Taxation and Human Capital Accumulation

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Abstract

How do taxes affect human capital accumulation? This question has been studied extensively in the context of two model classes: overlapping generations (OLG) and infinite horizon (IH) models. These embody very different assumptions about the intergenerational transmission of physical and human capital. OLG models typically abstract from intergenerational linkages, while IH models implicitly assume that new agents inherit human and physical capital from their parents. This paper investigates how such differences in intercohort persistence affect the responsiveness of human capital to taxation. A model is developed that nests OLG and IH models as special cases. The steady state and transitional effects of tax changes are computed for varying degrees of persistence. The main finding is that stronger intercohort persistence magnifies the long-run impact of taxation on human capital and leads to slower transitional dynamics. As a result, IH models generate systematically larger tax effects than OLG models, even if generations are altruistically linked. For the tax experiments studied here, the impact of persistence is large. Models with complete persistence generate steady state tax elasticities at least two times larger and transitional half-lives at least three times longer than do models without persistence.

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1. Introduction

A large share of total wealth in industrialized countries consists of human capital (Davies and Whalley 1991). Understanding how taxation affects human capital accumulation is therefore important for the evaluation of alternative tax schemes. A recent literature has addressed this issue in the context of two classes of models: overlapping generations (OLG) and infinite horizon (IH) models.[1] As noted by Lucas (1990), the two model classes have very different theoretical structures. In particular, they embody different assumptions about the intergenerational transmission of human and physical capital. In OLG models, it is typically assumed that the capital endowments of new agents are exogenously given. In this sense, there is no intercohort persistence.[2] I shall refer to such models as "pure OLG" models. An IH model, by contrast, may be thought of as an environment in which children inherit physical and human capital from their parents. Intercohort persistence is therefore complete. The findings obtained from both model classes therefore rely on extreme specifications of persistence.

The question addressed in this paper is how these differences in intercohort persistence affect the sensitivity of human capital to income taxation. To study this question, I develop a model that nests pure OLG and IH models as special cases and gives a precise meaning to the term "persistence." The model is parameterized to match selected U.S. post-war observations. Steady state and transitional effects of changes in wage and capital income taxes are computed for versions of the model that include pure OLG and IH models as well as intermediate cases.

The main finding is that stronger intercohort persistence of human capital increases the long-run sensitivity of human capital to wage and capital income taxes. An important implication is that IH models yield larger steady state tax elasticities than do OLG models with incomplete persistence, even if cohorts are altruistically linked. Stronger persistence also leads to slower convergence to the steady state following an unannounced tax change. For the tax experiments considered here, the implications of persistence are large. Models with complete persistence generate steady state tax elasticities at least two times larger and transitional half-lives at least three times longer than do models without persistence.

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[1] Important examples include Lucas (1990) and Trostel (1993) using IH models and Davies and Whalley (1991) using OLG models.

[2] The term "intercohort persistence" is used for the degree to which human capital acquired by parents is transmitted to their children. How this concept is related to the notion of persistence used in the literature on intergenerational mobility is discussed below.

These findings contrast sharply with the common view that IH models and OLG models "have very different theoretical structures, yet in practice, for the kind of tax problem under study here, seem to yield quite similar results" (Lucas 1990, p. 295). It is worth emphasizing, however, that the discrepancy between IH and OLG models is mostly a long-run phenomenon as the transition paths generated by all models studied here are quite similar for at least the first 10 years following an unannounced tax change. The reason is that the intergenerational transmission of human and physical capital mainly affects the cohorts entering the economy after the tax change takes effect.

The question how the intergenerational transmission of human capital modifies the effects of factor income taxes has received little attention in previous research. Hendricks (1999 and 2001 forthcoming) shows that the balanced growth effects of flat rate taxes depend in important ways on intercohort persistence. However, the results derived from endogenous growth models do not carry over to level effects of taxation in exogenous growth models. In particular, the key finding that longer horizons exacerbate the discrepancy between infinite horizon and life-cycle models does not hold for the class of models studied here. The main reason is that endogenous growth imposes linearity restrictions on the intergenerational transmission of human capital that cannot be imposed when growth is exogenous. The economic mechanism underlying the findings reported here is further investigated by Hendricks (2000a). This paper derives analytical solutions for the steady state effects of factor income taxes in a stylized version of the present model in order to obtain insights into the generality and the intuition underlying the interaction between intercohort persistence and tax effects. An additional issue that arises in the comparison of IH and OLG models is the role played by integenerational altruism. The results reported here are consistent with Engen et al.'s (1997) finding that altruism magnifies the steady state effects of income taxes.

The rest of the paper is organized as follows. Section 2 illustrates how intercohort persistence affects the tax sensitivity of human capital using an analytical example. Section 3 introduces the computational model. Numerical results are presented in section 4. The final section concludes.

2. An Analytical Example

This section develops an analytical example to illustrate how intercohort persistence affects the tax sensitivity of human capital. A simple pure OLG economy is described in which labor income taxes have no effect on human capital. I then add intercohort persistence to the model. If parents are altruistic towards their children, taxes reduce human capital. Allowing in addition for intercohort persistence of human capital further magnifies the effects of the tax. The example provides clear intuition for the mechanism through which the intergenerational transmission of human capital magnifies the impact of taxes on human capital accumulation.

Time is discrete and there is a single, perishable consumption good at each date. Households are the only agents. At each date a unit measure of households is born that live for two periods. Utility is derived from consumption at young and old ages according to $u(c_t^y)+\beta u(c_{t+1}^o)$. At birth, each young household receives an endowment of h_t^y units of human capital. Investing x_t units of the consumption good increases the human capital stock when old to

$$h_{t+1}^o = G(h_t^y, x_t) = (h_t^y)^{1-\varphi} x_t^{\varphi}, \quad 0<\varphi<1.$$

The household supplies one unit of labor inelastically at both ages which produces $w h_t^y$ units of goods when young and $w h_{t+1}^o$ units of goods when old, where $w > 0$ is a technological constant. However, a government imposes a wage tax on old agents so that the household only receives a wage rate of $w^o = (1-\tau) w$. Tax revenues are discarded.

I first study the effects of taxes on human capital in the absence of intercohort persistence. Young households solve

$$\max u(c_t^y)+\beta u(c_{t+1}^o)$$

s.t. $$c_t^y = w h_t^y - x_t, \;\; c_{t+1}^o = w^o\, G(h_{t+1}^y, x_t).$$

The Euler equation is

$$u'(c_t^y) = w^o\, G_x(h_t^y, x_t)\,\beta u'(c_{t+1}^o),$$

which has the standard interpretation that the marginal rate of substitution equal the rate of return. With log utility, $u(c)=\ln(c)$, this can be solved in closed form:

$$\frac{1}{w h_t^y - x_t} = \beta \frac{w^o\, G_x(h_t^y, x_t)}{w^o\, G(h_t^y, x_t)} = \frac{\beta\varphi}{x_t}$$

or $$x_t = w h_t^y (1+1/\beta\varphi)^{-1}.$$

Taxing old age earnings has no effect on human capital in this economy. This is immediate, if the human capital endowment is exogenous, but remains true even if the endowment of new cohorts is linked that of their parents. For example, if $h_t^y = \varepsilon (h_t^o)^{\rho}$, it is easy to show that the steady state level of h^o is independent of taxes. The strong neutrality of taxes depends, of course, on the functional forms chosen. Yet the point is general: Reducing wage taxes has offsetting income and substitution effects on savings, which cancel in the case of log utility. Large effects of taxes on human capital require large interest elasticities of saving.

Adding intergenerational altruism changes this result fundamentally, precisely because it implies that, regardless of preferences, the interest elasticity of saving is infinite. Assume that parents value not only their own consumption, but also their children's utility discounted at rate β. Preferences may then be written as

$$u(c_1^o) + \sum_{t=1}^{\infty} \beta^{t-1} \{u(c_t^y) + \beta u(c_{t+1}^o)\} .$$

All other model assumptions remain the same. Since parents and children pool their resources, the separate budget constraints for young and old age are replaced by

$$c_t^y + c_t^o + x_t = w h_t^y + w^o h_t^o .$$

It is easy to verify that optimality requires $u'(c_t^y) = u'(c_t^o)$ and

$$u'(c_t^y) = \beta u'(c_{t+1}^y) w^o G_x(h_t^y, x_t) .$$

In steady state, the rate of return must therefore equal the discount rate: $w^o G_x = 1/\beta$. This expression is familiar from infinite horizon models. With an operational bequest motive, the interest elasticity of saving is infinite and the after tax rate of return to saving therefore constant. Since $G_x = \varphi (h_t^y / x_t)^{1-\varphi}$, the solution for human capital investment is

(1) $$x_t = h_t^y (\varphi \beta w^o)^{1/(1-\varphi)} .$$

Old age human capital is therefore $h_{t+1}^o = h_t^y (\varphi \beta w^o)^{\varphi/(1-\varphi)}$. If the human capital endowment is exogenous, the elasticity of old age human capital with respect to (1−τ) then equals φ/(1−φ). Allowing for intercohort persistence in the human capital endowment magnifies this elasticity. Assume that $h_t^y = \varepsilon (h_t^o)^\rho$, 0 < ρ < 1. For simplicity, I assume that parents do not internalize the spillover. This may be motivated by interpreting h^o as average human capital of the parental generation. In steady state $h^y = \varepsilon (h^y)^{(1-\varphi)\rho} x^{\varphi\rho}$. Together with (1) this can be solved for the steady state level of the human capital endowment:

$$(h^y)^{1-\rho} = \varepsilon (\varphi \beta w^o)^{\rho\varphi/(1-\varphi)} .$$

The elasticity of h^o with respect to (1−τ) is therefore $\varphi/(1-\varphi)(1+\rho/[1-\rho])$. The first term reflects the effect of taxes on *x* (given h^y) and is the same as in the case of an exogenous endowment. The second term reflects the fact that a given change in *x* has a larger steady state effect on human capital, if the endowment is endogenous. The steady state tax elasticity of human capital is increasing in the degree of intercohort persistence of human capital (ρ). Note that the two model classes studied in the literature both make extreme assumptions about persistence: pure OLG models impose $\rho = 0$, whereas IH models implicitly assume $\rho = 1$.

In sum, the analytical examples suggests (though by no means proves) the following conclusions. Two features implicit in IH models contribute to large tax effects on human capital: intercohort persistence of human capital and of physical capital via altruistic bequests. It is therefore natural to expect the effects of taxes on human capital to be larger in IH models than in pure OLG models which abstract from all intergenerational links.[3]

3. The Model

This section develops a computable general equilibrium model that allows to quantify how intercohort persistence affects the magnitude of tax effects. The model may be thought of as a finite horizon version of a neoclassical growth model with human capital, which nests pure OLG and IH models as special cases. The economy is populated by competitive firms that produce a single good, by dynasties of finitely lived households, and by a government.

3.1 Households

At each date a unit mass of households is born which lives for I periods. Households are inactive until age T_1. They work and accumulate human capital until they retire at age T_R. At age t^* each household has one child, who receives its human capital endowment at parental age $T_H \geq t^*$. Parents leave a bequest (W_C) to the child at age I. The household's objective is to maximize discounted lifetime utility: [4]

(2) $$V(a_{T_1}, h_{T_1}, W) = \max \sum_{i=T_1}^{I} \beta^i u(c_i, l_i) + \beta_C \, \beta^{t^*} V(a_{T_1}^C, h_{T_1}^C, W_C)$$

subject to h_{T_1}, a_{T_1} given; $a_{I+1} = 0$

(3) $$a_{i+1} = (1 + r_i)\, a_i + w_i\, h_i\, (1 - l_i) - p_{V\,i}\, v_i\, h_i - p_{C\,i}\, c_i - p_{X\,i}\, x_i + \Gamma_i$$

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[3] It is possible to show that the intuition obtained from this example carries over to more general models; see Hendricks (2000a).

[4] Household variables are generally indexed by date t and age i. To simplify notation, date subscripts are omitted when there is no risk of confusion (e.g., h_i instead of $h_{t,i}$).

(4) $$h_{i+1} = (1-\delta_h)\,h_i + G(v_i h_i, x_i)\,.$$

(5) $$h_{T_1}^C = \varepsilon\left(\psi\, h_{T_H}^{\omega} + (1-\psi)\,\bar{h}_{T_H}^{\omega}\right)$$

The first term in the objective function (2) reflects the utility obtained from own consumption (c) and leisure (l), while the second term adds the utility of the child, which is valued with a weight of β_C. The household's endowments are a_{T_1} units of physical capital, h_{T_1} units of human capital and an inheritance of W received at age $t_b = I - t^* + 1$. Equation (3) is a flow budget constraint where the inheritance received is added at age t_b, and the bequest left (W_C) is subtracted at age I. The household's income consists of interest (r) earned on physical capital holdings, labor earnings, and lump-sum transfers (Γ). The household spends on consumption goods that cost p_C per unit and on goods inputs ($p_X\, x$) and time inputs ($p_V\, v$) in human capital accumulation. It receives a wage of w per efficiency unit of time spent in the market $[h(1-l)]$.

From age T_1 to T_R, the household works and engages in job-training, which augments human capital according to (4). After age T_R the household is retired, which imposes $l_i = 1$, $v_i = x_i = 0$. Equation (5) specifies that children inherit a weighted average of parental human capital (h_{T_H}) and of average human capital of the parent's cohort ($\bar{h}_{T_H}$). Of course, in equilibrium $h_{T_H} = \bar{h}_{T_H}$.

This specification allows to nest a number of interesting special cases. The model reduces to a pure OLG model, if the endowments of new cohorts are fixed exogenously ($\omega = 0$) and parents are not altruistic ($\beta_C = 0$). It reduces to an IH model, if parents are fully altruistic ($\beta_C = 1$), intercohort persistence is complete ($\omega = 1$), human capital is perfectly inherited ($\varepsilon = 1$), and parents take this inheritance into account when making investment decisions ($\psi = 1$). The specification chosen here also encompasses intermediate cases in which the intergenerational transmission of human capital is imperfect ($\omega < 1$) or only partly internalized ($\psi < 1$). Alternative ways of modeling intercohort persistence have been proposed in the literature, but do not nest the IH model as a special case.

A parameter of key importance for the subsequent analysis is the elasticity of the human capital endowment with respect to the human capital of the parental cohort (ω). It is tempting to relate ω to measures of intergenerational mobility commonly estimated from earnings regressions of the form $\ln y_i^P = \omega_0 + \omega_1 \ln y_i^C + \zeta_i$. Here, the unit of observation is a parent/child pair. y^P and y^C are measures of parental and child earnings, and ζ is a random error term. Depending on the measure of earnings used and on details of the estimation procedure, the estimates for ω_1 range from 0.2 to 0.6 (Mulligan 1997). Note how equation (5) could be used to motivate the empirical earnings equation. With $\psi = 1$, it reads

$\ln h_{T_1} = \ln \varepsilon + \omega \ln h_{T_H}$. It is therefore easy to conceive of versions of the model presented here in which estimates of ω_1 could be used to identify an empirically plausible range for ω. For this reason, as well as for lack of a better term, I call ω the degree of *intercohort persistence*.

However, it is also possible to construct versions of the model in which intergenerational mobility is not related to ω. For example, imagine a heterogeneous agent version of the model with ψ = 0, so that new agents start out with a fraction of average human capital as in Azariadis and Drazen (1990): $h_{T_1,i} = \varepsilon \bar{h}_{T_H}^{\omega} + \zeta_i$, where ζ is an iid random variable. Such a model has complete intergenerational mobility regardless of ω. What matters for tax effects is therefore not intergenerational mobility, but the degree to which human capital is transmitted from one cohort to the next. Identifying ways of measuring ω and ψ in the data is an important task for future research.

3.2 Firms

There is one good sold in a competitive market. A single representative firm rents capital (*K*) and labor (*L*) from households to solve a standard static profit maximization problem. The technology is $F(K,L) = AK^{\theta}L^{1-\theta}$ implying competitive rental prices for capital and labor of

(6) $$r^* = \theta A k^{\theta-1}$$

(7) $$w^* = (1-\theta) A k^{\theta},$$

where $k = K/L$. Measured output excludes aggregate job-training expenditures (I_X): $Y = F(K,L) - I_X$ (Prescott 1998).

3.3 Government

The structure of tax and transfer policies mimics Trostel (1993). The government levies taxes on capital income, labor income, and consumption to pay for government spending (*S*) and transfers (Γ). The budget is balanced in each period. The revenue from wage taxes is $\tau_L w^* L$. In addition, the household purchases goods inputs (I_X) in the market. A fraction d_X of these is tax-deductible, which reduces revenues by an additional $\tau_L d_X I_X$. The remainder is subsidized at rate s_X, which reduces revenues by an additional $(1-d_X) s_X I_X$. Capital tax revenues are $\tau_K K(r^* - \delta_K)$, which excludes tax-deductible depreciation. Finally consumption tax revenues are $\tau_C C$. Given these policies, the prices faced by households are

$$w = (1-\tau_L) w^*, \quad r = (1-\tau_K)(r^* - \delta_K), \quad p_C = 1 + \tau_C.$$

The price for purchased goods inputs in job training is

$$p_X = d_X(1-\tau_L) + (1-d_X)(1-s_X)\,.$$

Own time in training costs $p_V = (1-\tau_L)\,w^*$. Transfers are paid in equal amounts to all households aged T_1 or above. The government budget constraint is therefore

$$S_t + \Gamma_t\, N_t^A = \tau_C\, C + \tau_K\, K\,(r^* - \delta_K) + \tau_L\, w^* L - (1-p_X)\, I_X\,,$$

where $N_t^A = I - T_1 + 1$ is the size of the adult population.

3.4 Parameters

This section describes the choice of baseline parameters, summarized in table 1. In order to ensure that my findings are comparable to those in the literature, the choices are largely based on Trostel's (1993) widely cited study. However, since his is an IH model, a number of additional choices need to be made for the OLG versions of the model.

Demographics. As is common in the literature, I assume that agents enter the model at age T_1 = 20. They retire at age T_R = 64 and die at age I = 74. Children are born at parental age t^* = 28 and receive their human capital endowment in the middle of childhood at age 10 (T_H = 38).

Preferences. The utility function is of the standard iso-elastic form $u(c,l) = (c\, l^{\rho})^{1-\sigma}/(1-\sigma)$. The discount factor β is chosen to replicate a capital-output ratio of 2.5. The choice of σ = 2 is conventional. It is common to set ρ so as to replicate the share of time devoted to leisure. However, for the purpose of evaluating the outcomes of tax experiments, it is important to generate a sensible labor supply elasticity. Most econometric studies find negative labor supply elasticities for men and slightly positive ones for women (Killingsworth 1983). Lucas (1990, p. 306) considers an uncompensated labor supply elasticity of 0.1 an upper bound. I therefore set ρ = 0.5 which generates an uncompensated labor supply elasticity of 0.16.

Technology. I normalize A = 1 by choosing units of output. As in Trostel (1993), the parameter governing the capital share is set to θ = 0.25. The depreciation rate is chosen to match an investment share of I / Y = 0.17.

Human capital. For the human capital production function I assume the conventional functional form $G = B\,(v_t\, h_t)^{\alpha} x_t^{\varphi}$. B is normalized to one. The fraction of human capital that is transmitted from parent to child (ε) is chosen to generate wage growth of 60 percent between the ages of 25 and 48. As in Trostel (1993) the returns to private inputs, $\alpha + \varphi$, are set to 0.6. However, I deviate from Trostel's factor shares ($\alpha = 0.45$; $\varphi = 0.15$) based on Mincer's (1993) estimate that the cost shares of time and goods in job training are similar ($\alpha = \varphi = 0.3$).

The depreciation rate of human capital is $\delta_h = 0.04$ as in Trostel (1993). A range of values is explored for the parameters governing intercohort persistence (ω, ψ, β_C).

Policy parameters follow Trostel (1993) in setting $\tau_L = \tau_K = 0.4$, $\tau_c = 0$, and $S/Y = 0.15$. One quarter of goods inputs in job-training are purchased with foregone earnings ($d_X = 0.25$); the remainder is subsidized at a rate of $s_X = 0.6$. Transfers balance the budget.

[INSERT TABLE 2 HERE]

4. Numerical Experiments

This section examines how the balanced growth and transitional effects of tax changes depend on intercohort persistence. The policy experiments consist of permanent, unanticipated cuts in tax rates by ten percent (from 0.4 to 0.36) financed through reductions in lump-sum transfers. Tables 3 to 5 report the findings for changing wage, capital, and income taxation, respectively. The figures shown represent steady state tax elasticities of human capital, physical capital and output (*H, K, Y*).[5] All tables also report half-lives of *Y* for the transition between steady states, defined as the time period after which output remains within half the date-one distance from its new steady state value.

Finally, the tables show changes in steady state welfare levels, defined as the proportional change in consumption required to make the household indifferent between the initial steady state and the path following the tax change. More precisely, if the age profiles of consumption and leisure are $(\bar{c}_i, \bar{l}_i)$ in the initial steady state and (c_i, l_i) in the steady state under the new tax rate, then the welfare gain (Ω) is defined by

$$\sum_{i=T_1}^{I} \beta^i u(\Omega \bar{c}_i, \bar{l}_i) = \sum_{i=T_1}^{I} \beta^i u(c_i, l_i).$$

Each experiment is performed for a sequence of models, so as to decompose the differences between a pure OLG model and an IH model into the contributions of individual model features. Table 2 summarizes the assumptions for each model. Model M1 is a pure OLG model with finite horizons and no intergenerational links ($\omega = 0$; $\beta_C = 0$). Model M2 adds incomplete persistence of human capital ($\omega = 0.5$) which is not internalized by parents ($\psi = 0$). Model M3 allows for complete persistence ($\omega = 1$). Model M4 adds persistence of physical capital by allowing for altruistic bequests ($\beta_C = 1$). The intergenerational transfer of human

[5] The tax elasticities are defined as $E(X) = (dX/X)/(d\tau/\tau)$, where τ is the tax rate changed in the experiment.

capital is internalized in M5 ($\psi = 1$). Model M6, finally, is an IH model. Note that each model differs from the previous one only by a single feature. The exception is the IH model M6, which changes a number of demographic features at a time ($t^* = T_H = I + 1$, $T_R = I$, $\varepsilon = 1$). This setup allows to precisely identify the significance of each feature that distinguishes the IH from the pure OLG model. The fact that the properties of the IH model are very similar to those of model M4 verifies that it is indeed intergenerational links that drive the larger tax elasticities in the infinite horizon case and not any of the other features that differ implicitly between IH and OLG models.

[INSERT Table 2 HERE] [MODELS]

[INSERT TABLE 3 HERE] [WAGE TAX]

[INSERT TABLE 4 HERE] [CAPITAL TAX]

[INSERT TABLE 5 HERE] [INCOME TAX]

4.1 Balanced Growth Results

This section discusses the comparative balanced growth effects of tax reforms; transitional dynamics results are presented below.[6] The main findings may be summarized as follows:

1. Intercohort persistence strongly affects the response of human capital to tax changes. The steady state tax elasticities of H more than double for all tax experiments as persistence rises from none in M1 to complete in M3. Stronger persistence is generally associated with larger tax elasticities of H.

2. The two models familiar from the literature generate very different tax elasticities. In the IH model (M6), the changes in human capital, physical capital, and output are more than twice larger than in the pure OLG model (M1). This holds for all tax experiments.

3. Even with *complete* persistence of human capital, tax elasticities differ between OLG (M3) and IH models (M6). This discrepancy is almost entirely due to the bequest motive implicit in the IH model. Once a bequest motive is added to the OLG model with complete persistence (M4), its properties are similar to those of the IH model.

[6] Intuition for the signs of the tax effects and for their dependence on parameters is well-known and therefore not repeated here (see Trostel 1993; Engen, Gravelle and Smetters 1997).

4. Persistence has even larger effects on steady state welfare changes. In the wage and income tax experiments, increasing persistence from none to complete raises the welfare gains more than seven-fold.
5. In the capital tax experiment, the outcomes depend mostly on the presence of a bequest motive. In the models with bequests, the after-tax interest rate is fixed. Higher capital taxes then reduce the steady state *K/H* ratio. This depresses wages and human capital investment. By contrast, in the absence of a bequest motive, the after-tax interest rate drops so that the stock of human capital actually rises in response to higher taxes.
6. If the intergenerational transmission of human capital is internalized by the parent, the tax effects tend to be larger than in the case of an intergenerational human capital spillover (M5 vs. M4).

Taken together, these findings show that the intercohort persistence of both physical and human capital has an important impact on the predicted tax effects.

Varying the degree of persistence. To further clarify the relationship between persistence and tax effects, figure 1 shows steady state wage tax elasticities of human capital for the entire range of persistence levels ($\omega = 0$ to $\omega = 1$). Higher persistence is associated with larger (absolute) tax elasticities, even if the model allows for altruistic bequests. Since the relationship is slightly concave, the errors introduced by understating persistence are smaller than those introduced by overstating persistence. The impact of persistence is similar for capital taxes (see figure 2), even though the signs of the elasticities change when the bequest motive is introduced.

An important question is under which circumstances the models studied in the literature approximate an environment with *realistic* intercohort persistence. The answer depends on how persistent human capital is in the data. As pointed out earlier, it is easy to conceive of model versions in which the persistence parameter ω is related to measures of intergenerational earnings persistence. Empirical estimates of earnings persistence range from 0.2 to about 0.6 (Mulligan 1997). *If* one accepts that ω lies at the lower end of these estimates, then pure OLG models (perhaps augmented by a bequest motive) approximate environments with realistic persistence fairly well. For example, $\omega = 0.3$ yields a wage tax elasticity only 20% larger than the pure OLG model, whereas complete persistence implies an elasticity more than twice as large. However, this simple calculation should be viewed as merely suggestive. Not only is the range of estimates of intergenerational earnings persistence quite wide. It is at this point not even clear that relating ω to intergenerational earnings mobility is the right approach. More work is needed in order to determine how ω and ψ could be identified empirically.

[INSERT FIGURE 1 HERE] [VARYING PERSISTENCE: Wage tax]

[INSERT FIGURE 2 HERE] [VARYING PERSISTENCE: CAPITAL TAX]

Sensitivity analysis. To further investigate the robustness of the finding that persistence is an important determinant of tax elasticities, table 6 considers variations of the baseline parameter values. Steady state wage and capital tax elasticities of human capital are shown for three versions of the model: no persistence (M1), moderate persistence ($\omega = 0.5$; M2), and complete persistence (M3). The last two columns show by how much the higher persistence of models M2 and M3 increases tax elasticities. Each row changes just one parameter compared with the baseline case.

The first part of the table explores parameter changes that can be shown analytically to affect the importance persistence (see Hendricks 2000a). These include higher returns to scale in the production of human capital ($\alpha = \varphi = 0.35$), a lower share of goods inputs ($\alpha = 0.45$; $\varphi = 0.15$) as in Trostel (1993), and variations in the rate of human capital depreciation ($\delta_h = 0.01$ or $\delta_h = 0.1$). Finally, Hendricks (2000a) shows that tax elasticities rise with age. Therefore, the age at which children inherit their human capital should matter. The table shows the case where children are born late in their parent's life ($T_H = 54$). The second part of the table examines parameters that are known to be important for the magnitude of tax elasticities in IH models. These include preference parameters governing the intertemporal elasticity of substitution (σ) and the labor supply elasticity (ρ).

The general conclusion from the sensitivity analysis is that persistence remains important in all cases. The model with incomplete persistence yields wage tax elasticities between 22% and 59% greater than the pure life-cycle model. The tax elasticities in the complete persistence case are still larger by 29% to 154%. The only parameter change that substantially reduces the role of persistence compared with the baseline case is the higher depreciation rate of human capital. However, the value $\delta_h = 0.1$ is almost surely larger than reasonable in an OLG model.[7] The findings for capital taxes are similar (see table 7).

[INSERT TABLE 6 HERE] [SENSITIVITY ANALYSIS WAGE TAX]

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[7] Depreciation rates of human capital should be larger in IH models where the rate encompasses depreciation over the life-cycle as well as depreciation at death. Nonetheless, Stokey and Rebelo (1995) argue that even in IH models a depreciation rate of 0.1 is most likely too high.

[INSERT TABLE 7 HERE] [SENSITIVITY ANALYSIS CAPITAL TAX]

4.2 Transitional Dynamics

This section discusses how intercohort persistence affects the transitional dynamics following unannounced tax changes. For a cut in wage taxes from τ_L = 0.4 to 0.36, the time paths for output and aggregate human capital shown in figure 3. Three versions of the OLG model (M1: $\omega = 0$; M2: $\omega = 0.5$, and M3: $\omega = 1$) are compared with the IH model (M6).[8]

The main finding is that higher persistence not only raises the steady state tax elasticities but also reduces the speed of convergence to the steady state. The half-life of output rises from 13 years to 40 years as persistence increases from none to complete. The paths generated by the IH model closely resemble those of the OLG model with complete persistence. Note that all time paths are very similar for first 25 years because it takes 20 years until the first cohorts born after the tax change enter the model.

Convergence is relatively rapid in the pure life-cycle model because the endowments of physical and human capital received by new generations are independent of the economy's history. All cohorts born after the date of the tax change start out with endowments that are consistent with the new steady state. Their behavior differs from the steady state only because prices temporarily deviate from steady state levels. By contrast, if generations are linked in human capital, the endowments of new cohorts initially deviate from their long-run levels.

[INSERT FIGURE 3 HERE] [PATHS OF Y AND H. WAGE TAX]

Welfare changes by cohort are shown for the OLG models (M1 through M3) in figure 4. Reducing the wage tax hurts the initial old because they do not receive earnings any more, but their lump-sum transfers are cut in order to balance the government budget. Younger cohorts, by contrast, benefit from the lower tax distortion. Welfare gains rise over time and are larger for higher persistence. The reason is that human capital endowments rise more strongly when persistence is higher.

[INSERT FIGURE 4 HERE] [WELFARE CHANGES. WAGE TAX]

Transition paths following a capital income tax cut are shown in figure 5. As in the wage tax case, transitions are more drawn out when persistence is higher. A striking observation is the

[8] For OLG models with bequest motive transition paths are not computed because they do not converge to steady states after a permanent tax change. The reason is that consumption does not regress to the mean within a dynasty.

large discrepancy between the OLG models and the IH model. The response of human capital to the tax change is larger, if altruistic bequests prevent adjustments in the after-tax interest rate that could stabilize human capital investment.

The welfare changes due to a capital tax cut are shown in figure 6. The initial old cohorts benefit because their capital return increases. The initial young, who were born before the tax change, are hurt by lower wages; they overinvested in the past. The young born later on benefit from the reduced distortion. In the presence of intercohort persistence, the welfare changes exhibit a distinct saw-tooth pattern reflecting the fact that children's welfare depends on the exact cohort of their parents via their human capital endowments. This unusual dynamics is an artifact of two model features: all agents within a cohort are identical and human capital is transmitted from parent to child not over a period of years but at a point in time.

[INSERT FIGURE 5 HERE] [PATHS OF Y AND H. CAPITAL TAX]

[INSERT FIGURE 6 HERE] [WELFARE CHANGES. CAPITAL TAX]

5. Conclusion

The question how taxes affect human capital accumulation has been studied extensively in the context of two classes of models: in overlapping generations (OLG) and in infinite horizon (IH) models. These two model classes embody very different assumptions about the intergenerational transmission of human and physical capital. In pure OLG models, the endowments of new agents are exogenous and there is no intercohort persistence. By contrast, persistence is implicitly complete in IH models. The question addressed here is how these differences in persistence affect the responsiveness of human capital to wage and capital income taxation.

The main finding is that stronger persistence implies larger steady state tax elasticities and slower rates of convergence to the steady state. An important implication is that the two model classes studied previously generate very different predictions about long-run tax effects. IH models generate larger tax elasticities than OLG models with incomplete persistence, even if cohorts are altruistically linked. For the tax experiments studied here, these differences are large. Models with complete persistence yield tax elasticities at least twice larger and half-lives at least three times larger than do models without persistence. These findings contrast sharply with the common view that IH models and OLG models "have very different theoretical structures, yet in practice, for the kind of tax problem under study here, seem to yield quite similar results” (Lucas 1990, p. 295). While it is true that OLG models with *complete* persistence of human and physical capital yield results very similar to IH models, the two model classes do not generally have similar properties.

An important task for future research is therefore to measure intercohort persistence in the data. One promising approach, pursued further in Hendricks (2000b), is to relate the model’s persistence parameters (ω, ψ) to measures of intergenerational mobility commonly estimated in the literature (e.g. Mulligan 1997). However, alternative theories of the intergenerational transmission of human capital are conceivable, in which persistence is not related to intergenerational mobility. This paper has maintained the assumption, implicit in IH models, that children essentially inherit human capital from their parents. This allows the IH model to be nested as a special case, but modeling alternatives need to be explored.

The findings presented in this may have important implications beyond the study of tax policies. For example, a recent branch of the growth literature studies whether human capital augmented growth models can account for two important development facts: for the large cross-country income differences observed in post-war data and for the observed speed of convergence to steady state.[9] This question has been studied in the context of infinite horizon models in which tax distortions cause some countries to underinvest in human and physical capital. The findings here suggest that the extent to which the standard growth model can account for both observations should depend on intercohort persistence.

[9] See Mankiw, Romer, and Weil (1992); McGrattan and Schmitz (1998); Prescott (1998).

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7. Appendix

7.1 Household Optimization

The Lagrangean for the household problem can be written as

$$V(a_{T_1}, h_{T_1}, W) = \max \sum_{i=T_1}^{I} \beta^i u(c_i, l_i) + \beta_C \beta^{t^*-1} V(a_{T_1}^C, h_{T_1}^C, W_C)$$
$$+\lambda \left[\frac{a_{T_1}}{R_{T_1-1}} + \frac{W}{R_{t_b}} - \frac{W_C}{R_I} + \sum_{i=1}^{I} R_i^{-1} \left\{ w_i\, h_i\, (1-l_i) - p_{V\,i}\, h_i\, v_i - p_{C\,i}\, c_i - p_{X\,i}\, x_i + \Gamma_i \right\} \right]$$
$$+ \sum_{i=T_1}^{I} \frac{\mu_i}{R_i} \{(1-\delta_h)\, h_i + G(v_i, h_i, x_i) - h_{i+1}\}$$

where $R_i = \prod_{s=1}^{i} (1+r_s)$ is a cumulative discount factor. It is assumed that the solution is always interior and satisfies $l_i > 0$, $v_i > 0$, $l_i + v_i < 1$, $x_i > 0$, except during retirement The first-order conditions are:

(8) $$\beta^i\, u_c(i) = p_{C\,i}\, \lambda / R_i$$

(9) $$\beta^i\, u_l(i) = w_i\, h_i\, \lambda / R_i$$

For the job-training period ($i = T_1, \ldots, T_R - 1$):

(10) $$p_{V\,i}\, h_i\, \lambda = \mu_i\, G_v(i)$$

(11) $$p_{X\,i}\, \lambda = \mu_i\, G_x(i).$$

During retirement $v_i = x_i = \mu_i = 0;\ i \geq T_R$. The first-order condition for h_i is

(12) $$\frac{\mu_{i-1}}{R_{i-1}} = \frac{\mu_i}{R_i}[1 - \delta_h + G_h(i)] + \frac{\lambda}{R_i}[w_i\,(1-l_i) - p_{V\,i}\, v_i].$$

If the intergenerational spillover is internalized, the marginal value of augmenting child human capital must be added to (12) at age t^*: $\beta_C\, \beta^{t^*-1} V_h(a_{T_1}^C, h_{T_1}^C, W_C)\, \psi\, \omega\, \varepsilon\, h_{t^*}^{\omega-1}$. The derivative of the value function is

$$V_h(a_{T_1}, h_{T_1}, W) = \frac{\lambda}{R_{T_1}} \left(w_{T_1}\,(1-l_{T_1}) - p_{VT_1}\, v_{T_1} \right) + \frac{\mu_{T_1}}{R_{T_1}} \left(1 - \delta_h + G_h(T_1)\right) = \frac{\mu_{T_1-1}}{R_{T_1-1}}.$$

Finally, bequests are determined by

$$\beta_C \, \beta^{t^*-1} \, V_W(a^C_{T_1}, h^C_{T_1}, W_C) = \frac{N_C \, \lambda}{R_I}, \tag{13}$$

where $V_W = \dfrac{\lambda}{R_{t_b - 1}}$. This can be simplified to

$$u_c(c_I, l_I) = \beta_C \, u_c(c^C_{t_b}, l^C_{t_b}). \tag{14}$$

7.2 Equilibrium Conditions

Define the following aggregates:

Aggregate job training investment: $I_{X,t} = \sum_{i=T_1}^{T_R} x_{t,i}$

Aggregate time input in training: $L_{h,t} = \sum_{i=T_1}^{T_R} v_{t,i} \, h_{t,i}$

Aggregate investment: $I_t = K_{t+1} - (1 - \delta_K) K_t$

The market clearing conditions are then:

Goods market: $A K_t^{\theta} L_t^{1-\theta} = C_t + I_t + I_{X,t}$

Rental market for capital: $K_t = \sum_{i=T_1}^{I} a_{t,i}$

Labor market: $L_t = \sum_{i=T_1}^{T_R} (1 - v_{t,i}) h_{t,i}$

An equilibrium is a sequence of prices $(w_t, r_t, w_t^*, r_t^*, p_{X,t}, p_{V,t}, p_{C,t})$, aggregate quantities $(K_t, L_t, C_t, I_t, I_{X,t}, L_{h,t})$, an endogenous policy variable, and household age profiles $(a_{t,i}, h_{t,i}, x_{t,i}, v_{t,i}, c_{t,i}, v_{t,i}, l_{t,i})_{i=1}^{I}$ and bequests W_i that satisfy

1. $a_{t,1} = 0$ and h_{t,T_1} obeys the conditions for intergenerational links
2. Firms maximize profits.
3. Prices are determined as described in the government section.
4. The government budget constraint is met at each date.

5. Households maximize utility, given paths of prices. I.e., age profiles obey (8) through (13), the laws of motion, and the present value budget constraint.

6. Markets clear.

7.3 Balanced Growth Conditions

On a balanced growth path, all per capita variables are either constant or grow at rate γ. The equilibrium conditions in that case simplify as follows. Dropping generation subscripts, denote by h_i the age profile of the household born at date 1. The bequest condition (14) becomes $u_c(c_I, l_I) = \beta_C\, u_c(c_{tb}, l_{tb})$. This implies a condition familiar from IH models: $[\beta(1+r)]^{t^*-1}\,\beta_C = 1$. The aggregate bequest flow per period is $W = W_C$. Since parents are $t^* - 1$ years older than their children, the intergenerational spillover condition becomes

$$h_{T_1} = \varepsilon\left(h_{t^*+T_1-1}\right)^{\omega}.$$

Aggregate investment in job-training is $I_X = \sum_{i=T_1}^{I} x_i$. Analogous conditions hold for aggregate consumption, and the supplies of capital. Investment in physical capital obeys $I = \delta_K K$. The market clearing conditions remain unchanged.

Tables

Table 1. Baseline parameters

Demographics		Human capital technology	
$I = 74$	Lifetime of 74 years	ε	Replicates earnings growth over the life-cycle
$T_1 = 20$	First year of work	$\delta_h = 0.04$	Trostel (1993)
$T_R = 64$	Last year of work	$B = 1$	Normalization
$t^* = 28$	Date of child birth	$\alpha = 0.3$ $\varphi = 0.3$	Mincer (1993)
$T_H = 38$	Age at which human capital is transmitted to children		
Preferences		**Firms**	
$\rho = 0.5$	Matches labor supply elasticity (Lucas 1990)	$A = 1$	Normalization
β	Matches capital-output ratio of 2.5	$\theta = 0.25$	Trostel (1993)
$\sigma = 2$		δ_K	Replicates $I/Y = 0.17$

Notes: Policy variables are described in the main text.

Table 2. Model specifications

Model	Parameters
M1. Pure life-cycle model	Exogenous endowments: $h_{T1} = \varepsilon$ No altruism: $\beta_C = 0$
M2. Incomplete persistence	$\omega = 0.5$; $\psi = 0$
M3. Complete persistence	$\omega = 1$
M4. Altruistic bequest	$\beta_C = 1$
M5. Internalized persistence	$\psi = 1$
M6. Infinite horizon model	No retirement: $T_R = I$ Child born when parent dies: $t_b = T_H = I+1$ Endowment inherited from parents: $\varepsilon = 1$

Table 3. Effects of wage tax reduction

Model	Tax Elasticities *H*	*K*	*Y*	*Welfare*	Half-life
M1. Pure life-cycle model	-0.295	-0.274	-0.349	0.006	13
M2. Incomplete persistence	-0.408	-0.460	-0.489	0.021	19
M3. Complete persistence	-0.691	-0.932	-0.843	0.056	40
M4. Altruistic bequest	-0.681	-0.927	-0.847	0.056	––
M5. Internalized persistence	-0.724	-1.004	-0.896	0.041	––
M6. Infinite horizon model	-0.729	-1.008	-0.892	0.053	51

Notes: The table shows the effects of a reduction in wage taxes from 0.4 to 0.36. Shown are the steady state tax elasticities of aggregate human capital (*H*), physical capital (*K*), and output (*Y*), the steady state welfare change as described in the main text, and the half-life of the transition to the steady state.

Table 4. Effects of capital income tax reduction

Model	Tax Elasticities *H*	*K*	*Y*	*Welfare*	Half-life
M1. Pure life-cycle model	0.016	-0.250	-0.077	0.001	19
M2. Incomplete persistence	0.024	-0.237	-0.068	0.000	7
M3. Complete persistence	0.042	-0.207	-0.046	-0.003	92
M4. Altruistic bequest	-0.098	-0.482	-0.209	0.012	––
M5. Internalized persistence	-0.153	-0.600	-0.305	0.005	––
M6. Infinite horizon model	-0.110	-0.530	-0.233	0.013	58

Notes: See table 3.

Table 5. Effects of income tax reduction

Model	Tax Elasticities *H*	*K*	*Y*	*Welfare*	Half-life
M1. Pure life-cycle model	-0.277	-0.537	-0.431	0.007	13
M2. Incomplete persistence	-0.382	-0.712	-0.561	0.020	19
M3. Complete persistence	-0.644	-1.155	-0.890	0.053	38
M4. Altruistic bequest	-0.784	-1.451	-1.072	0.068	––
M5. Internalized persistence	-0.833	-1.566	-1.134	0.054	––
M6. Infinite horizon model	-0.845	-1.589	-1.143	0.066	52

Notes: See table 3.

Table 6. Sensitivity analysis: Wage tax elasticities

	No persistence	Incomplete persistence	Complete persistence	Incomplete vs. no persistence	Complete vs. incomplete persistence
Baseline model	-0.295	-0.408	-0.691	38%	69%
$\alpha = \varphi = 0.35$	-0.371	-0.552	-1.119	49%	103%
$\alpha = 0.45$; $\varphi = 0.15$	-0.225	-0.303	-0.483	35%	59%
$\delta_h = 0.01$	-0.184	-0.285	-0.725	55%	154%
$\delta_h = 0.1$	-0.414	-0.506	-0.654	22%	29%
$T_H = 54$	-0.295	-0.470	-0.781	59%	66%
$\sigma = 1.2$	-0.338	-0.452	-0.702	34%	55%
$\sigma = 4$	-0.251	-0.358	-0.674	43%	88%
$\rho = 1$	-0.382	-0.523	-0.869	37%	66%

Notes: The table shows steady state tax elasticities of human capital for models M1 through M3. The last two columns show the percentage change in the percentage differences in tax elasticities generated by M2 vs. M1 and by M3 vs. M2.

Table 7. Sensitivity analysis: Capital tax elasticities

	No persistence	Incomplete persistence	Complete persistence	Incomplete vs. no persistence	Complete vs. incomplete persistence
Baseline model	0.016	0.024	0.042	50%	75%
$\alpha = \varphi = 0.35$	0.018	0.029	0.059	61%	103%
$\alpha = 0.45$; $\varphi = 0.15$	0.025	0.035	0.059	40%	69%
$\delta_h = 0.01$	0.015	0.025	0.065	67%	160%
$\delta_h = 0.1$	0.010	0.012	0.015	20%	25%
$t_b = 54$	0.016	0.023	0.034	44%	48%
$\sigma = 1.2$	-0.005	-0.007	-0.010	40%	43%
$\sigma = 4$	0.038	0.058	0.114	53%	97%
$\rho = 1$	0.010	0.014	0.023	40%	64%

Notes: The table shows steady state tax elasticities of human capital for models M1 through M3. The last two columns show the percentage change in the percentage differences in tax elasticities generated by M2 vs. M1 and by M3 vs. M2.

Figures

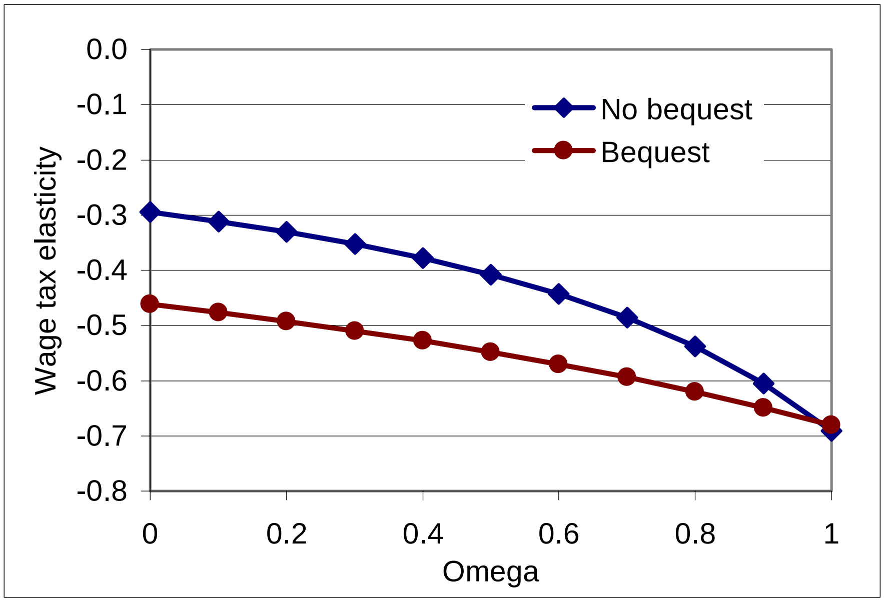

Figure 1. Wage tax elasticities of human capital

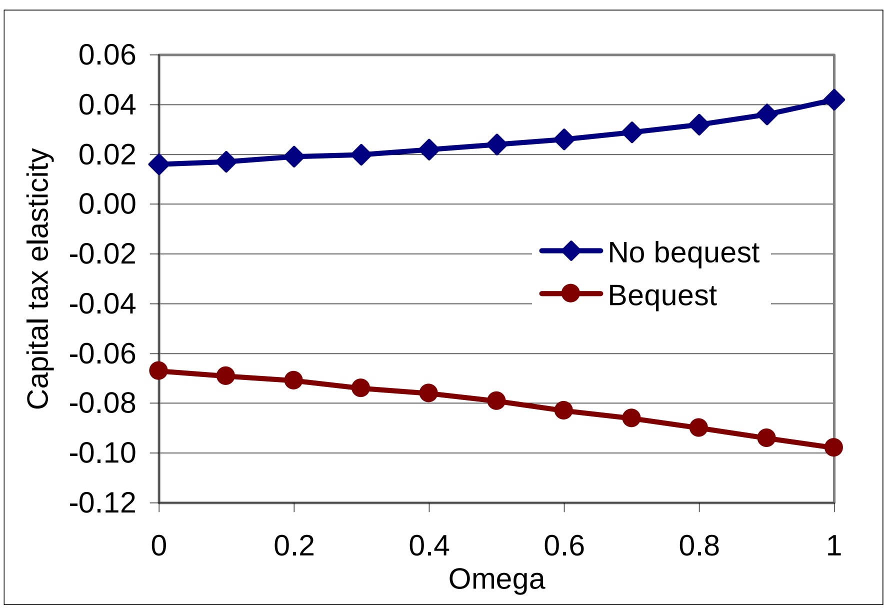

Figure 2. Capital tax elasticities of human capital

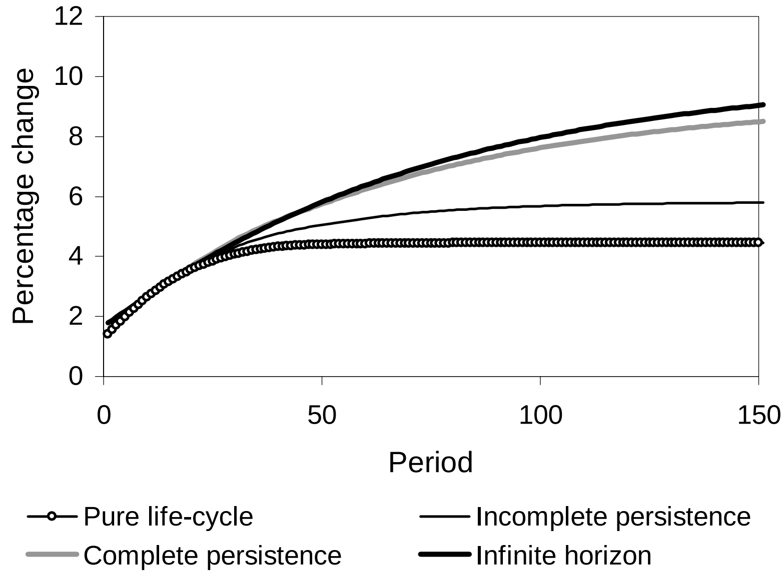

Figure 3a. Time-paths of output following a reduction in wage taxes

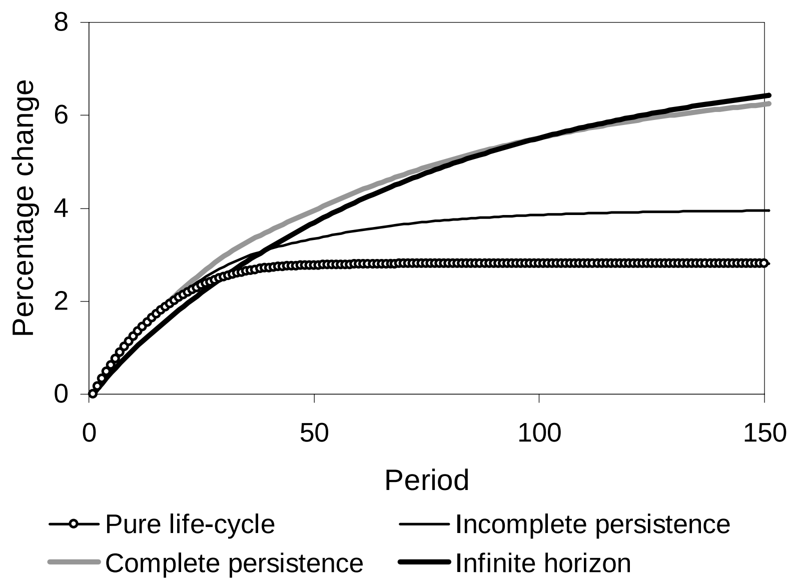

Figure 3b. Time-paths of aggregate human capital following a reduction in wage taxes

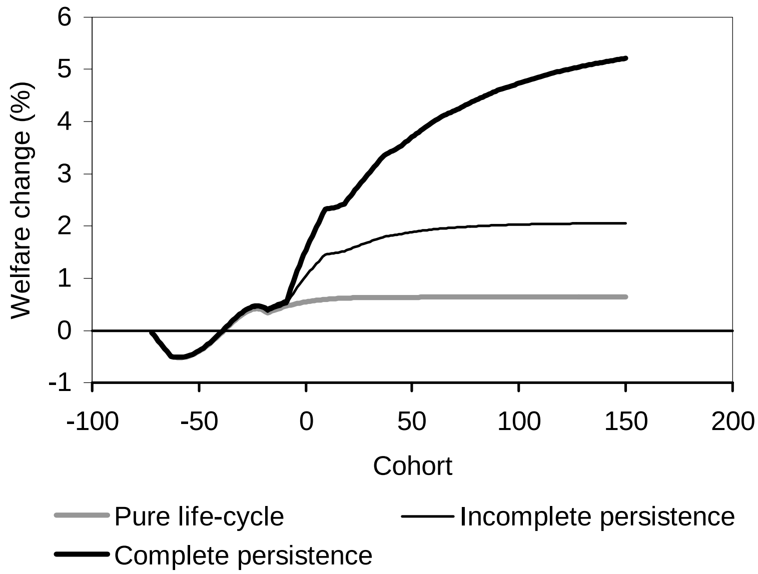

Figure 4. Welfare effects of a wage tax reduction

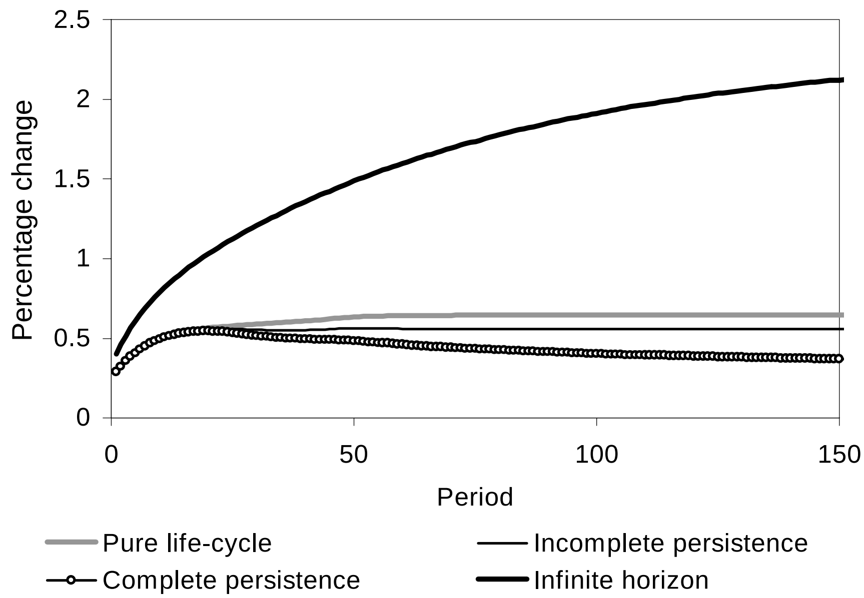

Figure 5a. Time paths of aggregate output following a reduction in capital taxes

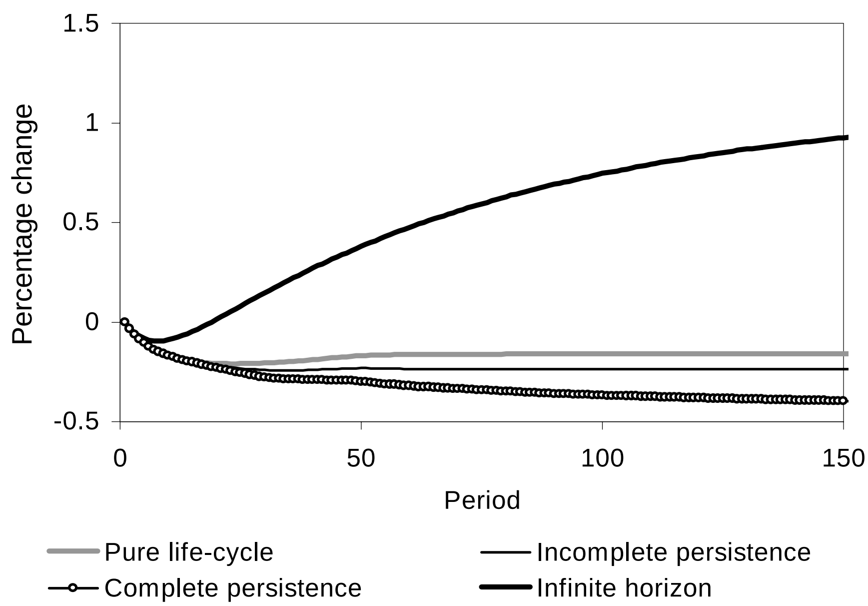

Figure 5b. Time paths of aggregate human capital following a reduction in capital taxes

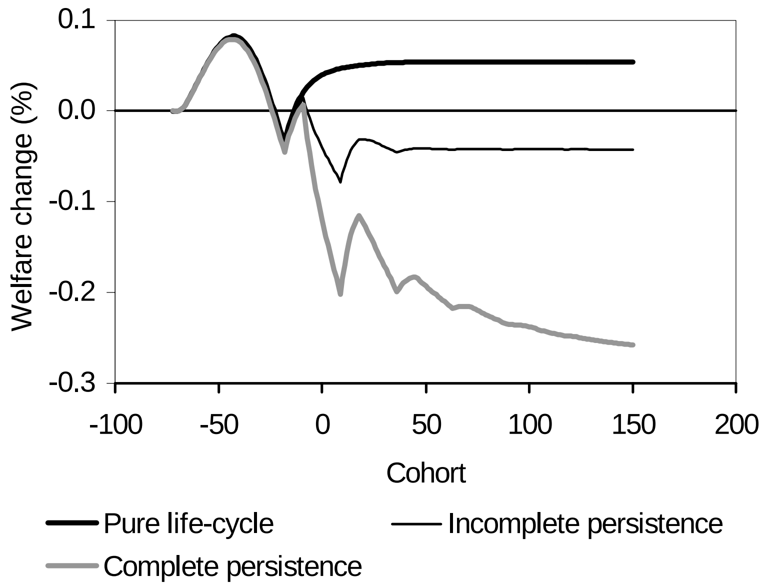

Figure 6. Welfare effects of a capital tax reduction

Technical Appendix

The Infinite Horizon Case

The firm's problem and the government budget constraint are the same as in the baseline model. They yield three equilibrium conditions: the government budget constraint and the cost-minimizing factor prices.

Households

A standard infinite horizon model emerges as a special case of the baseline model, if the following assumptions are maintained:

1. Children are born after the parent has died ($t^* = I+1$).
2. Households do not retire ($T_R = I$).
3. Generations are linked by an altruistic bequest motive with $\beta_C = 1$.
4. Parents can bequeath not only physical but also human capital ($\omega = 1$, $\psi = 1$, $\varepsilon = 1$), so that $h_1^C = h_{I+1}$.

Under these conditions, the parent's problem simplifies to

$$V(a_1, h_{T_1}, W) = \max \sum_{i=1}^{I} \beta^i u(c_i, l_i) + \beta^I V(a_1^C, h_{I+1}, W_C)$$
$$+\lambda\left[\frac{a_1}{R_0} + \frac{W}{R_0} - \frac{W_C}{R_I} + \sum_{i=1}^{I} R_i^{-1}\left\{w_i\, h_i\,(1-l_i) - p_{V\,i}\, h_i\, v_i - p_{C\,i}\, c_i - p_{X\,i}\, x_i + \Gamma_i\right\}\right]$$
$$+\sum_{i=1}^{I} \frac{\mu_i}{R_i}\{(1-\delta_h)\, h_i + G(v_i, h_i, x_i) - h_{i+1}\}$$

where $R_i = \prod_{s=1}^{i}(1+r_s)$ is a cumulative discount factor. Since the child inherits all human capital, the laws of motion for h and a collapse to their infinite horizon versions. Similarly, since $V(a_1^C, h_{I+1}, W_C) = \max \sum_{i=1}^{I}\beta^i\, u(c_{I+i}, l_{I+i})$, the parent's utility function collapses to the infinite horizon version $\sum_{i=1}^{\infty}\beta^i\, u(c_i, l_i)$.

Since the child is born after the parent dies (at parental age I+1), it receives the bequest at the beginning of its first period. Equivalently, it receives the bequest plus interest in the first period. Therefore, the first-order condition for the bequest left is

$$u_c(c_I, l_I) = (1 + r_{I+1})\beta u_c(c_1', l_1') .$$

Instead of $\mu_I = 0$, we have $\mu_I / R_I = \beta^I V_{h_{T1}}(.')$. Households then solve the problem

(1) $$\max \sum_{i=1}^{\infty} \beta^i N_i u(c_i, l_i)$$

subject to a_1, h_1 given

(2) $$a_{i+1} = (1 + r_i) a_i + w_i h_i (1 - l_i) - p_{V\,i} v_i h_i - p_{C\,i} c_i - p_{X\,i} x_i + \Gamma_i$$

(3) $$h_{i+1} = (1 - \delta_h) h_i + G(v_i, h_i, x_i) .$$

This specification slightly generalizes the one derived as a reduced form of the life-cycle model with bequests in that it allows for population growth. It is assumed that the solution is always interior and satisfies $0 < l_i < 1$, $0 < v_i < 1$, $x_i > 0$. It is further assumed that a present value budget constraint holds:

(4) $$\frac{a_1}{R_0} + \sum_{i=1}^{\infty} \frac{1}{R_i} [w_i h_i (1 - l_i) - p_{V\,i} v_i h_i - p_{C\,i} c_i - p_{X\,i} x_i + \Gamma_i] = 0 ,$$

where $R_i = (1 + r_1) \cdot \ldots \cdot (1 + r_i)$ and $R_0 = 1$. Write the Lagrangean as

$$\sum_{i=1}^{\infty} \beta^i u(c_i, l_i) + \lambda \left\{ \sum_{i=1}^{\infty} \frac{1}{R_i} \{w_i h_i (1 - l_i) - p_{V\,i} v_i h_i - p_{C\,i} c_i - p_{X\,i} x_i + \Gamma_i\} + a_0 \right\}$$
$$+ \sum_{i=1}^{\infty} \frac{\mu_i}{R_i} \{(1 - \delta_h) h_i + G(v_i, h_i, x_i) - h_{i+1}\}$$

The first order conditions are

(5) $$\beta^i u_c(i) = p_{C\,i} \lambda / R_i$$

(6) $$\beta^i u_l(i) = w_i h_i \lambda / R_i$$

(7) $$p_{V\,i} h_i \lambda = \mu_i G_v(i)$$

(8) $$p_{X\,i} \lambda = \mu_i G_x(i)$$

(9) $$\frac{\mu_{i-1}}{R_{i-1}} = \frac{\mu_i}{R_i} [1 - \delta_h + G_h(i)] + \frac{\lambda}{R_i} [w_i (1 - l_i) - p_{V\,i} v_i] .$$

Define $\eta = \mu / \lambda$. Then (7) through (9) can be rewritten as

$$p_{V\,i}\, h_i = \eta_i\, G_v(i)$$

$$p_{X\,i} = \eta_i\, G_x(i)$$

(10) $$\frac{\eta_{i-1}}{R_{i-1}} = \frac{\eta_i}{R_i}[1-\delta_h + G_h(i)] + \frac{1}{R_i}[w_i\,(1-l_i) - p_{V\,i}\, v_i]$$

For computational purposes, it is useful to reduce the dimensionality of the problem. Taking ratios of (5) and (6) yields

(11) $$\frac{u_c(i)}{u_l(i)} = \frac{l}{\rho\, c} = \frac{p_{C\,i}}{w_i\, h_i}.$$

Similarly, the ratio of (7) and (8) is

(12) $$\frac{G_v(i)}{G_x(i)\, h_i} = \frac{\alpha}{\varphi}\frac{x_i}{v_i\, h_i} = \frac{p_{V\,i}}{p_{X\,i}}.$$

Substituting (8) into (10) allows to eliminate η:

(13) $$\frac{p_{X,i-1}}{G_x(i-1)}(1+r_i) = \frac{p_{X,i}}{G_x(i)}[1-\delta_h + G_h(i)] + w_i\,(1-l_i) - p_{V,i}\, v_i$$

Equilibrium

Define the following aggregates: $K = a$, $L = (1-l-v)\,h$, $C = c$, $X = x$, $I_t = K_{t+1} - (1-\delta_K)\,K_t$. Given initial conditions (a_1, h_1), an equilibrium is a sequence of prices $(r, w, r^*, w^*, p_X, p_V, p_C)$ and quantities (K, L, a, h, v, x, c) plus one endogenous policy variable that satisfy

- the firm's first-order conditions;
- the definitions of the prices given policy variables;
- the household's first-order conditions shown above together with the present value budget constraint and the laws of motion for h and a shown in the household problem;
- the aggregation conditions;
- the government budget constraint;
- the market clearing condition (which is redundant):

$$F(K, L) = C + I + X + S.$$

Balanced Growth

Along a balanced growth path, quantities, prices and shadow prices (λ, μ) are constant. $R_i = (1+r)^i$. The endogenous variables (*a, h, x, v, c, l*, μ, λ) are determined by the household first-order conditions (5) through (9), the laws of motion for *a* and *h*, where it is imposed that both grow at rate γ, and the present value budget constraint (4). In addition, one policy variables is determined endogenously to satisfy the government budget constraint. The law of motion for *h* becomes

(14) $$1 = (1-\delta_h) + G/h .$$

The first-order condition for *c* implies the usual Euler equation: $\beta = 1+r$.