Exam 1. Econ420. Fall 2009

Professor Lutz Hendricks

Instructions:

- Answer all questions.
- *Explain* your answers – do not just state them.
- *Show* your derivations – do not just state the final result.
- The total time is 75 minutes.
- The total number of points is 100.

1 Measurement

Take as given the following figures. Year 1990:

Good	Quantity	Price
Computers	50	\$1,000
Bread	100,000	\$2

Year 2000:

Good	Quantity	Price
Computers	200	\$400
Bread	120,000	\$2.20

1. [5 points] Calculate nominal GDP in both years.
2. [10 points] Calculate real GDP in 2000 with 1990 prices and vice versa.
3. [10 points] Calculate the average annual growth rate of real GDP for both cases.
4. [5 points] Imagine that the quality of computers improves over time. Qualitatively, how might you revise your estimates of real GDP and inflation?

2 Model of Production

Consider two countries: the U.S. with $Y/L = \$42,000$ and $K/L = \$100,000$ and China with $Y/L = \$3,000$ and $K/L = \$6,000$. Assume the production function $Y = \bar{A}K^{1/3}L^{2/3}$.

1. [5 points] The actual output gap between the U.S. and China is $42/3 = 14$. Which output gap does the model attribute to the fact that K/L in the U.S. is 16 times higher than in China?
2. [5 points] How large is the ratio of $\bar{A}$ of the U.S. relative to China implied by the model?
3. [15 points] Plot the production functions of the two countries (not to scale). Assume that both are in steady state. Show the contributions of K/L and $\bar{A}$ to the Y/L gap between the 2 countries.

3 Solow Model

Consider the following modified Solow model:

$$\begin{aligned} Y_t &= \bar{A}K_t^{\alpha}L_t^{1-\alpha} - X \\ K_{t+1} - K_t &= sY_t - dK_t \\ Y_t &= C_t + I_t \\ L_t &= \bar{L} \\ I_t &= sY_t \end{aligned}$$

with $\alpha, s, d, \bar{L}, \bar{A}, X$ given. The new part is X. I am assuming that a constant amount of output, X, is lost in each period.

1. [5 points] Plot sY_t and dK_t against K_t.

2. [5 points] Define a steady state. In your graph, how many steady states are there?

3. [15 points] Discuss the dynamics of K_t. For any given K_0, what happens to K_t over time? Which of the steady states are stable (i.e.: if the economy starts with K_0 near the steady state, it converges towards it)? Explain your answer.

4 Short Questions

1. [5 points] If $Y_t = K_t^{1/3}$ and K_t grows at 6% per year, what is the growth rate of Y_t?

2. [5 points] If nominal GDP grows at 5% per year and real GDP grows at 2% per year, how much inflation is there over 20 years, i.e., calculate [GDP deflator at $t+20$] / [GDP deflator at t]. Show the steps of your calculations.

3. Imagine that $x(t)$ grows at the constant rate g.

   (a) [5 points] Plot $\ln(x(t))$ against time t. Explain the key features of the graph.

   (b) [5 points] Now suppose that the growth rate increases permanently, starting in year 50, to the constant rate $\hat{g} > g$. How does this change the plot of $\ln(x(t))$?

5 Answers: Exam 1, Econ420, Fall 2009

5.1 Answer: Measurement

1. $GDP_{1990} = \$50,000 + \$200,000$. $GDP_{2000} = \$80,000 + \$264,000 = \$344,000$.
2. $GDP_{1990}^{2000} = 50 \times \$400 + 100,000 \times \$2.20 = \$240,000$. $GDP_{2000}^{1990} = 200 \times \$1,000 + 120,000 \times \$2 = \$440,000$.
3. Average growth rate with 1990 prices: $(440/250)^{1/10} - 1 = 5.82\%$. With 2000 prices: $(344/240)^{1/10} - 1 = 3.67\%$.
4. True inflation is lower and true real gdp growth is faster.

5.2 Answer: Model of Production

1. Start from the production function $y = Ak^{\alpha}$. $y_{US}/y_{CHN} = (k_{US}/k_{CHN})^{1/3} = 16^{1/3} = 2.52$. Of, if you use exact numbers: $(100/6)^{1/3} = 2.55$.
2. Solve the production function for y and plug in numbers. Or, more easily, $14 = 2.5 \times A_{US}/A_{CHN}$ so that $A_{US}/A_{CHN} = 5.6$. Or, if you use exact numbers: $A_{US}/A_{CHN} = (42/3)/(16^{1/3}) = 5.56$.
3. See figure 1.

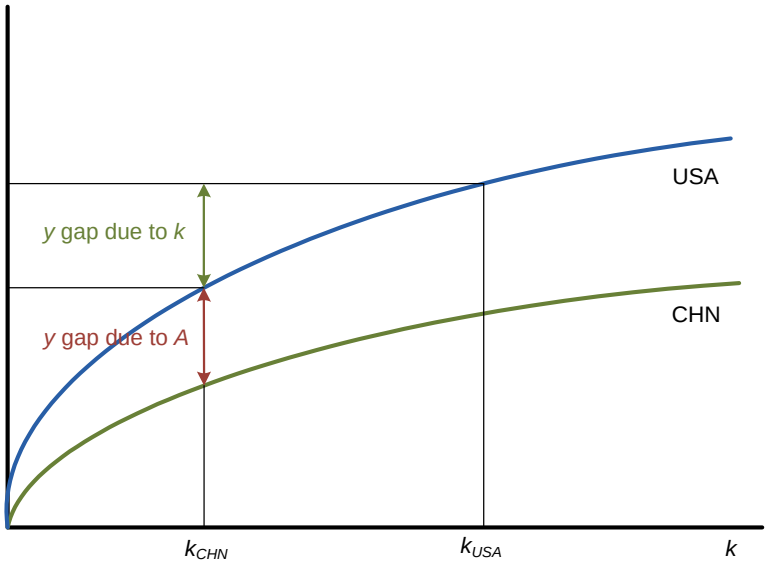

Figure 1: Decomposition of output gaps

5.3 Answer: Solow Model

1. See figure 2. I plotted in per capita variables (k, y) but this does not make any difference. The only change relative to the standard Solow model is that the sy curve is shifted down by a constant.

2. A steady state is an equilibrium in which all per capita (real) variables are constant. There are 2 steady states: k^* and k^{**}.

3. Anything to the right of k^{**} behaves like the usual Solow model. k^{**} is unstable. To the left of k^{**} the economy collapses to the point where output is less than X. In all of this, is it ok (and probably better) to assume that $y = 0$ for k so low that production is less than X. Then there is a 3rd (trivial) steady state at $k = 0$. Nothing else changes.

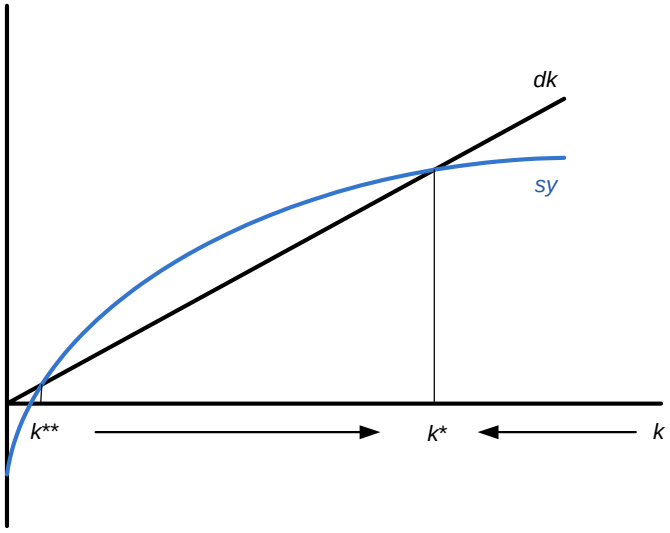

Figure 2: Modified Solow Model

5.4 Answer: Short Questions

1. 2%. Or, more precisely, $1.06^{1/3} - 1$.

2. Inflation rate: 3% per year. Growth of prices over 20 years: $1.03^{20} = 1.8$.

3. $x(t)$ grows at rate g.

   (a) straight line with slope g.

   (b) this adds a kink at $t = 50$.