \left(\frac{\bar{A}_{US}}{\bar{A}_{poor}} \right) \left(\frac{s_{US}}{s_{poor}} \right)^{1/2} \quad (12)$$

$$32 = 16 \times 2 \quad (13)$$

- Why factor 2 for saving rates?
- Because s (or K/Y) differs by a factor near 4.

How important is K/L for cross-country Y/L gaps?

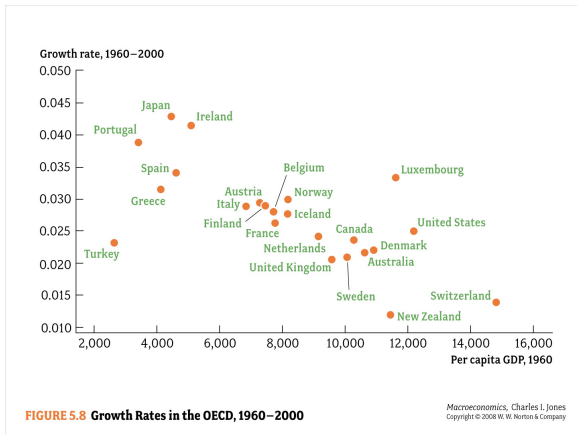
This is a central and robust result:

Capital accumulation accounts for only a small fraction of cross-country income gaps.

Long-run Growth

- What does the Solow model imply for long-run growth?
- Main result: countries grow faster when they are far below their steady state.
- What is the evidence for this?
- One exercise: if countries have similar steady states, their income levels should converge over time
 - initially poor countries should grow faster

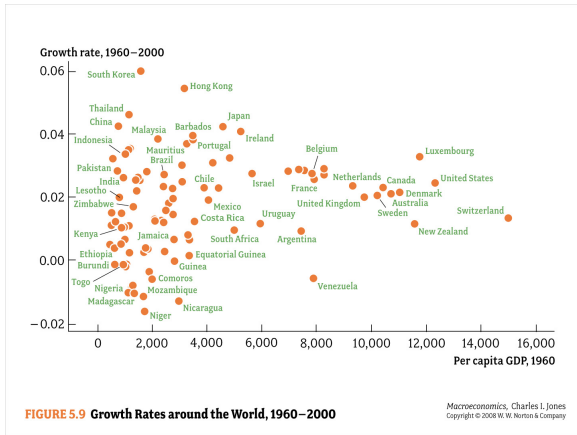
Convergence: Evidence



Among OECD countries: those that were initially poor grew faster.

- Should we conclude that transitional growth explains cross-country differences in output growth?
- No!
- Figure 5.8 only shows OECD countries - mostly rich Western European countries + North America.

Empirical Evidence



No luck for a broad set of countries.

- But figure 5.9 is the wrong experiment!
- The Solow model does not say: "poor countries grow faster"
- It says: "countries that are poor **relative to their steady states** grow faster."
- That is true in the data.

- One hypothesis one often reads:
 - Countries that share similar "fundamentals" (e.g. OECD countries) converge to similar steady states.
 - For them the principle of transition dynamics explains post-war growth.
 - Other countries have different fundamentals, so the poorer ones need not grow faster.
- This is the **wrong explanation** for post-war growth.

Convergence Clubs?

- The Solow model makes a quantitative prediction about growth rates.
- If you work out the numbers, countries should converge fairly quickly to their steady states (perhaps within 20 years).
- Then they all should grow at almost the same rates.

Fact

The Solow model cannot explain why countries grow at different rates for long periods of time.

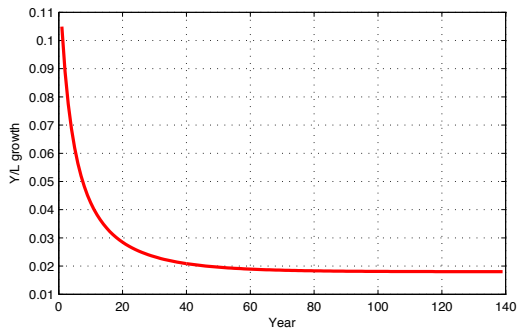
- Post-war growth may look like Solow convergence to a common steady state, but it is not.
- Post-war growth is driven by growth in TFP, not by growth in K/L .

Exercise

- Take a spreadsheet.
- Fix parameters at plausible empirical values: $\alpha = 1/3$, $d = 0.08$, $s = 0.2$.
- Compute the steady state.
- Fix K_0 at some multiple of K^* .
- Compute the transition path for K_t by iterating over $K_{t+1} = sY_t + (1 - d)K_t$.
- Plot the growth rate of Y_t against time.
 - You should see that growth is very high initially, if K_0 is small. But growth slows dramatically very quickly.
- Now plot the growth rate of Y_t against over 40 years against initial Y_t - this is the model analogue of figure 5.8.
 - You should see that the model relationship is much flatter than the data relationship.
 - The model fails to explain large variation in 40 year average growth rates.

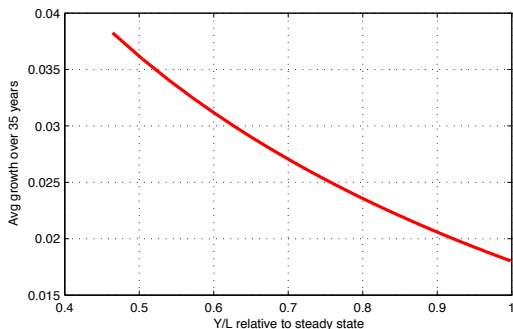
Simulating the Solow Model

Example: Start an economy from 1/10th of steady state k^* .



Simulating the Solow Model

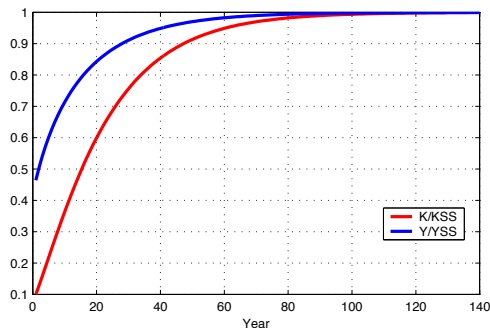
Growth varies less with y/y^* than in the data.



This suggests that part of growth is not transitional in the data.

Simulating the Solow Model

Speed of convergence



Convergence is too fast.

In the data, the "**half-life**" is about 30 years – 10 years in the model.

Convergence is even faster when the saving rate is endogenous.

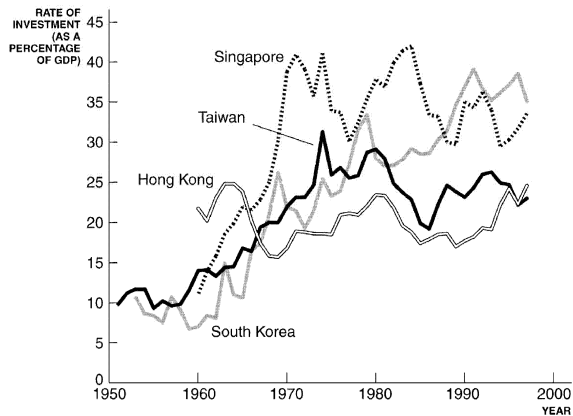
Did we just invalidate the Principle of Transition Dynamics?

- No, we did not.
- Countries grows faster when their capital stocks are low.
- But this does not account for the observed differences in long-run (40 year) growth rates across countries.
- It does account for growth rates at business cycle frequencies.

The Tigers

- There are a few countries that sustained growth by capital accumulation for a long period of time.
- How?
- It cannot work with a constant saving rate s - the Solow model shows this.
- Such countries must have saving rates that **rise over time**.
- Examples are: South Korea, Singapore, Hong-Kong.

The Tigers

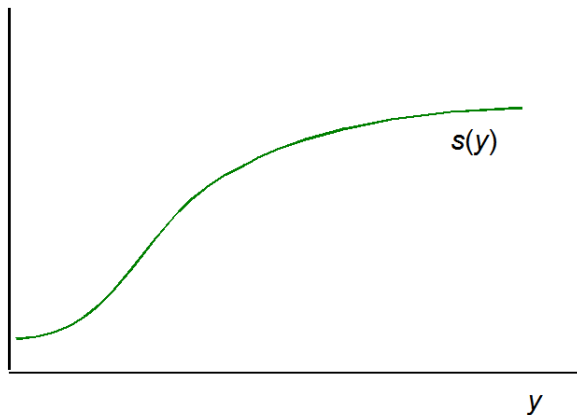


Exercise: Rising saving rate

- Simulate the Solow model with a saving rate that rises from 10% to 40% (Singapore).
- Start the model in steady state: $K_0 = K^*$.
- Show that the growth rate of y stays positive for a long time.
- You could now compare that growth path with data for Singapore and convince yourself that a large share of Singapore's spectacular growth since 1960 is indeed due to capital accumulation (as shown by Alwyn Young).

Exercise: The Saving Rate Depends on Income

- Consider an alternative version of the Solow model.
- The saving rate depends on income.
- What happens?



Conclusion: Is the Solow Model Useful?

- As a model of growth or large cross-country income differences, the model is a failure.
- But its failure contains important insights:
 - 1 Capital does not drive growth.
 - 2 Capital does not drive large fractions of cross-country income gaps.
- Both findings are surprising - and often not understood in the policy debate.

Conclusion: Is the Solow Model Useful?

- But the main significance of the Solow model itself is as a **building block** for macro models.
- We always have to keep track of how capital is accumulated.
- A Solow block is therefore part of virtually every model.
- The same logic extends to other accumulated factors: human capital, knowledge capital, organization capital.
- The Solow transition dynamics is an important piece for understanding business cycle dynamics.

- Jones, Introduction to Economic Growth, ch. 2, 3

Advanced Reading:

- McGrattan, Ellen; James Schmitz (1998). “Explaining cross-country income differences.” Federal Reserve Bank of Minneapolis Staff Report #250. Section 3.1.
- Hall, Robert E.; Charles I. Jones (1999). “Why do some countries produce so much more output per worker than others?” *Quarterly Journal of Economics*, 114: 83-116.