Review Problems: Long-run Labor Market
Prof. Lutz Hendricks. March 4, 2010

Jones, Macroeconomics, problems 7.1, 7.2, 7.4-7.8, 7.9, 7.10.

1 Basics

1. What are the definitions of labor force, unemployment, unemployment rate?

2. Why is it hard to measure unemployment? Why might unemployment be overstated or understated in the data?

2 Walrasian Model

1. What shifts labor supply / labor demand?

2. For the production function $Y = K^{\alpha} + L^{1-\alpha}$, derive the labor demand curve. What is the effect of higher K on labor demand? Why does it differ from the Cobb-Douglas case $Y = K^{\alpha}L^{1-\alpha}$?

3. Analyze the effects of shocks on employment and wages: $A \uparrow$, $K \uparrow$, wage taxes, expected increase in future productivity.

4. Analyze the effects of a minimum wage. Explain why it is inefficient. Note the general point: it is a bad idea to redistribute income by distorting prices.

5. What is the effect of a stock market rally on equilibrium wages and employment? Assume that stock prices rise without any changes in fundamentals (current or future firm profits or dividends).

6. What is wrong with the following statement: Higher productivity reduces employment because fewer workers are needed to produce the same amount of output.

7. Most economies experience persistent growth in A, which increases labor demand. Why do we not observe that employment rises over time in those countries?

3 Unemployment

1. Explain main reasons why there may be involuntary unemployment: efficiency wages, contracts, search/matching, centralized wage bargaining.

4 Europe vs. the U.S.

1. How could downwardly rigid wages explain the European unemployment experience?
2. Explain how higher taxes could account for the fact that Europeans work fewer hours than Americans.

5 Value of Education

1. You should be able to calculate present values.
2. Suppose you want to have \$1m in retirement wealth at age 65. Assume the interest rate is 3% per year.
   (a) How much would you have to save at age 25 to achieve this goal?
   (b) Starting at age 25, how much do you have to save *per year* to achieve this goal?
   (c) If you started with \$200,000 at age 25 and saved nothing in the future, what interest rate would you have to earn to retire with \$1m at age 65?
3. In our calculation of the value of a college degree we assumed that wages stay constant over time. In reality, we expect wages to grow at 2% per year (in real terms). How does this change the value of education? Assume a real interest rate of 3% and that college graduates work between the ages of 22 and 67. High school graduates work between the ages of 18 and 67.
4. How could one explain that the wages earned by college graduates rose faster than those earned by high school graduates, even though the supply of college graduates rose over time?

5.1 Answers: Value of Education

1. See slides.
2. Answers:
   (a) You save x today and accumulate interest for $T = 65 - 25 = 40$ years. The amount needed today has the future value \$1m: $x\left(1+r\right)^{T} = \$1m$. I.e.: x is the present value of \$1m. With $T = 65 - 25$ and $r = 0.03$ we have $x = 306,000$.
   (b) Just to get the details right, assume that the first amount is saved at the end of age 25 and accumulates 39 years of interest. So you save x in each of 40 years. The last amount saved contributes x to retirement wealth. The one in the year before contributes $x(1+r)$, and so on. The first value saved contributes $x(1+r)^{T-1}$. Now x satisfies: $x + x\left(1+r\right) + x\left(1+r\right)^{2} + ... + x\left(1+r\right)^{T-1} = \$1m$. Recall that the sum evaluates to $\frac{(1+r)^{T}-1}{(1+r)-1}$. Plug in values for T and r. Solve for $x = \$13,800$. A quick plausiblity check:

$40 \times 13,800 = 552,000$. On average, your money is invested for 20 years and doubles due to interest.

i. Consider a slight variation of the question. Assume that the payments start now, not at the end of this year. Then there is an easier way to answer the question. We are looking for a payment stream with future value \$1m. That is the same as looking for a payment stream with present value $\$1m/(1+r)^T$. The present value formula then says:

$$x\frac{1/(1+r)^T - 1}{1/(1+r) - 1} = \frac{\$1m}{(1+r)^T} \tag{1}$$

ii. That means there is an easier way of doing the original question as well: We are looking for a payment stream, starting next year, with future value \$1m. That is the same as looking for a payment stream with present value $\$1m/(1+r)^T$. The present value formula then says:

$$\frac{1}{1+r}x\frac{1/(1+r)^T - 1}{1/(1+r) - 1} = \frac{\$1m}{(1+r)^T} \tag{2}$$

Why do I need the $1/(1+r)$ in front of x? Because each payment is made a year later than the present value formula assumes. The payment stream is not $x, \frac{x}{1+r}, ..., \frac{x}{(1+r)^{T-1}}$ but $\frac{x}{1+r}, ..., \frac{x}{(1+r)^T}$.

(c) We are looking for the interest rate that satisfies $20,000 \times (1+r)^T = \$1m$ or $1 + r = (\$1m/20,000)^{1/T} = 50^{1/40} = 1.103$. You need to earn 10.3% interest. Good luck!

3. Now the stream of payments you receive is w, $w\frac{1+g}{1+r}$,$w\left(\frac{1+g}{1+r}\right)^2$, ..., up to $w\left(\frac{1+g}{1+r}\right)^{T-1}$. Relative to the case where wages did not grow, all that has changed is that the discount factor $\frac{1}{1+r} = \frac{1}{1.03}$ is replaced by $\frac{1+g}{1+r} = \frac{1.02}{1.03}$. Let's call the number R. So the present value is given by $PV = \frac{R^T-1}{R-1} \times w$. In the example, $T = 67 - 22 + 1$ for college graduates.