General Notes

Econ602. Spring 2007. Lutz Hendricks

This course draws in large parts on the excellent Econ702 course I took at the University of Pennsylvania from William English and Fumio Hayashi.

1. Definitions and Terms

Balanced growth path

A path along all variables grow at time-invariant (although possibly different) rates.

Calibration

Picking parameters and functional forms for a theoretical model so that it can be solved numerically. The choices are generally based on independent micro-econometric evidence (e.g., preference parameters). Other parameters are chosen so that the model replicates certain features of the data (e.g., the long-run average growth rate of the U.S. economy). See Prescott (1986, 1996).

CARA utility function

CARA means "constant absolute risk aversion." General form: $u(c) = -e^{-\alpha c}$. Note that $u'(c) = \alpha e^{-\alpha c}$ and $u''(c) = -\alpha^2 e^{-\alpha c}$. The coefficient of absolute risk aversion is therefore α.

CES production function

Constant elasticity of substitution. Cobb-Douglas is a special case with elasticity of one. See BS ch. 1 appendix.

Cobb Douglas production function

For the case of capital and labor inputs: $Y = K^{\alpha} L^{1-\alpha}$. Exercise: Show that the factor shares are α and $1-\alpha$. By factor shares I mean the fractions of total output received by capital and labor.

CRRA utility function

CRRA means "constant relative risk aversion". Also called isoelastic utility function because the elasticity of substitution is constant. General form: $u(c) = c^{1-\sigma}/(1-\sigma)$. Elasticity of substitution is then 1/σ. The limit for $\sigma \to 1$ is log utility. Note that $u'(c) = c^{-\sigma}$ and $u''(c) = -\sigma c^{-(1+\sigma)}$. The coefficient of relative risk aversion is therefore $CRR = -c\,u''(c)/u'(c) = \sigma$.

Engel curve

Aka expansion path. Consider a consumer who faces constant relative prices of all consumption goods. Now vary the consumer's income. The curve that is traced out by the consumption choices as income varies is called an Engel curve.

Equilibrium

A term that is commonly abused to mean many different things. Here we shall mean a "competitive equilibrium," i.e., an allocation and a price system such that all agents solve their respective maximization problems and all markets clear.

Exchange economy

An economy in which there is no production. Instead, agents receive an endowment of goods ("manna") in every period. Also called "endowment economy."

Friedman Rule

According to the Friedman rule, monetary policy should target a zero nominal interest rate. The logic is that money is costless to produce. Therefore its opportunity cost (the nominal interest rate) should also be zero.

Homogeneity

A function is homogeneous of degree λ, if $f(k\,x) = k^{\lambda} f(x)$. Linear homogeneity ($\lambda = 1$) is called constant returns to scale. $\lambda > 1$ is called increasing returns.

Homotheticity

A function $f(x, y)$ is homothetic, if the ratio of partial derivatives f_x / f_y depends only on the *ratio* x / y, not on x and y separately. The most common application where homotheticity is invoked is preferences. If preferences are homothetic, Engel curves are linear.

Inada Conditions

Conditions often imposed to ensure that a problem has an interior solution. $f'(0) = \infty$, $f'(\infty) = 0$, $f' > 0, f'' < 0$.

Numeraire

In an economy with N goods, it is always possible to normalize the price of one good to one. This good is called the numeraire. What does it exactly mean to normalize a price? It really means choosing the unit of account. For example, suppose we want to make the price of a specific good in the U.S. economy the numeraire, let's say we choose one ounce of gold. This costs about $300. So we define a unit of account to be equivalent to $300 and make the price of the numeraire (gold) equal to one.

Optimal policy function

Consider any optimization problem. The optimal choice will be a function (or correspondence) of some state variables (prices, household income, …). This function, "optimal choice = f(state variables)" is called the optimal policy function.

Pareto optimality

An allocation is pareto optimal, if it is not possible to improve welfare of at least one agent without making another one worse off.

Representative Agent

If all agents are identical, it is possible to assume that there is only one representative agent in the economy who behaves competitively.

Ricardian Equivalence

Roughly speaking, the insight that the timing of lump-sum taxes does not affect the equilibrium allocation, if the present value of tax payments remains unchanged. See MW, p. 71, for details.

Risk aversion

There are many measures of risk aversion. For our purposes we need only two. Given a utility function $u(c)$, the coefficient of *relative* risk aversion is $CRR = -c\,u''/u'$, which is the elasticity of marginal utility with respect to consumption. The coefficient of *absolute* risk aversion is $CAR = -u''/u'$.

Seignorage

By printing more money the government can gain additional purchasing power. The amount of real revenue raised this way is called seignorage.

State variables

Consider an optimization problem (or even an equilibrium description for an entire economy). The state variables are all variables that determine the optimal choices (or equilibrium outcomes) of the current period. Example: Typical state variables for a household problem are prices and endowments. There is a complication: nothing prevents us from adding extraneous state variables such as the number of suspots. These have intrinsically nothing to do with the equilibrium outcomes, but if all agents believe that sunspots matter, then it is possible that they do matter in equilibrium. We will ignore this possibility throughout, but there is an extensive literature on the issue (known as sunspots).

Steady State

An equilibrium path along which all variables are constant. I.e., a balanced growth path with a growth rate of zero for all variables.

Sunspots

See notes on state variables.

Walras' Law

If there are n markets in an economy and we know that $n-1$ markets clear and that all agents satisfy their budget constraints, then we can be sure that the last market clears as well. That means, we can drop one market clearing condition or one budget constraint from the system of equations when we solve for a competitive equilibrium.

2. Problems To Watch Out For

The following are a couple of points that commonly cause problems.

Market clearing conditions only contain quantities

A market clearing condition is simply "quantity supplied equals quantity demanded." So there can't be anything in the equation other than quantities. No tax rates either. And certainly no prices, except as arguments of supply or demand functions.

Planning problems don't have prices

The planner chooses quantities subject only to resource constraints. There are no prices, markets, budget constraints, or assets. Never ever.

Households take aggregates as given

Make it a habit to distinguish in the notation between aggregates and individual variables. That reduces the risk of implicitly having a household or firm choosing aggregates.

The interest rate does not always equal the marginal product of capital

Firms are best thought of as paying a rental price (q) to households, which equals the marginal product of capital. Households then receive the rental price plus undepreciated capital for each unit of capital rented to firms (net of any income taxes). So the household budget constraint will usually have a capital income term of the form $k(q+1-\delta)$.

The issue becomes more complicated if the price of capital is not constant over time; then there are capital gains terms as well. Assume that the price of capital is p while the price of consumption is one. Then the budget constraint might read

$$p_t\, k_{t+1} = w_t - c_t + k_t\, q_t + p_t\, k_t\, (1-\delta)\,.$$

The first two terms are earnings and consumption spending. In addition, the household enters the period with capital stock k_t, which it rents to firms. At the end of the day the household then has capital income of $q_t\, k_t$ (rental price · capital rented out). The household also owns a stock of wealth equal to $p_t\, k_t\, (1-\delta)$, which is the undepreciated capital valued at the current price.

So what is the interest rate? The interest rate tells us how much the household can consume tomorrow if it reduces consumption today by one unit. Giving up a unit of c_t allows to buy $1/p_t$ units of capital. So k_{t+1} rises by $1/p_t$. Next period the household can eat the capital income q_{t+1}/p_t

and the undepreciated capital, which buys $p_{t+1} / p_t (1-\delta)$ units of c_{t+1}. The gross rate of return is therefore: $R_{t+1} = p_{t+1} / p_t (1-\delta) + q_{t+1} / p_t$.