

## 1 Growth rate calculations

1. If  $y_{1950} = 100$  and  $y_{2000} = 150$ , what is the average growth rate of  $y$ ?
2. If  $y$  grows at rate  $g = 0.02$ , by how much does  $y$  grow over 50 years?
3. Ethiopia's per capita income in 2000 was \$635. Compute its per capita income in 2050 for growth rates of 1%, 2%, 4%, 6% per year. (Note that the same calculations would tell you how much your retirement savings increase over your working life for different rates of return on your investments.)
4. Plot (log) per capita GDP for the following scenarios:
  - (a) Growth of 5% per year for 70 years.
  - (b) Growth of 2% for 70 years, followed by 7% for 20 years, followed by 5% for 28 years.
  - (c) Growth of 7% for 50 years, followed by 1% for 140 years.
5. Assume  $g(x) = 5\%$  and  $g(y) = 2\%$ . Calculate:
  - (a)  $g(xy)$
  - (b)  $g(x/y)$
  - (c)  $g(x^{1/2}y^{1/2})$
  - (d)  $g(x^{-1/3}y^{1/2})$

### 1.1 Answer:

1.  $(1 + g)^{50} = \frac{y_{2000}}{y_{1950}}$
2.  $(1 + g)^{50}$ .
3. To be written.
4. Plotting log GDP:
  - (a) Let's normalize log GDP at the beginning of time to 1 ( $\log(1) = 0$ ). After 70 years, GDP grows to  $1.05^{70} = 30.4$  so that  $\log(GDP) = 1.48$ . So we get a straight line from (1,0) to (70,1.48).
  - (b) First 70 years: log GDP goes from 1 to  $\log(1.02^{70}) = 0.60$ . Etc. The general point: we have straight lines with slopes equals to the growth rates.

## 2 Investment Fees

Suppose you put away \$100,000 at age 25. At age 65 you withdraw the funds with interest. The return on investment is 5% per year.

1. How much wealth do you withdraw at age 65?
2. How does your answer change if you pay your investment advisor 1% of the portfolio's value in fees?
3. Assume that the inflation rate is 2%. What is the real value of your portfolio at age 65 (without fees)?
4. Assume that your nominal capital income is taxed each year at a flat rate of 30%. The inflation rate is 2%. What is the real value of your portfolio at age 65?
5. Now assume that the inflation rate rises to 4%. Calculate the real value of your portfolio at age 65. Why is it less than \$100,000, even though the real rate of return (5%-4%) is positive?

### 2.1 Answer: Investment fees

1. The funds grow to  $\$100,000 \times (1.05)^{40}$ .
2. Now the rate of return (growth rate of funds) is 4%.
3. The real value is [nominal value]/[price index]. The price index today is 1 (a normalization). In 40 years it is  $1.02^{40}$ . The real value is  $\$100,000 \times 1.05^{40}/1.02^{40}$ .
4. Capital income is 5% of the portfolio value. After tax, it is  $0.05 \times (1 - 0.3)$ . Same equation as #3 with the new rate of return.

## 3 Log scale

1. Assume that GDP ( $y_t$ ) grows at a constant rate of 3% per year for 50 years and then at a constant rate of 1% per year for 20 years. Plot  $\log(y_t)$  against time  $t$ . This need not be to scale.
2. How does your graph change if the growth rate for the first 50 years rises to 4% per year?