The Romer Model

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Issues

- Productivity $(\bar{A})$ seems important for cross-country income differences (and for growth).
- The dominant view today holds that TFP growth is due to **innovation** - the production of ideas.
- We need to understand how ideas are produced.
- We study a simplified version of a model due to Paul Romer.

The Romer Model

- The economy produces rival goods (Y) and non-rival ideas (A).
- The rival inputs are labor only (no capital)
  - Relaxing this assumption does not change the main points.
- Labor can be used to produce goods (L_{yt}) or ideas (L_{at}).
- The **production functions** are
  - for goods:
$$Y_t = A_t L_{yt} \tag{1}$$
  - for ideas:
$$\Delta A_t = A_{t+1} - A_t = \bar{z} A_t L_{at} \tag{2}$$
- $\bar{z}$: a productivity parameter.

- Important **assumptions**:
  1. Constant returns to rival inputs (L_{yt}, L_{at}).
  2. Constant returns to ideas: double A_t - double ΔA_t.
- For rival inputs: constant returns come from replication
  - 2 factories produce twice as much as one.
- For ideas: there is **no good reason** for constant returns.
  - We'll get back to that.

- Resource constraint for labor:

$$\bar{N} = L_{yt} + L_{at} \tag{3}$$

  - $\bar{N}$: fixed supply of labor.

- Still missing: a "rule" for allocating labor between sectors.
- A simple assumption for starters:

$$L_{at} = \ell \bar{N} \tag{4}$$

- Why would we like to relax this assumption?

Model Summary

- Production functions:

$$\begin{aligned} Y_t &= A_t L_{yt} & (5) \\ \Delta A_t &= A_{t+1} - A_t = \bar{z} A_t L_{at} & (6) \end{aligned}$$

- Resource constraint:

$$\bar{N} = L_{yt} + L_{at} \quad (7)$$

- Allocation of labor:

$$\begin{aligned} L_{at} &= \ell \bar{N} & (8) \\ L_{yt} &= (1 - \ell) \bar{N} & (9) \end{aligned}$$

- Endogenous variables (determined in the model): Y_t, L_{yt}, L_{at}, A_t.
- Exogenous (given): A_0, $\bar{N}$, $\bar{z}$, ℓ.

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- Ideas are produced like goods.

Solving the Romer Model

- Labor inputs are fixed exogenously:

$$L_{at} = \ell \bar{N} \quad (10)$$
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- Ideas grow at a constant rate:

$$\Delta A_t = \bar{z} A_t L_{at} \Longrightarrow \tag{14}$$

$$\mathbf{g} = \Delta A_t / A_t = \bar{z} \ell \bar{N} \tag{15}$$

- $y_t = A_t (1-\ell)$ grows at rate g as well.

Solving the Romer Model

- Solving for A_t in closed form...
- Since A_t grows at the constant rate g:

$$A_t = A_0 \ (1+g)^t \tag{16}$$

- Since the production function implies that $y_t = A_t(1-\ell)$:

$$y_t = A_0 \ (1+g)^t (1-\ell) \tag{17}$$

- with

$$g = \bar{z}\ell\bar{N} \tag{18}$$

- All this work to find that the growth rate is exogenous and constant??

Summary: Equilibrium in the Romer Model

- The growth rate is constant over time (if parameters are fixed):

$$g = \bar{z}\ell\bar{N} \tag{19}$$

- Output per capita (y) and ideas (A) grow at rate g:

$$A_t = A_0 \ (1+g)^t \tag{20}$$
$$y_t = A_0(1-\ell) \ (1+g)^t \tag{21}$$

Why is there growth in the Romer Model?

- What differs from the Solow model?

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- What differs from the Solow model?
- The Solow model has **diminishing returns** in the accumulated factor (k):

$$\Delta K_t = \bar{A} K_t^{\alpha} \bar{L}^{1-\alpha} - dK_t \tag{22}$$

  - As K grows, MPK falls, and growth slows.
- The Romer model has **constant returns** in the accumulated factor (A):

$$\Delta A_t = \bar{z} A_t L_{at} \tag{23}$$

  - As A grows, the marginal product of A in producing A does NOT fall.
  - Growth continues.

Why is there growth in the Romer Model?

Exercise:

- Write down a Solow model in which the *MPK* never falls below a certain value.
- For example:
  - $Y_t = \bar{A} K_t$.
  - more complicated, but same result: $Y_t = \bar{A}[K_t + K_t^{\alpha}]$.
- Show that the model implies persistent growth.

Why is there growth in the Romer Model?

Fact

There is persistent growth in the Romer model because we have assumed constant returns to A in the production of A.

- There is a (partial) **justification** for this assumption:
  - We expect constant returns to scale in rival factors (K and L).
  - Therefore, diminishing returns in K only (Solow).
  - This argument does not apply to non-rival A (Romer).
- But: I have not given a good reason why there should be CONSTANT returns to A!

Balanced growth

Definition

A balanced growth path is an equilibrium in which all (real, per capita) variables grow at constant rates.

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- Note: A **steady state** is a special case of a balanced growth path with a growth rate of 0.
- In the Romer model, per capita variables always grow at a constant rate (g).
- This is not true in the Solow model: the growth rate varies over time as the economy approaches the steady state.

Experiments in the Romer Model

Experiments in the Romer Model

- What happens if we change parameters?
- We will always study this to gain insight into how the model works.
- It is also useful for checking whether the model makes empirical sense.

Increase in Population

Experiment: $\bar{N}$ increases permanently by a constant amount.

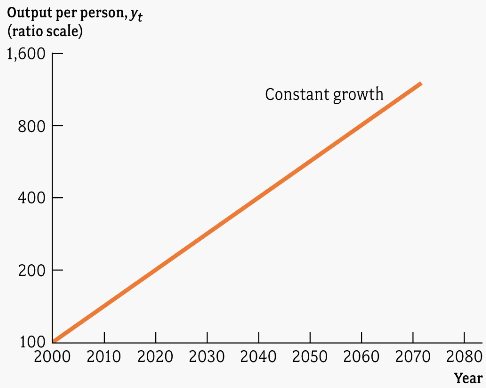

FIGURE 6.2 **Output per Person in the Romer Model**

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- Higher $\bar{N}$ implies faster growth.
- Does this prediction make sense empirically?

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- Higher $\bar{N}$ implies faster growth.
- Does this prediction make sense empirically?
- We need to think of this as a model of world growth.
- Countries benefit from each other's research.
- Even then: world population grows over time - the model implies ever accelerating growth!
- This is called a **scale effect**.

Increasing Share of Labor in R&D

Experiment: ℓ rises permanently.

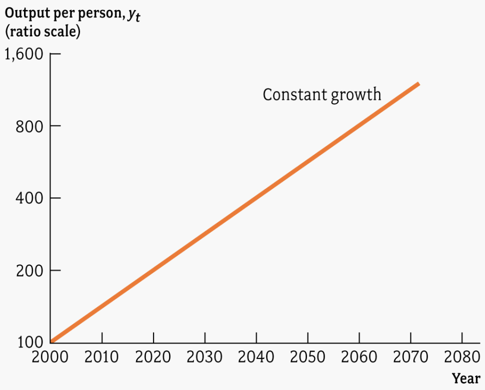

FIGURE 6.2 Output per Person in the Romer Model

Macroeconomics, Charles I. Jones

What if returns to A are diminishing?

$$\Delta A_t = \bar{z} A_t^{\rho} L_{at}$$
$$0 < \rho < 1$$

Then the growth rate is given by

$$\Delta A_t / A_t = \bar{z} A_t^{\rho - 1} \ell \bar{N} \quad (24)$$

Plot that ...

Diminishing returns to A

- With diminishing returns $(\rho < 1)$ the model has lost its ability to sustain growth.
- It behaves a lot like a Solow model.
- But what if the population grows over time?

Diminishing returns to A

- Assume

$$N_t = (1+n)^t \tag{25}$$

- The growth rate is given by

$$\Delta A_t / A_t = \bar{z} A_t^{\rho - 1} \ell \, (1+n)^t \tag{26}$$

- Can we have constant growth?

Balanced growth

- Let's call the constant growth rate of A $g(A)$.
- Constant growth then requires

$$A_t^{1-\rho} = \frac{\bar{z}\ell\left(1+n\right)^t}{g\left(A\right)} \tag{27}$$

- Take the growth rate of that:
  1. LHS: $g\left(A_t^{1-\rho}\right) = (1-\rho)\, g\left(A\right)$.
  2. RHS grows at rate n.
- Equate LHS = RHS:

$$g\left(A\right) = \frac{n}{1-\rho} \tag{28}$$

Balanced growth

Fact

With diminishing returns of A in the production of A the Romer model still generates persistent growth. (Robustness)

Fact

But the long-run growth rate no longer depends on R&D investment. It only depends on the population growth rate.

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What is the intuition for these results?

Intuition

- Higher N implies faster growth right now - more labor is allocated to R&D.
- But over time, growth slows due to diminishing returns to A. (Like the Solow model)
- Without population growth, no long-run growth in A.

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- With population growth: N rises each period.
- This pushes up the growth rate for some time.
- Since N rises each period, growth gets a push in each period and remains high.

What do we make of this?

- The model with constant returns to A is an implausible **special case**.
- With $\rho < 1$, long-run growth does not depend on R&D spending.
  - But short-run growth still depends on ℓ.
- Growth is sustainable, as long as population growth increases R&D inputs.

What do we make of this?

- In practice, the distinction may not matter that much.
- If ρ is close to 1, the model behaves similarly to the $\rho = 1$ model for a long time.
- E.g., an increase in ℓ may raise $g(A)$ for hundreds of years, but not forever.
- The point: $\rho = 1$ looks really special. For very long-run questions it is special. Otherwise, it does not matter.

Reality check

1. The model says: constant population - no growth.
   - But we are still producing new ideas all the time.
   - How can we reconcile this?
2. What if the population shrinks over time?
   - Is the long-run growth rate negative?

Policy Issues

Policy Issues

- There is no reason to believe that "the market" chooses the **optimal level of R&D**.
  1. R&D is only profitable if the innovator earns rents. This requires patents (government intervention).
  2. Innovators are monopolists for some time (patents). Monopoly pricing is not efficient.
  3. Innovation has knowledge spillovers - other innovators benefit.
- Which policies induce efficient innovation is an easy question in theory, but hard in practice.

Free Riding

- Most countries seem to invest almost nothing in R&D.
- They free-ride on innovations in the leading countries (U.S., Japan, Germany).
- One implication: it is not clear how much an increase in U.S. R&D would increase U.S. productivity.
- In the long-run, the effect could be quite small.

A Caveat

- Most economists believe that growth is due "R&D" type innovation.
- But we have little evidence to support this belief.
- Alternative views:
  1. Innovation could unintentional or driven by not-for-profit science.
  2. Human capital accumulation could be important.

Recap: Economic Growth

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- Facts:
  1. Large cross-country income gaps - factor 25.
  2. Large cross-country growth rate gaps.
  3. Great Divergence.
- Cross-country differences are largely due to productivity.
  - K and H each account for a factor of 2 (Solow model)
  - Productivity accounts for a factor of 6 (roughly)

Recap: Economic Growth

- Evidence suggests that institutions account for a good share of productivity differences.
- Capital (diminshing returns) cannot sustain growth.
- Productivity growth seems to come from innovation – Romer model.
- Since ideas flow across borders, it is hard for (small) countries to change long-run growth.
- In the very long-run, growth depends on population growth (Romer model).
    - But for practical horizons, changing R&D does affect growth.

Reading

- Jones, *Macroeconomics*, ch. 6
- Jones, Introduction to Economic Growth, ch. 5.