

# Review Problems: A Model of Production

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Jones, Macroeconomics, exercises 4.3, 4.5, 4.6.

## 1 Methodology

Suppose you want to find out how income taxes affect aggregate consumption. One approach would be to get data on income tax rates ( $\tau_t$ ) and on aggregate consumption since 1950 ( $C_t$ ). Then one could run an OLS regression of the form

$$C_t = \alpha + \beta\tau_t + \varepsilon_t \quad (1)$$

1. Intuitively, what does an OLS regression do?
2. What is the interpretation of  $\beta$ ?
3. Why does  $\beta$  not answer the question: a 10% increase in taxes would reduce consumption by  $\beta \times 10\%$ ?
4. How could one answer the question: how do taxes affect consumption?

## 2 Production function

1. What properties of the Cobb-Douglas production function,  $Y = AK^\alpha L^{1-\alpha}$ , lead us to believe that it is a good approximation of the data?
2. How could one estimate the important parameter  $\alpha$ ?
3. For the production function  $Y = AK^\alpha L^\beta$  find the marginal products of capital and labor.
4. If  $\alpha + \beta = 1$ , what share of income goes to capital and labor? The rest goes to pure profits. What is the profit share? Assume that capital and labor are paid their marginal products.

5. If  $\alpha + \beta < 1$ , what share of income goes to capital, labor, and profits?
6. If  $\alpha + \beta = 1$ , by how much does doubling  $K/L$  increase  $Y/L$ ? By how much does a 10-fold increase of  $K/L$  increase  $Y/L$ ? If  $\alpha = 0.3$ , why is the effect of the 10-fold increase so much less than 10 times the effect of doubling  $K/L$ ?
7. Repeat the previous exercise for  $\alpha = 0.8$ . How does your answer change?

## 2.1 Answers: Production function

1. Constant returns to scale and constant capital and labor shares.
2. Show that capital receives fraction  $\alpha$  of total output. In the data, the share of GDP that goes to capital is about 1/3. See the slides for details.

## 3 Measuring Productivity

1. Given data on capital, labor, and output, how can the production model be used to measure total factor productivity ( $A$ )?
2. Why is the value of  $\alpha$  critical for answering the question: How important is capital for cross-country income gaps?

### 3.1 Answers: Measuring Productivity

1. Assume a production function. For reasons we discussed, a Cobb-Douglas function makes sense:  $Y = AK^\alpha L^{1-\alpha}$ . Get data on  $Y, K, L$ . Solve the production function for  $A$ :  $A = \frac{Y}{K^\alpha L^{1-\alpha}}$ . Plug in the data values to estimate  $A$  for each country.
2. Low  $\alpha$  means quickly diminishing MPK. A given cross-country gap in capital implies a small gap in output. The opposite is true with high  $\alpha$ .