The Life-cycle Model

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Topics of this Section

- We want to understand
  - fiscal policy / the effects of government deficits
  - saving: why has the U.S. saving rate declined?
  - inequality
- We need a model...

Model Elements

- Households
  - make consumption / saving decisions
  - make work / leisure decisions
- Government
  - imposes taxes / transfers
  - issues debt
  - pays retirement income

Environment

- Demographics:
  - There are many (N) identical households.
  - Each lives for 2 periods - young / old.
- Endowments:
  - Each agent works one unit of time when young.
  - No labor / leisure choice yet.

Environment: Preferences

- Households want to consume when young c and when old c'.
- Lifetime utility: $u(c) + \beta u(c')$
- $\beta > 0$ is the discount factor
- u has nice properties (increasing, concave)
- Example: $\ln(c) + 0.95 \ln(c')$.

Notation convention:

Tomorrow's variables are indicated by primes: c'.

Environment: Markets

- Households trade
  - **goods** at price 1
  - one period **bonds** at price 1 with real interest rate $R = 1 + r$
  - Households are price-takers.

Environment: Government

- Imposes lump-sum taxes t, t'.
- The young pay t
- The old pay t'
- Later we have to worry about how the government balances its budget.

Household Behavior

Household Behavior

- Households max utility $u(c)+\beta u(c')$
- subject to budget constraints
- Result: consumption functions that give c as function of prices, endowments, tax rates

Young household

- Income is earnings net of taxes: $y - t$.
- Budget constraint:
$$y - t = c + s \tag{1}$$
- The choice variable: $s > 0$ is saving. $s < 0$ is borrowing.

Old household

- The household makes no choices: simply consume all income.
- Expenditure: c' – households do not save.
- Income:
  - from labor: $y' - t'$.
  - from capital: $(1+r)\ s$.
- Budget constraint:

$$y' - t' + (1+r)s = c' \tag{2}$$

- r is the **real interest rate** (what is that?).

Household problem

$$\max_{c,c',s} u(c) + \beta u(c')$$

$$s.t.$$

$$y - t = c + s$$

$$y' - t' + (1+r)s = c'$$

Household problem

Sub budget constraints into utility function:

$$\max_{s} u(y - t - s) + \beta u\left(y' - t' + [1 + r]s\right)$$

First-order condition:

$$u'(c) = \beta u'(c')[1 + r]$$

Solution to the household problem: c, c', s that satisfy

1. first-order condition
2. 2 budget constraints

Intuition

$$u'(c) = \beta u'(c')[1+r]$$

- The first-order condition is called the **Euler equation**.
- It equates marginal benefit and marginal cost of a small change in saving s.
- Marginal cost is:
- Marginal benefit is:

Lifetime Budget constraint

Lifetime budget constraint

- The household really chooses between c and c'.
- The **lifetime budget constraint** tells us all combinations of (c, c') the household can afford.
- An insight emerges: the **Permanent Income Hypothesis**.

Lifetime budget constraint

Start from the 2 period budget constraints:

$$\begin{aligned} c + s &= y - t \quad (3) \\ c' &= y' - t' + (1+r)s \quad (4) \end{aligned}$$

Solve the period 2 constraint for s:

$$s = \frac{c' + t' - y'}{1+r} \quad (5)$$

Substitute into period 1 constraint, eliminating s:

$$y' - t' + (1+r)\underbrace{(y - t - c)}_{s} = c' \quad (6)$$

Lifetime budget constraint

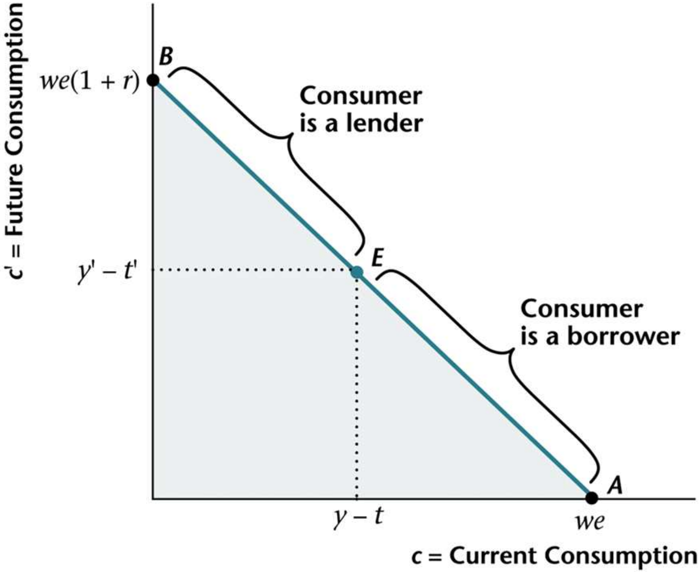
Rearrange:

$$\underbrace{c + \frac{c'}{1+r}}_{\text{Present value of c}} = \underbrace{y + \frac{y'}{1+r}}_{\text{Present value of y}} - \underbrace{t + \frac{t'}{1+r}}_{\text{Present value of t}}$$

Call the right hand side **lifetime wealth** (we).

$$c' = (1+r)(we - c) \tag{7}$$

Lifetime budget constraint



Source: Williamson, Macroeconomics

Present values

Definition

The present value of a payment x received at t is the amount $PV(x_t; t, r)$ which, when invested at interest rate r, grows to the value x_t by date t.

Example

Invest \$100 in 2007 with interest rate $r = 0.1$. In 2008, the investment has grown to

$$\$100\ (1 + r) = \$110$$

Thus, $PV(\$110, 1, 0.1) = \$100$.

Present values

Example

Invest \$100 for 10 years in 2007 with $r = 0.1$. This grows to

$$\$100\ (1+r)^{10} = \$260$$

Thus, $PV(\$260, 10, 0.1) = \$100$.

Note: The present value depends on

1. Interest rate
2. Time elapsed
3. Amount to be received.

Present values

An investor who can borrow / lend at interest rate r should not care when a given present value of payments is received.

Example

Which payment stream finances $(C, C') = 100$ with $r = 0.1$?
$(100, 100)$.
$\left(100 + \frac{100}{1.1}, 0\right)$: Save $100/1.1$ today. Receive 100 tomorrow.
$(0, 100 + 110)$: Borrow 100 today. Pay back 110 tomorrow.

For any payment stream with the same present value, the investor can borrow / lend to finance the same (C, C').

Household Problem

With present value budget constraint

$$\max_{c,c'} u(c) + \beta u([1+r][we - c])$$

First-order condition: ...

Intuition

- Lifetime budget constraint: household can exchange c' for c at a rate of $1+r$.

$$c' = (1+r)(we - c)$$

- Graph...

Intuition: Moving along the budget constraint

- Think about moving along the lifetime budget constraint.
- Give up a little c. Marginal cost:
- Raise s by $\Delta s = -\Delta c$.
- Tomorrow: earn additional income of $(1+r)\Delta s$.
- Marginal benefit: ...
- Plot marginal cost and marginal benefit against Δc...

The same with indifference curves

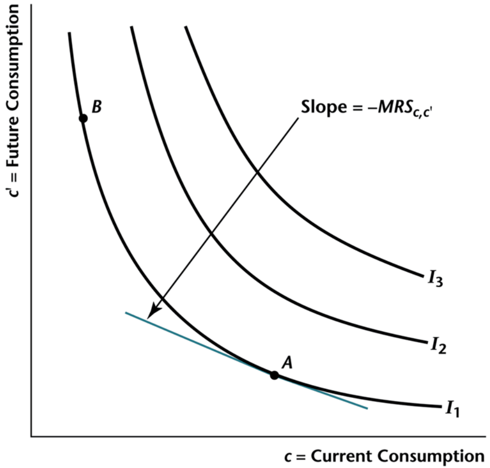
Definition

An indifference curve shows all combinations of (c, c') that yield the same utility.

Properties of indifference curves:

1. Downward sloping
2. Higher level $\rightarrow$ higher utility
3. Convex

Indifference curves



The slope of the IC is the marginal rate of substitution, $MRS_{c,c'} = u'(c)/[\beta u'(c')]$.

Consumption smoothing

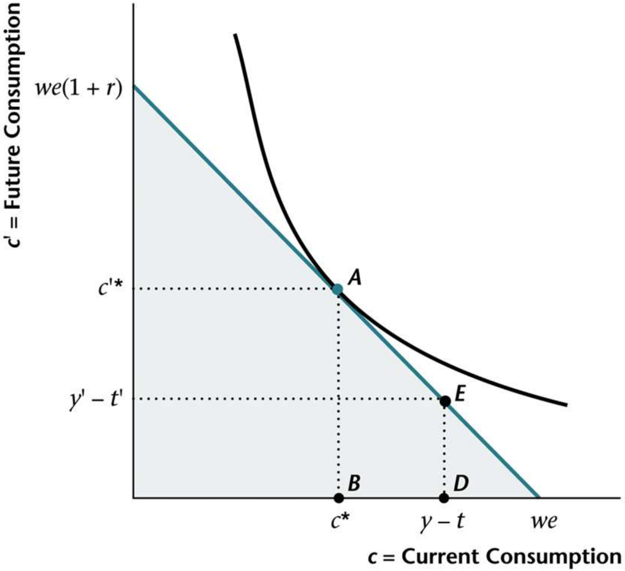
- A consequence of convex indifference curves:
  - households prefer "smooth" consumption over "unequal" consumption
- That means:
  - if (c_A, c'_A) and (c_B, c'_B) are on the same indifference curve
  - then the household prefers any average

$$[\lambda c_A + (1-\lambda)c_B, \lambda c'_A + (1-\lambda)c'_B]$$

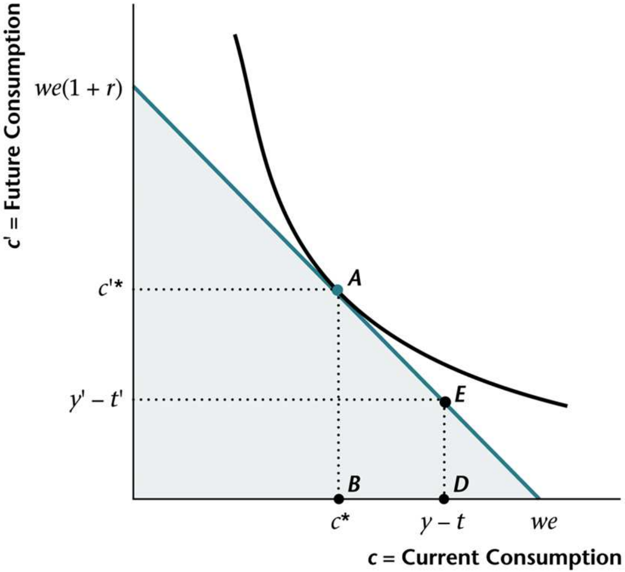
- Graph...

Optimal consumption-saving choice

Tangency of IC and budget constraint implies: $MRS_{c,c'} = 1 + r$.



A positive income shock (*we* rises)



Consumption smoothing

- Current income shocks are only partly consumed: $\Delta c < \Delta y$.
- The household prefers smooth consumption:
- Note: It does not matter for (c, c') whether current or future income is higher!

Permanent vs Transitory Income Shocks

- How do they differ?
- Draw a picture...
- What does it imply for tax policy?

Permanent Income Hypothesis

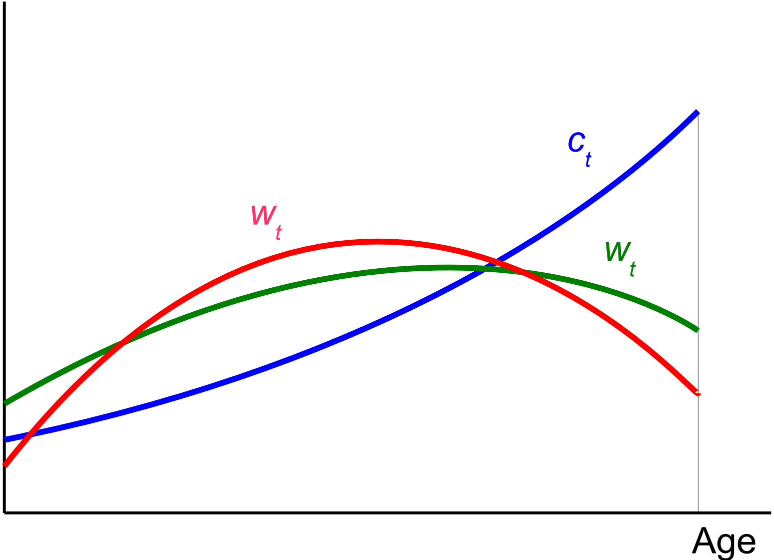
Permanent Income Hypothesis

Consumption only depends on the **present value** of lifetime income, not on its time profile.

- The PIH follows from the fact that a **lifetime budget constraint exists**.
- It does not depend on preferences (with a few exceptions).

Permanent income hypothesis

Two households with different wage profiles $\{w_t\}$ but with the same lifetime income choose the same age consumption profiles.



Permanent Income Hypothesis: Implications

A **temporary** income shock (y rises today) has a smaller effect on consumption than a **permanent** income shock (y rises today and in all future periods).

- Graph this...

This is important for **fiscal policy**:

- Stimulating consumption with temporary tax cuts is often not very effective.
- Households view tax cuts as transitory - government only postpones revenue collection.
- Therefore, refund checks are largely saved, not consumed.

[Compare with IS/LM or AS/AD]

Household: Example

- Assume log utility, $u(c) = \ln(c)$:

$$\max \ln(c) + \beta \ln(c') \tag{8}$$

- Then

$$u'(c) = 1/c \tag{9}$$

- Euler equation:

$$\frac{1}{c} = \beta [1 + r] \frac{1}{c'} \tag{10}$$

- Or, in terms of consumption growth:

$$1 + g(c) = \frac{c'}{c} = \beta [1 + r] \tag{11}$$

- Consumption growth only depends on the **interest rate**.

Household: Example

Permanent income hypothesis

How does the household choose (c, c')?

Think about a household with many periods.

For each pair of periods:

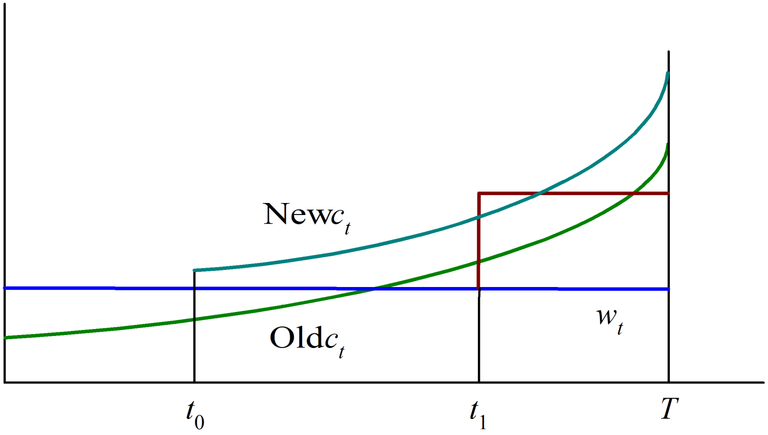
$$c'/c = \beta(1+r) \tag{12}$$

Step 1: Figure out the slope of the age consumption profile from the Euler equation.

Step 2: Pick the highest profile the household can afford.

Response to shocks

- How does the household respond to shocks?
- Imagine the household learns at date t_0 that his earnings will be higher from date t_1 onwards.



- This is supported by the data.

Response to shocks

- The household reoptimizes when the news arrives (at t_0).
- Because he wants smooth consumption, the entire c_t profile shifts up, starting at t_0, even though the additional income will not be received until age t_1.
- The household runs down his assets to finance the higher consumption early on.

Response to shocks

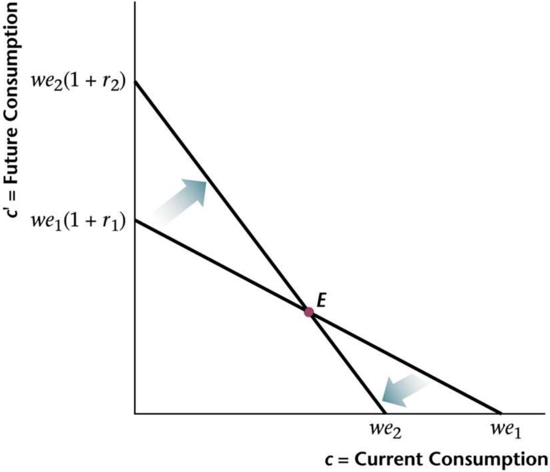
- This has two important implications:
  1. **Expected shocks have current effects**. For example, an innovation that occurs today but does not affect productivity for some time will increase consumption today.
  2. Households spread the effects of transitory shocks over time in an attempt to smooth consumption.
- This is important for understanding the business cycle properties of the model.

Interest rate shocks

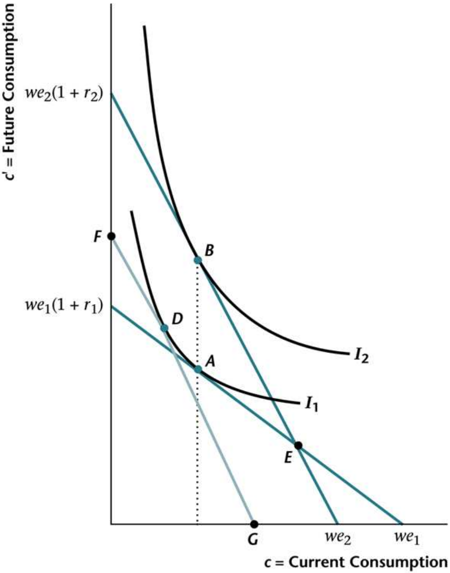
$r \uparrow$. The lifetime budget constraint

$$(y - t - c)\ (1 + r) = y' - t' - c' \tag{13}$$

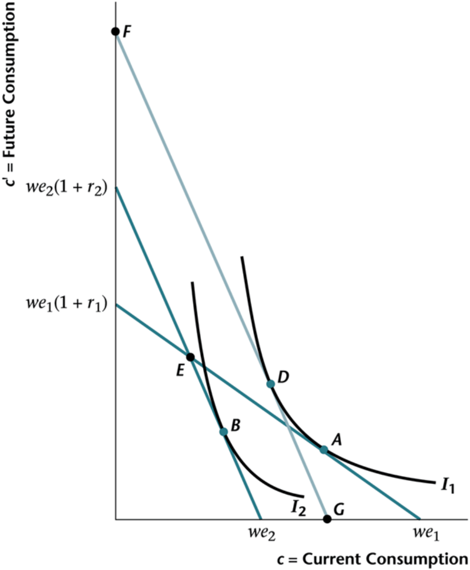
rotates around the endowment point.



A lending household



A borrowing household



c unambiguously falls. *s* rises.

Aggregate consumption function

Aggregate consumption (C) depends on:

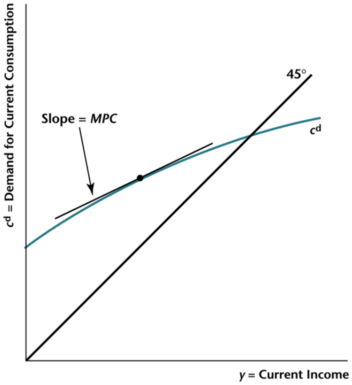
- the present value of current and future income net of taxes $(+)$,
- interest rate $(?)$,
- wealth $(+)$.

This is the payoff from micro-foundations: We know exactly what the properties of the consumption function are.

Aggregate consumption function

The **marginal propensity to consume** out of current income (MPC)

- is less than 1,
- varies with current income, future income, interest rate, and wealth.



Labor Supply

Household Problem

$$\max_{c,c',l,l'} u(c) + \beta u(c') + v(l) + \beta v(l') \tag{14}$$

subject to lifetime budget constraint

$$c + \frac{c'}{R} + t + \frac{t'}{R} = w(1-l) + \frac{w'(1-l')}{R} \tag{15}$$

where:

- $R = 1 + r$: gross interest rate
- l: leisure
- $1 - l$: work time

First-order conditions

The easiest approach: a Lagrangian:

$$\max_{c,c',l,l'} u(c) + \beta u(c') + v(l) + \beta v(l') \tag{16}$$

$$+\lambda \left[w(1-l) + \frac{w'(1-l')}{R} - c - \frac{c'}{R} - t - \frac{t'}{R} \right] \tag{17}$$

First-order conditions:

$$u'(c) = \lambda \tag{18}$$

$$v'(l) = \lambda w \tag{19}$$

Or:

$$\frac{v'(l)}{u'(c)} = \frac{w}{1} \tag{20}$$

Intuition

Marginal rate of substitution = relative price

- the price of consumption is 1
- the price of leisure is w

Note how leisure is treated like any other consumption good.

Effect of higher wage today

- $w \uparrow$
- Income effect:
- Substitution effect:

Note:

The effect of wages on current labor supply is ambiguous.

Effect of higher future wage

- $w' \uparrow$
- Income effect:
- Substitution effect:

Effect of permanently higher wage

- $w, w' \uparrow$
- Income effect:
- Substitution effect:

Permanent vs transitory shocks

Note the difference:

- transitory shock: small income effects; substitution effects dominate
- permanent shock: larger income effects

Lesson:

Permanent shocks can have qualitatively different effects from transitory shocks.
(Not just larger, but opposite sign.)

Reading

- Williamson, Macroeconomics, ch. 8
- Romer, Advanced Macroeconomics, 4e, 2.8-2.9, 8.1, 8.4