Practice Problems: Life-cycle Model

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1 Household

1.1 Tax cuts

Compare the effects of several tax cuts on consumption (c, c') and saving of an individual household. Can you also figure out what happens in equilibrium?

1. Transitory tax cut: $\Delta t = -1,\ t'$ unchanged.
2. Future tax cut: t unchanged, $\Delta t' = -(1+r)$. Note that this raises the same amount of revenue, in present value terms, as does the transitory tax cut.
3. Permanent tax cut: $\Delta t = -0.5,\ \Delta t' = -0.5\,(1+r)$. Note that this also raises the same amount of revenue as the two previous examples.
4. Tax deferral: $\Delta t = -1,\ \Delta t' = +(1+r)$. This raises no additional revenue, in present value terms.

1.1.1 Answer: Tax cuts

The first three cases have the same effect on the present value budget constraint. Therefore, the effects on consumption are the same.

1. See our analysis of a pure income effect. c, c' both rise. Saving must rise, too, to finance higher c'.
2. Same change in c, c'. But now s must fall to finance higher c.
3. Same change in c, c'. Whether s rises or falls depends on preferences.
4. No change in present value budget constraint. Therefore no change in c, c'. $\Delta s = -\Delta t$: the household completely saves the tax refund.

Equilibrium effects: You cannot figure out what happens in equilibrium without more information. What you do know is: the answers to #1 to #3 are the same, if the changes in G and G' are the same. Note that the quantity changes depend only on the (exogenous) changes in G and G': $\Delta C = -\Delta G$ and $\Delta C' = -\Delta G'$. This follows from goods market clearing.

1.2 Interest rate changes

Suppose that the interest rate rises and the household's future income increases. Show how these changes affect current consumption. Decompose the changes into income and substitution effects. Your answer will depend on whether the household is a borrower or a lender.

1.3 Borrowing constraint

Assume that the household can borrow at most a fixed amount x.

1. How does affect the household's budget constraint? Graph it.
2. Graph the household's consumption demand function as current income y changes. How does it differ from the case without a borrowing constraint? What does this imply for the Permanent Income Hypothesis?

1.3.1 Answer: borrowing constraint

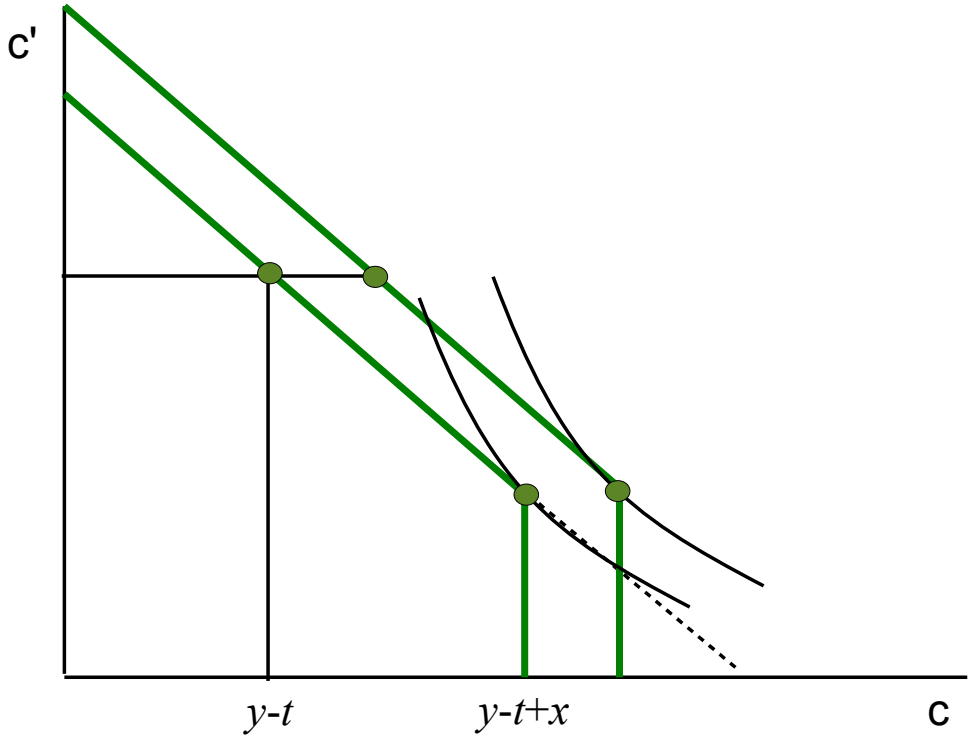
1. Period 1 budget constraint becomes

$$c = y - t - s \leq y - t + x \tag{1}$$

   where x is the borrowing limit. The budget constraint is "cut off" at $y - t - x$. See figure 1.

2. As y rises, the budget constraint shifts right / up. For low y, the borrowing constraint binds: the household is at the kink of the budget constraint. Rising y is then entirely consumed. The household stays at the kink of the budget constraint. The MP_C is one for low y. The PIH fails when households are borrowing constrained.

Figure 1: Borrowing constraint



1.4 Lending rate

Assume that the household has to borrow at a high interest rate (r_B) but lends at a low rate (r_L).

1. Show how this affects the household's budget constraint.
2. How does this affect the household's consumption as the lending interest rate changes?

1.4.1 Answer: Lending rate

1. The budget constraint has a kink. For points to the right of $c = y - t$ (borrowing), it is steep (high interest rate). For point to the left (saving) it is flat.
2. For households who are not borrowing, the usual arguments apply. For households who borrow, nothing happens. There will be range of interest rates for which the household is stuck at the corner without borrowing or lending.

2 Competitive Equilibrium

2.1 Ricardian Equivalence

Consider a special case of our two period economy where the household consumes a constant fraction of lifetime wealth in each period, regardless of the interest rate. Suppose the fraction consumed when young is $\alpha = 0.5$. This turns out to be the case if the household's utility function is of the form

$$U(c_1, c_2) = \alpha \ln(c_1) + (1 - \alpha) \ln(c_2).$$

Assume that endowments are $y_1 = y_2 = 1$ and there are no taxes and government spending. There are $N = 1,000$ households.

1. Calculate the competitive equilibrium allocation (c_1, c_2, S) and prices (r).

2. Now imagine the government pays a transfer of $t = -0.2$ to the young, financed by issuing bonds: $B = tN$. Recalculate the competitive equilibrium.

2.1.1 Answer: Ricardian Equivalence

1. Allocation. Young consumption is a fixed fraction of lifetime income:

$$c_1 = \alpha \left[y_1 + \frac{y_2}{1+r} \right] \tag{2}$$

Then

$$c_2 = (1-\alpha) \left[y_1 + \frac{y_2}{1+r} \right]$$

by the budget constraint. $s = y_1 - c_1$. In equilibrium, we need $s = 0$; this is the capital market clearing condition. Substitute in c_1 from (2) to obtain

$$1 + r = \frac{\alpha}{1-\alpha} \frac{y_2}{y_1}$$

Now we can calculate consumption from (2).

2. Transfers. The consumption function is now

$$c_1 = \alpha \left[y_1 + t + \frac{y_2 - t(1+r)}{1+r} \right]$$

because the old age tax must be $t\,(1+r)$. Note that the t-terms cancel. For any given interest rate, c_1 and c_2 are unchanged. To find the interest rate, use capital market clearing. This is now

$$s = t$$

because s is household saving and t is government borrowing. Solve for the interest rate to see that it has not changed. This proves Ricardian equivalence for the example.

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