Review Problems: Solow Model
Prof. Lutz Hendricks. September 17, 2009

Jones, Macroeconomics, exercises 5.1-5.1, 5.7, 5.10.

1 Levels accounting

1. Why does the Solow model imply a larger role for productivity in explaining cross-country income gaps than does the production model?
2. Graph the effect of a higher $\bar{A}$ on steady state output. Decompose it into a direct effect of $\bar{A}$ on y^* and an indirect effect going through k^*.

2 Properties of the Solow Model

1. Derive that law of motion for $k = K/L$ in the Solow model. What is the interpretation?
2. Show that the Solow model has a unique steady state using a diagram.
3. Show that the economy converges to the steady state from any initial $\bar{k}_0$. Show that the growth rate declines as the economy approaches the steady state.
4. Consider two countries with identical technology parameters and production function $Y = K^\alpha \ (AL)^{1-\alpha}$. The only difference is that the saving rate is $s_A = 0.05$ and $s_B = 0.2$. $\alpha = 1/3$. How large is the gap in per capita income between A and B? How does your answer change if $\alpha = 0.8$?
5. In what sense does the Solow model predict that poor countries grow faster?
6. Does the Solow model explain the growth experiences of countries over the post-war period? Why or why not?

2.1 Answer: Properties of the Solow model

1.-3. We did that in class.

5. We did these calculations in class.

3 Comparative Dynamics

1. Examine the effect of a one-time **increase in the labor force** (L) in the Solow model without technical change. Plot the time paths of Y/L, wages, and interest rates. [Jones 2.2] [Answer: the same as a decrease in K or an increase in A. The reason: only $\bar{k}$ appears in the law of motion.]

2. **Income tax:** Examine the short-run and long-run effects of an income tax. Instead of receiving $wL + rK$, households receive $(1 - \tau)(wL + rK)$. The government consumes the tax revenues. [Jones 2.3] [Answer: the same as a decline in s from its original value to $s(1 - \tau)$].

   (a) How does your answer change if the government instead invests the tax revenues? [Answer: the same as a rise in s.]

4 Comparative statics

Derive the effects of the following events in the Solow model. Assume that the economy starts in the steady state. Plot the transition path of $k = K/L$ following the shock.

1. A permanent increase in s.
2. A permanent increase in the size of the population (not in the growth rate n).
3. A permanent increase in productivity growth, g.

5 Solow Model With Subsistence

Consider a version of our Solow model where households require a subsistence level of consumption. Here are the details. The production function is

$$Y = K^{\alpha} \ (AL)^{1-\alpha} \tag{1}$$

and capital is accumulated according to

$$K_{t+1} = s \ Y_t + (1 - \delta) \ K_t \tag{2}$$

Labor input grows at a constant rate n (set $n = 0$, since we did not talk about it in this class). Productivity is constant at A. The new part is the saving rate. While income is lower than a threshold $\bar{y}$ the household does not save. Of the income above $\bar{y}$ the household saves a constant fraction $\bar{s}$. Per capita saving is therefore given by $sy = 0$ if $y < \bar{y}$ and

$$sy = \bar{s}\ (y - \bar{y}) \tag{3}$$

if $s \geq \bar{y}$.

1. Graph the saving rate against y.

2. Plot the Solow diagram for this model. By this I mean a diagram depicting sy and $(n + \delta)\, k$. For comparison also plot the sy line without subsistence ($\bar{y} = 0$).

3. How many steady states does the model have? Assume that $\bar{y}$ is not too large – otherwise the economy has no steady states with $k > 0$.

4. For various values of initial capital, characterize which steady states the economy may converge to.

5.1 Answer: Solow Model with Subsistence

1. $s = \max\{0, \bar{s} - \bar{y}/y\}$. This starts at 0 until y reaches $\bar{y}$. Then it rises towards $\bar{s}$.

2. Solow diagram: Note that $sy = \bar{s}\ k^{\alpha} - \bar{s}\ \bar{y}$. The sy line is simply shifted down by a constant. It crosses the x-axis where $y = \bar{y}$. See figure 1.

3. The model has 3 steady states (including $k = 0$).

4. The highest steady state is similar to the regular Solow model and locally stable. The steady state at $k = 0$ is also stable. The middle steady state is unstable. The model has a poverty trap.

6 Convergence

6.1 Galton's Fallacy

[Based on Jones 3.4]

1. Imagine you have data on the height of mothers (h_m) and daughters (h_d). You plot the change in daughter's height ($h_d - h_m$) against mother's height and find a negative relationship. Tall mothers tend to have daughters that are less

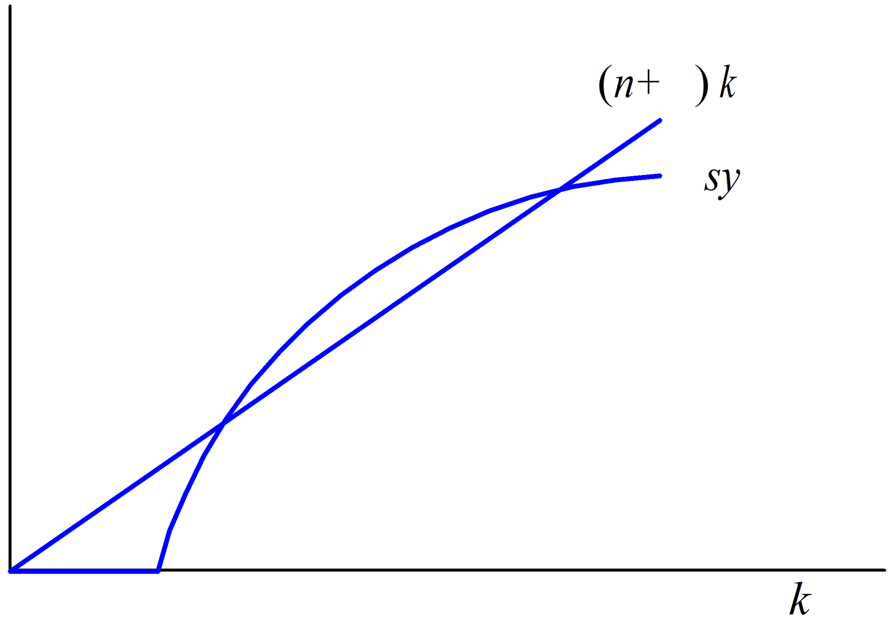

Figure 1: Solow diagram

tall than their mothers. Does this imply that all persons converge over time to the same height? Why or why not?

2. Now imagine that mother's height and daughter's height are drawn independently from the same distributions. What will you find if you plot the change in height against mother's height?

3. What does all this imply for the interpretation of cross-country growth regressions that find a negative relationship between initial income and subsequent income growth?

6.1.1 Answer: Galton's Fallacy

1. Heights need not converge. Counter-example: each person draws her height from a fixed distribution, independently of mother's height. The reason why the change in height is small when the mother is tall is simply that everyone has (on average) average height daughters. Tall mothers therefore have daughters that are shorter than themselves.

2. See #1.

3. In some samples of countries we find that $g(y)$ is negatively related to y at the beginning of the sample. This does not imply σ convergence.

7 Growth Accounting

1. Explain how growth accounting works. Given the production function $Y = A\ K^{\alpha}\ L^{1-\alpha}$, derive the key growth accounting equation

$$g(Y/L) = g(A) + \alpha\ g(K/L) \tag{4}$$

   and explain what it means.

2. The postwar data for Hong Kong are: $g(Y/L) = 0.073$ and $g(A) = 0.027$. What do these figures tell you about how the high growth rate of Y/L was achieved and whether it is sustainable in the future?

3. Why does it make sense to say: "Ultimately all growth in Y/L is due to productivity"? Doesn't this flatly contradict the growth accounting equation?

7.1 Answer: Growth accounting

1. This is on the class slides.

2. In the Solow model eventually $g(Y/L) = g(A)$. The production function implies

$$(1-\alpha)\,g(y) = \alpha\ g(k/y) + (1-\alpha)\,g(A) \tag{5}$$

   Temporarily $g(y)$ can be above $g(A)$. This requires a rising k/y, which in turn requires rising investment. When I/Y levels off, $g(y)$ drops again to $g(A)$, suggesting that the rapid growth of Hong Kong is not sustainable.

3. On the balanced growth path $g(Y/L) = g(A)$. In that sense, all long-run growth is due to A. The growth accounting equation also captures transitional growth due to changing K/Y.